NUMERICAL ANALYSIS OF OLD MASONRY BUILDINGS: A DEBATING ISSUE

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ABSTRACT

The structural members of old masonry buildings are essentially plane walls, arches and vaults; both types of structures are usually designed to support vertical loads only. Today's engineers are often confronted with the task of studying, to the best of their ability, the behaviour of structural members of old masonry buildings, and, in doing so, they must face at least three very different situations.

The first is the need to check the safety of the structural parts against their original design loads; the second is the need to check the safety of the structural parts against different and new loading conditions, because of new destinations of the building; the third occurs when an old, possibly historical building happens to be in a seismic zone.

The features of masonry, seen as a material, make it necessary to resort to numerical analysis to try and understand its actual behaviour. The paper summarises what can be done by applying computational techniques to study old masonry buildings; in doing so, we will make reference to the Finite Element Method as the standard numerical tool, even if, with the aim of modelling the complex texture of masonry, other techniques might be employed, such as, for instance, the Boundary Element Method.

The paper underlines that, in any case, when dealing with the task of interpreting the result of a FEM analysis of an old structure, a mosaic of different assumed structural models and analysis methods have to be compared with each other. Old masonry structures can reach different, sometimes unpredictable limit states, with different safety coefficients. Such complexity of behaviour requires a broad spectrum of numerical analysis tools, including extensions of simplified methods proposed in the past both for the elastic and for the limit analysis.

The difficulty of performing numerical analyses in the non-linear range, specially in the presence of brittle behaviour, with the aim of obtaining reasonably accurate information and possibly useful from the engineering viewpoint are illustrated.

Then by means of examples concerning both walls and arches, some results obtained by means of the numerical analysis methods of old masonry buildings are compared.

Key words: old masonry, numerical models, constitutive laws, FEM analysis.

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INTRODUCTION

If one wants to categorise the different structural problems, with reference to the relevant analysis techniques, one might obtain the following picture.

1. Masonry walls loaded in their plane.
   Very little can be done, analytically, to understand their elastic behaviour, whereas limit analysis tools can be employed to obtain usually lower bound estimates of the collapse load [Sacchi et al., (1984)].
   Numerical methods may provide information both about the elastic behaviour and about the inelastic and limit behaviour, at the price of meeting several difficulties, some of which will be illustrated in the next paragraph.

2. Arches and vaults.
   Here it is possible to perform several types of analytical investigations, both in the elastic and in the inelastic range. It is in fact possible to study the elastic and post-elastic behaviour of arches by means of the "thrust line" method, which gives information about the structural behaviour up to collapse. The techniques of limit analysis can be employed to obtain, with relatively little effort, upper bounds to the collapse load. However, it is often difficult to take into account, in the analytical models, details such as the presence of spandrel walls. The numerical analysis is a useful alternative, which should complement whatever results can be obtained from hand computations.

3. There are other types of structural problems which may be encountered when dealing with old buildings, owing to the design and construction modalities. One of them is the analysis of box-like structures, in which the geometry and loading conditions force the engineer to consider an assembly of walls as a fully three-dimensional structure. In most cases the walls are studied as isolated from each other, under the assumption that the main loading acts in their own plane. However, this might not be true; even if here we will not consider such cases, the reader can find some information about the modeling of masonry walls loaded as plates in [Lee et. al., (1996)].

4. Other topics of interest in the analysis of masonry structures are the stability and the structural strength under seismic actions. This last topic has a special relevance, since masonry buildings, be they old masterpieces such as the gothic cathedrals or be they of minor architectural relevance but of great historical importance anyway, were not built for withstanding horizontal actions. Finally, a very specific but important field of work is the assessment of the load bearing capacity of old masonry when the restoration design implies a new use for the buildings (such as in the case of museums, etc.), and therefore heavier loading than for the original structure.

One can never stress enough the difficulty of performing numerical analyses in the nonlinear range, specially in the presence of brittle behaviour, with the aim of obtaining reasonably accurate information and possibly useful from the engineering viewpoint.
Both the difficulty of defining accurate constitutive laws for masonry, and the difficulty of modeling geometrical details which have great importance for the overall structural behaviour, are reasons which have so far substantially limited the progress of research and understanding in this field.

The paper examines the possible alternatives among different choices in modelling the problem. which one has to choose when one has to analyse an old masonry building by means of a FEM code. Then, by means of examples concerning both walls and arches, some results obtained by means of the numerical analysis methods of old masonry buildings are compared.

ISSUES IN THE NUMERICAL MODELLING OF OLD MASONRY

Geometry of the numerical model

When dealing with any real solid, and in particular with masonry walls, the first problem is whether to use a two-dimensional or a three-dimensional model. When studying arches or vaults there is a double alternative: between a continuum (with the choice between solid two- or three-dimensional elements) or a structural (choice between beam and shell elements) approach.
If one considers the extremely large amount of simplifying assumptions underlying any such analysis, one is lead to use the simplest possible model, but this might not always be a feasible alternative. In the writers' experience, continuum elements are to be preferred over structural ones, even if they provide information sometimes difficult to interpret.

Discrete numerical model

The numerical analysis of old masonry structures can be approached from several different viewpoints.
The first is to consider masonry as a homogeneous material, at the scale of the full structure, governed by a suitable number of elastic constants (see, for instance, Anthoine (1995)) and references therein quoted) and a suitable, more or less "standard" elastic-plastic model for describing its "homogenised" behaviour in the nonlinear range. We will come back later to this last topic, crucial to this chapter (see references [Caddemi (1992); Maier et al. (1991); Pietruszczak et al. (1992); Gambarotta et al. (1997b); Lourenco et al. (1997)]. More refined techniques imply the discretisation of the full geometry (so called "micromodels"), taking into account the single blocks and mortar beds. Such an approach seems not to be feasible for practical purposes, owing to the burden of the discretisation and to the size of the resulting problem.
Finally, some researchers have developed specific models for dry masonry, in which the blocks are treated as rigid (so called "discontinuous deformation analysis"). This way of working might have a strong relevance in the field of old structures; examples of these models can be found in [Ma (1995)], with the main reference to limit analysis.

Loading and boundary conditions

Both the analysis of masonry walls and the analysis of arches and vaults often imply difficult decisions concerning the modeling of boundary conditions and loading. We don't even consider the usual difficulties that arise, for instance, when reducing a three-dimensional structure to an assembly of plane parts. Specific problems arise, for instance, from the modelling of the usually unknown foundation structures. Similar problems arise when trying to understand what types of loading a structure has withstood during the centuries; in particular, it is very difficult to take into account the effect of atmospheric agents, such as wind, snow, temperature and humidity changes, traffic effects, briefly what researchers call “effects due to sustained loads”.

Choice of formulation and finite elements

Once the engineer has decided his model, he must also decide which FEM code he wants to use. The choice of a specific FEM code implies both the choice of the problem formulation and the available options about finite element and material model choices. There are several different finite element formulations and implementations, both with reference to the basic variational approach and with reference to solution techniques, especially in the nonlinear range. There is no room here to discuss about this choice, which is treated in [Genna et al. (to appear)], where also an extensive bibliografy on the issues can be found.

Once the geometrical model and the formulation to be used are defined, one has to decide what type of finite element is best suited for the analysis. Here we are concerned mainly with nonlinear behaviour, and care must be taken, when choosing finite elements, in several respects. If masonry is modelled as a homogeneous continuum, there is a large choice of elements; however, if one is interested in performing an analysis up to collapse, one must prefer a selected group of elements, i.e., those with the ability to correctly catch the collapse mechanism. There are several examples of very poor performances of standard finite elements in situations of diffused yielding. In general, for continuum elements, the basic four-noded elements, either fully integrated or with reduced integration, should not be used in analyses pushed close to structural collapse. Even constant stress triangles may prove troublesome. In the authors' experience, a good element, in this respect, is the under integrated eight-noded quadrilateral, able to correctly represent most collapse modes, even those implying strain rate concentration along thin bands.

Analogous care must be exerted both in the modeling of bricks and mortar as separate constituents and in the choice of interface elements

Choice of the constitutive model

The choice of the constitutive model, in the case of the analysis of old masonry buildings, presents several different, and conflicting, alternatives. It must decide whether one prefers simplicity and low cost, or sophistication – whatever this can mean in this field - coupled with high cost and uncertainty about the results.

In the nonlinear range there are several categories of constitutive laws from which to choose, namely (i) standard plasticity, (ii) damage, possibly coupled with plasticity, (iii) viscoelasticity, (iv) softening and (v) nonlinear fracture mechanics models.

The choice of a very sophisticated constitutive model has meaning when the main objective of the analysis is to follow with the best possible accuracy the post-failure behaviour at all points of a structure. In dealing with a full scale old structure, however, it is impractical to try and catch all the equilibrium path details, putting too much emphasis on post-cracking behaviour. One should consider oneself happy if one can correctly predict the onset of global failure and, to this purpose, a standard plasticity model may be accurate enough.

Several standard elastic-plastic constitutive laws have been developed to model "brittle behaviour", starting with the simple "no tension" law, described and used, for instance, in [Maier et al. (1990)]. Other plasticity models commonly used in analysing masonry are the Drucker-Prager and Mohr-Coulomb laws, possibly with modifications, such as those described in [Andreaus (1996)]; the simple Galileo-Rankine model, used in [Lucchesi et al. (1996)]. Even the von Mises constitutive model has sometimes been used, such as, for instance, in [Boothby et al. (1998)]. Quite popular is the use of a generalised "no tension" model, with limited compressive strength, described in terms of bending moment and axial force in [Heyman (1966)] with reference to the limit analysis of arches.

Damage constitutive models have only recently been used for studying masonry. Examples can be found in [Maier et al. (1991); Gambarotta et al. (1997a)].

Creep has been recognised to be a possible cause of collapse by some researchers. Works on the effect of aging on the mechanical behaviour of masonry can be found in [Binda et al. (1991); Anzani et al. (1995)].

The importance of fracture mechanics concepts, when trying to reproduce the phenomenon of brittle rupture, which fully characterises the ultimate behaviour of masonry, has been recognised since the early seventies. Nowadays several FEM codes incorporate suitably designed elements and constitutive models able to follow crack initiation and propagation in a number of significant situations. A recent review of these models can be found in [Weihe et al. (1998)].

As one can easily appreciate, there is indeed a huge variety of proposed models from which to choose. Each has advantages and disadvantages; it is safe to say that the most sophisticated constitutive laws are, at the present stage of development of the available hardware, essentially suited for academic work only.

In the following sections we will therefore illustrate some analyses performed on masonry structures by means of standard plasticity models only, with the aim of showing what type of results one can obtain, and what is the order of magnitude of the various material parameters involved by the models used. To conclude this subsection, however, we will point out the alternative between the use of a rate formulation and the use of a nonlinear elastic (holonomic) one, assuming that the available code gives the possibility of choosing between the two. The use of a reversible model, written in terms of finite quantities, is definitely convenient when performing limit analysis calculations, since the collapse load factor in limit analysis is independent on the actual loading path, and a holonomic analysis is much cheaper than the integration of the rate equations. Also, if one is interested in estimating cracking by means of elastic-plastic equations, one might argue that fracture can be modeled – although only very crudely - by means of reversible plastic strains, therefore concluding that a holonomic model would be fully adequate to the purpose.

Choice of the material parameters

Once the analyst has decided that a specific constitutive law offers the best compromise for his purposes, he must face the problem of feeding into the model the material parameters data. One has to bear in mind that, in any case, either in the analysis of masonry vertical walls or in that of curved elements, the variety of construction modalities, such as the ratio between the thickness of bricks and mortar layers, or the inhomogeneity of the masonry texture in the thickness of the structural element, has a great influence on the overall "macroscopic" values of material parameters (see, for instance, [Tiraboschi et al. (1995); Falter et al. 1998)]).

There are some non-destructive tests which might be employed to obtain data about the stiffness of old masonry; the most used is the flatjack technique, originally applied to measure the in-situ stress level, but then extended to determine the Young modulus of the material. The results given by this technique present difficults to interpret; recent work has suggested a way to depurate the effect of plastic strains induced by the test modalities [Ronca (1996a)].

To have the values of material parameters one might resort to identification techniques, such as discussed in [Morbiducci (1998)], with specific reference to the model described in [Gambarotta et al. (1997b)].

Such techniques, however, are still in a too early stage of development and testing, at least with reference to masonry and masonry-like materials, to be considered as a feasible tool for use by practising engineers. This alone is a sufficient reason to justify resorting to standard elastic-plastic models, whose strength parameters - usually uniaxial strengths, or cohesion and internal friction angle are difficult enough to obtain from experimental tests.

As latter point of the present discussion

it can be pointed out that often FEM analyses results are difficult to interpret from a practising engineering viewpoint. The situation is relatively simple for arches, whose structural nature allows one to define generalised variables such as bending moments and axial forces which can be easily represented even as a result of a continuum FEM model. For two-dimensional structures, such as masonry walls, one usually sees a lot of colour contour plots of stress components, or simply deformed shapes. These pictures give a qualitative idea of where stresses concentrate, but are hardly easy to use as a design tool. A better idea can be obtained from illustrations of the so called isostatic lines, showing the direction of principal components of tensors at some points of the structure. Not all the computer codes can draw such lines.

EXAMPLES OF NUMERICAL ANALYSES

In this section we summarise results obtained from the analysis of two different masonry walls. Both walls had structural problems, which led to extensive cracking and, eventually, to the need for restoration. Both analyses have been done considering the situation both before and after the restoration work; the first analysis has also been done with the specific purpose of comparing results given by several different elastic-plastic constitutive models.

Here there is not enough space to write down and explain all the equations describing each model; for this reason the reader is encouraged to go to the original published work [Genna et al. (1998)]. Here we will try and give a brief summary of the main relevant results, and the conclusions which can be drawn from them.

A wall of the San Faustino cloister in Brescia, Italy

The wall is part of the monumental complex of the San Faustino Maggiore Monastery in Brescia, Italy. Figure 1 shows a view of the structure, a masonry wall about 87 m long, 8.25 m high and 30 cm thick. The wall exhibits a static anomaly, in that it does not reach the ground, but it rests on both transversal diaphragms and on low rise arches, 60 cm thick, which also support the ceilings of the friars' cells.

All these components are made of clay masonry; the average brick size is 28x12x6 cm and the thickness of the mortar layers ranges from 1 to 2 cm. The main construction for this part of the complex was completed by the end of the XVI century.

Different models describing the full wall have been analysed [Genna et al. (1998)]. The model represented here describes the portion of the structure exhibiting the worst cracking pattern, and at the same time being of a size which allowed the performing of several nonlinear analyses. In the model of Figure 2 the three main structural elements - wall, arch and supporting diaphragms – have been physically separated, and suitable finite elements have been introduced at their interfaces, in order to allow the simulation

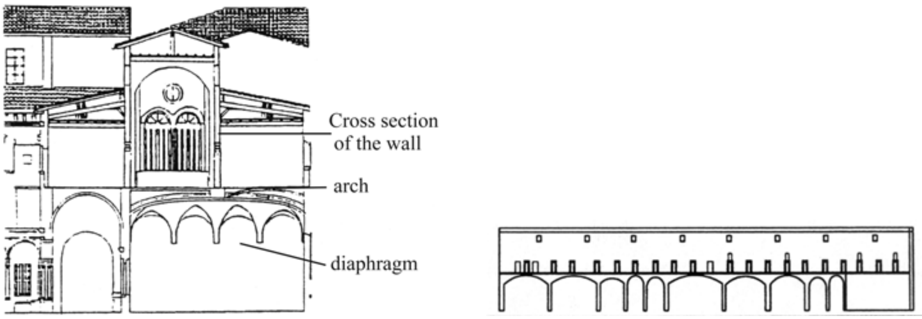

Figure 1 – Cross section and lateral view of the San Faustino cloister wall

of the interface behaviour. Such modeling is essential to avoid the prediction of an excessive collaboration between the structural components.

The material has been assumed to behave as elastic-perfectly plastic, with the exception of the supporting diaphragms, whose main stress components lie in their plane, i.e., in the plane orthogonal to that containing the plane model of Figure 2. With such a model it is worthless to try and catch any nonlinear phenomenon in the diaphragms, which have therefore been considered as linear elastic. The flow-laws have always been considered as associated; this should not create many problems for most analyses, whereas it might lead to inaccurate results in the case of limit analyses. The loading condition has been dead load (self weight and weight of the various roof slabs resting on the studied wall), as well as live load given by snow. All the details concerning these data can be found in [Genna, et al. (1998)].

The material models used in the analyses are as follows:

1. Mohr-Coulomb friction law with cohesion, for the interface elements;
2. Galileo-Rankine elastic-plastic model;

3. no-tension elastic-plastic model;
4. Drucker-Prager elastic-plastic model;
5. a tension cracking constitutive model, as described in [Franchi et al. (1991)];
6.

In the analyses we have used the following sets of data.

- Interface elements. In a first group of analyses, they have been treated as linear elastic, in order to check only the effect of modeling of the continuum elements. After these analyses we have reduced to two the set of plastic models to be used for the continuum, and have modeled also the interfaces as nonlinear. After several preliminary tests, we have decided to use a zero cohesion value, and a friction angle $\Phi = 50°$.

- Wall masonry. Young modulus E = 5000 MPa, Poisson coefficient $\nu = 0.2$; the material has been treated as isotropic.

- Galileo-Rankine model. Strength in tension $\sigma_t = 0.3$ MPa; strength in compression $\sigma_c = 1.0$ MPa.

- Drucker-Prager model. Cohesion $k = 0.23$ MPa, friction angle $\Phi = 56°$; these data correspond to $\sigma_t = 0.2$ MPa and $\sigma_c = 1.5$ MPa;

- Tension cracking model. Uniaxial strength in tension $\sigma_t = 0.3$ MPa; fracture energy $G_f = 0.02$ N/mm. This last value has been guessed, in the absence of other evidence, as about 1/5 of that suggested for plain concrete. The choice of a low value for the fracture energy, together with the large size of the structure, should magnify possible local instability phenomena due to sudden elastic energy release when cracking occurs.

All these values have been individuated after several numerical tests, and after comparison of the predicted cracked zone with surveyed results, such as those shown in Figure 3.

The numerical results are illustrated and extensively discussed in [Genna et al. (1998)]. Here we focus the attention on Figure 4, which compares results of the analyses performed considering linear elastic interfaces, for the 4 constitutive model used for the continuum elements. Figure 4a shows plastic strains for the Galileo-Rankine model, Figure 4b for the Drucker-Prager model, Figure 4c for the no tension model and Figure 4d for the cracking model.

The most striking feature here is the great difference between the no-tension results and the three others, which exhibit more or less similar features. The no-tension model predicts displacements about three times larger than the other models, and compressive stresses about two times larger than the other models.

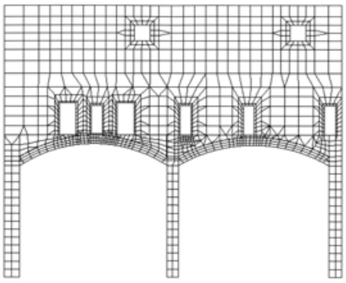

Figure 2 – Two dimensional mesh of the studied part of San Faustino cloister wall

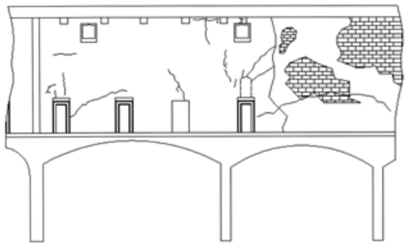

Figure 3 – Surveyed crack pattern in a portion of San Faustino cloister wall

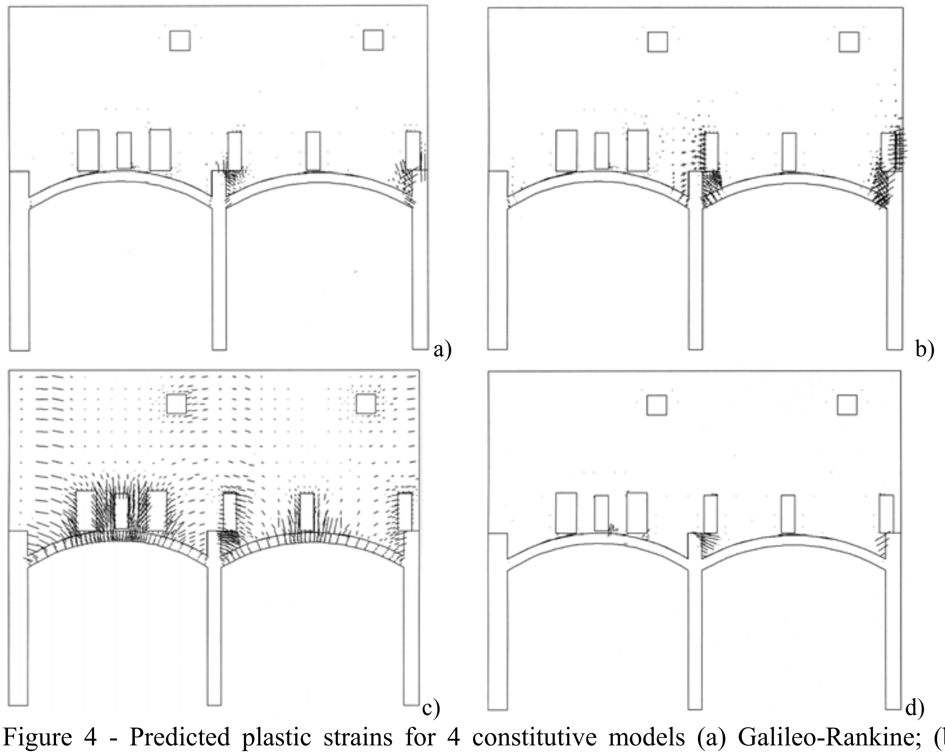

Figure 4 - Predicted plastic strains for 4 constitutive models (a) Galileo-Rankine; (b) Drucker-Prager; (c) no tension; (d) cracking model

Also the geometry of the predicted "cracked" zone is quite different, in Figure 4c, from all the others. These results confirm what has already been found in [Genna (1994); Genna et al. (1993)] about the possible lack of meaning of results given by a no-tension model when applied to the analysis of large brittle structures.

Figures 5 refers to an analysis in which wall and arches are modeled by means of the Drucker-Prager law, and the interfaces are treated as nonlinear, as described above. Figure 5 displays the displaced configuration (5a), which shows the detachment between some portions of the wall and arches, the predicted plastic zone (5b), and the isostatic lines of tension (5c) and compression (5d). These last confirm, in terms of stress distribution, what is visible in Figure 5a. The load applied just above the arch in the left portion of the wall cannot be supported by the arch itself, and the above wall behaves like a simply supported slab subjected to distributed load applied to its lower edge. On the contrary, the right portion of the wall still rests for the most part on the arch, and its stress pattern resembles that of a slab with distributed load applied on the top.

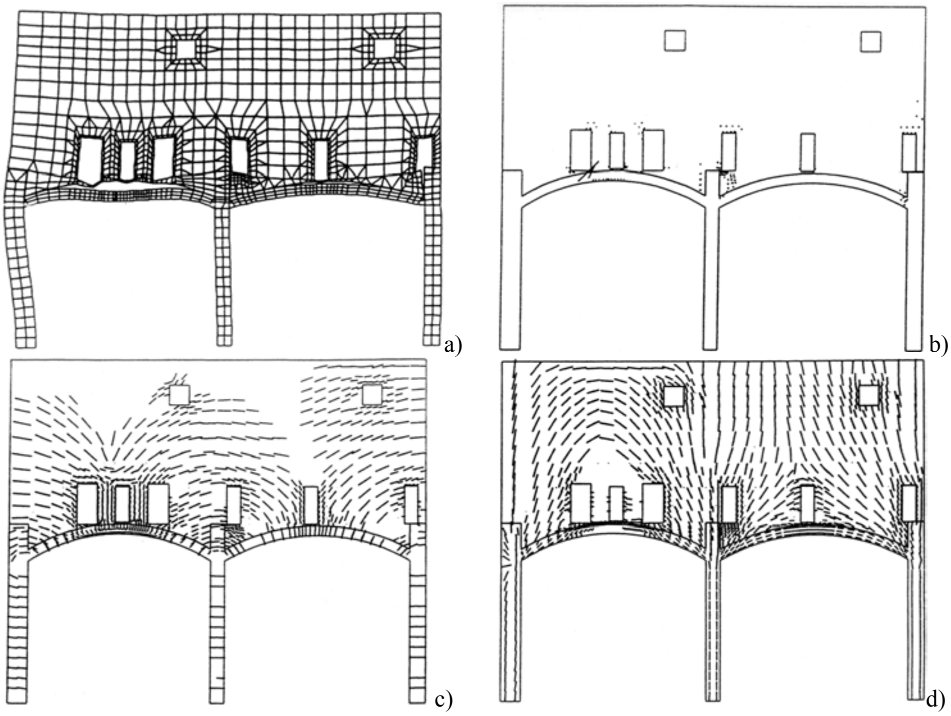

Figure 5 – Results of the analysis with nonlinear interfaces, Drucker-Prager law. (a) deformed shape; (b) principal plastic strains; (c) isostatic lines of tension; (d) isostatic lines of compression

The FEM code used is STRUPL-2 [Franchi et al. (1984)], based on the LCP formulation of rate plasticity for discrete structures introduced in [Maier 1970)].

To conclude this section, results obtained do actually furnish useful information to the engineer who wants to design a repair for the structure. All the analyses pinpoint problems due to the lack of horizontal restraint at the abutments of the low rise arches, which indicates the need of either placing a tie rod connecting the abutments of the arches or placing a tie rod inside the wall, connecting the top extremities of the supporting diaphragms. The analyses also indicate the need for improving the mechanism of stress diffusion in the proximity of the lower parts of the openings, where significant "cracking" is predicted.

A shaped arch-wall of the church "Chiesa della Disciplina" in Verolanuova, Italy

We will here briefly comment the results of structural analyses performed on a main arch-wall of the Chiesa della Disciplina in Verolanuova (Italy), a rather small church with a single nave along which 5 walls with ogival arches support the ceiling. The arches, probably built in the XIV century, are placed at a distance of 4.5 m from each other, and were initially equipped with tie rods connecting their abutments. In the XVII century, during some remodeling work, the tie rod was removed from the first arch. This has caused severe structural weakness, which became evident in the following decades when extensive cracking, involving both the arch and the wall above it, took place.

The geometry of this first arch and wall is shown in Figure 6, where all the dimensions are given in centimeters. The thickness of the ogival arch is of 74 cm. The walls which support the arch have thickness of 74 cm on the right side of Figure 6, and of 90 cm on the left side. The thickness of the wall above the arch, which transmits the load from the ceiling to the arch itself, is of 74 cm. An alarming crack picture suggests that a static collapse of the structure has already happened (the church has in fact been closed to the public for a long time), and that it was certainly due to the lack of restraint of the arch abutments against horizontal displacements. More details about this analysis can be found in [Carini, (1999)]. We have analysed this structure both in the linear and in the nonlinear range, both before and after some restoration works, aimed at restoring its full static effectiveness. The applied load consists of permanent and live load. Again, all the FEM analyses have been performed using the STRUPL-2 code.

The material models and parameters used are as follows.

- Masonry has been considered as isotropic elastic-plastic. Young's modulus $E = 750$ MPa, Poisson coefficient $\nu = 0.2$. The yield condition used has been De Felice (1994), which proved very effective in the context of the previous analysis. The relevant material parameters are cohesion $k=0.05$ MPa, friction angle $\Phi = 30°$, tensile strength $\sigma_t = 0.05$ MPa, compressive strength $\sigma_c = 1.0$ MPa, ratio m between brick width and height $m=5$ for the walls and $m=0.2$ for the arch. The lateral walls have been assumed as linear elastic.

- Interface elements. Their elastic properties are the same as those of masonry. The nonlinear behaviour has been defined in terms of a Coulomb model with no cohesion and a friction angle of $\Phi = 20°$. A strength limit in compression has also been added, with $\sigma_c = 0.2$ MPa.

Several analyses have been performed for this structure. Here we show results for only two, concerning the state before the consolidation work, and the state after it. Figure 7 shows the distribution of plastic strains in the structure before the repair work, under permanent loading only. The plastic strain suggests both detachment between arch and wall, and large damage occurring in proximity of the zones where actual cracking has occurred. The arch is severely strained, with large zones undergoing bending.

Figure 8 shows the mesh used to study the structure after the rehabilitation works, which consisted of restoring the tie rod connecting the arch abutments and in placing several other steel pins through the upper part of the wall, connecting a concrete girder, added on the top of the wall, with the arch.

Figures 9 illustrate the numerical results in term of plastic strain obtained in this new situation under the full live loading. The vertical displacement at the arch keystone is 0.95 cm; the stress in the steel tie rod is 95 MPa.All the stresses in the masonry are limited

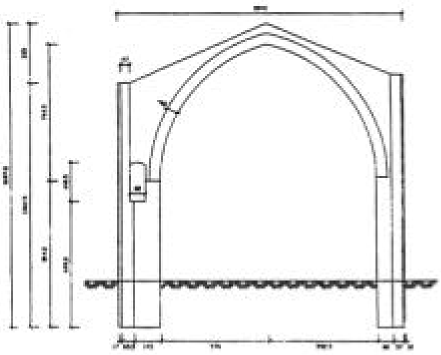

Figure 6 – Geometry of the second arch of the church “Chiesa della Disciplina” in Verolanuova, Italy.

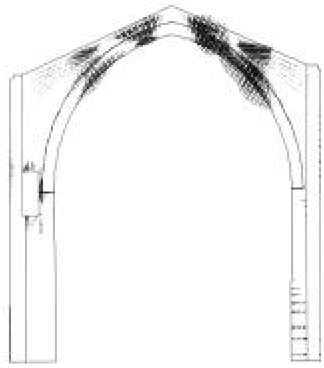

Figure 7 – Results of the analysis under permanent loads only. Principal plastic strains

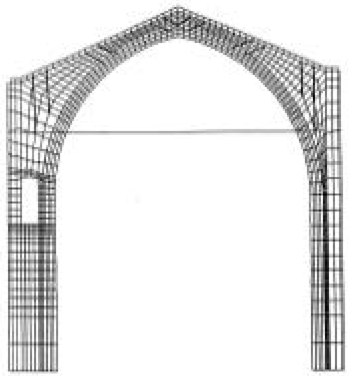

Figure 8 – Mesh of the wall and arch after the consolidation work

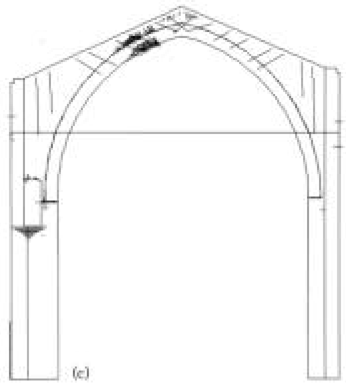

Figure 9 – Results of the analysis after the consolidation work. Principal plastic strains

within the range $-0.5 \leq \sigma \leq 0.07$ MPa; the arch is practically subjected to compressive stresses in its entire span and there is indication of very little "yielding". All this is obtained without pretension of the tie rod, which, if applied, would probably eliminate any residual nonlinearity in the behaviour of the structure.

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