DETERMINATION OF EI_{eff} FOR CMU WALLS

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ABSTRACT

The effects of a variety of parameters on the effective flexural rigidity of plain and single-layer reinforced concrete masonry walls at failure were investigated. A computerized finite element technique was developed to account for effects of both material and geometric nonlinearities on the behaviour of walls. Effects of stress – strain relationships for masonry in compression, masonry tensile cracking, and longitudinal reinforcing steel are included directly in the moment – curvature relationship which is used in the determination of element stiffness at successive load increments. The variation of flexural rigidities of masonry walls with eccentricity for various slenderness ratios and subjected to single and double curvatures were obtained and discussed. Accordingly, a lower bound equation was developed to calculate the effective flexural rigidity to be used in determining the critical buckling load.

Key words: Walls, Finite Element, Moment - Curvature, Eccentricity, Slenderness, Beam-Column, Rigidity

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INTRODUCTION

In the design of slender masonry walls, the P-Delta effect must be included in a rational way. This effect, also referred to as a moment magnifier effect is characterized by the recurrent compounding of bending moment generating deflection, which in turn becomes the source of additional moment. In most design methods developed to account for this effect the difficulty to deal with the complexities associated with both material and geometric nonlinear responses results in empirical or semi-empirical criteria. The Canadian masonry design code (S304.1-M94) suggests a load - displacement method or a moment magnifier procedure similar to that used in other areas such as, for example, steel design. In calculating displacement for the load - displacement method and buckling load for the moment magnifier method, an effective flexural rigidity, EI_{eff}, is employed. Two equations for calculating EI_{eff} for both plain and reinforced walls are also recommended in S304.1. Although these equations have the advantage of being simple to apply, they may not account for slenderness effects with a consistent level of safety.

A masonry wall cross-section subjected to increasing combined axial load and significant lateral bending, undergoes a reduction in both the modulus of elasticity, E_m, and the moment of inertia, I, during the loading history. The compounding effect of axial load and lateral deflection results in growth of tension cracks which reduces the effective moment of inertia and elevates the level of stress, which leads to a reduced value of E_m due to a non-linear masonry stress - strain relationship. Moreover, for a slender wall, the P-Delta effect has a significant role in reducing the capacity of the wall. The coupling effect of the two sources of nonlinearity and a realistic recognition of a potentially large range of material properties and geometries, make it difficult to develop a rational design method with wide applicability.

Several authors have published notable work in this general area. Among these are Yokel et al. (1971), Fattal and Cattaneo (1976), Hatzinikolas and Warwaruk (1978), MacGregor et al. (1974), and Drysdale et al. (1994). The available literature shows a disparity amongst several suggested values of flexural rigidity as recommended by various researchers. Proposed equations have been either empirically or semi-empirically founded with questionable agreement, if any, between experimental and theoretical results.

In this paper, a numerical analysis based on finite element techniques is developed to predict the capacity of reinforced masonry walls. The effects of a variety of parameters on the effective flexural rigidity have been investigated. The coupling effects of both material and geometric nonlinearity are accounted for in the analytical modeling. An equation for calculating EI_{eff} based on a non-uniform modulus of rigidity is presented.

NUMERICAL TECHNIQUE

The Priestly and Elder model was adopted to estimate the stress - strain relationship for concrete masonry in compression (Ref. 7). According to their results, the model showed good agreement with experimental data. This model is represented in Fig. 1 and subsequent equations defining the stress, σ_m, in the masonry at compressive strain, ε_c.

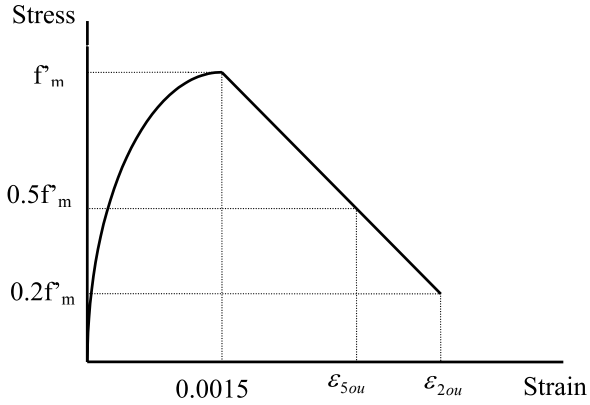

Figure 1: Assumed Stress - Strain Curve for Masonry in Compression

$$\sigma_m = 1.067 f'_m \left[\frac{2\varepsilon_c}{0.002} - \left(\frac{\varepsilon_c}{0.002} \right)^2 \right] \qquad \varepsilon_c < 0.0015 \qquad (1)$$

$$\sigma_m = f'_m \left[1.0 - Z(\varepsilon_c - 0.0015) \right] \qquad 0.0015 \le \varepsilon_c \le \varepsilon_{2ou} \qquad (2)$$

$$\sigma_m = 0.0 \qquad \varepsilon_c > \varepsilon_{2ou} \qquad (3)$$

where f'_m is the maximum compressive stress of masonry and $Z = 0.5/(\varepsilon_{5ou} - 0.002)$, where $\varepsilon_{5ou} = (3 + 0.29 f'_m)/(145 f'_m - 1000)$. Also ε_{5ou} and ε_{2ou} are the values of strains corresponding to σ_m =0.5 f'_m and σ_m =0.2 f'_m, respectively.

A moment - curvature relationship was used to evaluate the effective flexural rigidity,EI, as a single entity for a combination of axial load and moment. In the procedure, a value of the curvature and an extreme fiber compressive strain of a cross-section are first assumed. The corresponding compression zone depth is obtained and divided ideally into n elements which have the same height, X/n. For each element, the compressive stress is obtained using the stress – strain relationship. By considering equilibrium of the applied force and internal forces resisted at a cross-section, the assumed strain can be checked. This process is repeated until the assumed strain and calculated strain converge thus satisfying equilibrium for the assumed curvature. The corresponding internal moment about the section centroid is then calculated and a point on the moment - curvature curve is thus established. By increasing the value of the curvature and repeating this procedure, another point on the moment - curvature curve can be determined. The moment - curvature relationship recognizes the nonlinear stress - strain nature of masonry in compression, the presence of steel reinforcing, and the effects of cracking.

The thickness of the cross - section, t, is not necessarily constant due to possible crushing at the compressive area when the applied compressive load is high and the ultimate compressive strain has been reached. As there is a dearth of available literature on the subject of the ultimate compressive strain for masonry, a maximum compressive strain of 0.003 was adopted according to the CSA S304.1-M94. Therefore, any compressive area with strain exceeding 0.003 is considered as crushed and the resulting reduced thickness, the original thickness minus the crushed height of compressive zone, is used in the equations. This accounting of the decrease in cross - section thickness results in the falling branch of the moment - curvature relationship.

Since the analytical method evaluates and integrates the behaviour of individual elements, it is possible to track flexural rigidity changes and stress changes along a member and to provide more realistic estimations of overall effective flexural rigidity, buckling capacity, and specimen response. In this procedure, a masonry wall is divided into m line elements of equal length along its height. Element stiffness matrices are determined and assembled into a structure stiffness matrix, [K]. Loads are assembled into a nodal force vector to establish an equilibrium equation of the form [K] {w} = {F} where {w} is a nodal displacement vector. At this point, boundary conditions may be applied and the resulting equations solved using a modified Cholesky method. A combined incremental and iterative technique is used to obtain the entire load vs. deflection curve. The technique involves a process of incrementing external loads along positive stiffness portions of the loading curve and incrementing deflections when the stiffness is negative. During the process, stability and material failures are checked at each increment of load after the convergence criterion is satisfied and stiffness matrices are updated as required. This process is repeated until a complete load - deflection curve is obtained. Stability failure is checked as well as material failure. Calculation of the buckling load and lateral deflection includes the effects of modulus deterioration and cracking along the member at each load step. Using the calculated critical buckling load, P_{cr}, at failure, an effective flexural rigidity for the whole member is determined as:

$$EI = \frac{P_{cr}(kh)^2}{\pi^2} \tag{4}$$

where k is the effective length factor and the other terms are as defined above.

Verification of the analytical technique was conducted by comparing analytical results with reported experimental findings including tests of walls under both eccentric loading and combined axial and lateral loading. The comparison shows that the analytical technique is capable of predicting the behaviour of masonry beam-columns with a broad range of physical properties and loading conditions (Ref.8).

PARAMETRIC STUDY

Plain and single-layer reinforced walls subjected to both eccentric compressive loading and combined axial and lateral loading were considered. For eccentric compressive loading, the effects of slenderness, with values of h/t of 6, 12, 18, 24, 30 and 36, were used. Eccentricity ratios, e/t, with values of of 0.1 to 1.0 with an increment of 0.1 for reinforced walls and 0.1 to 0.3 with an increment of 0.05 for plain walls were included. Moment gradients represented by e_1/e_2 values of 1, 0, -1 were investigated. e_1/e_2 = 1 defines single curvature bending while e_1/e_2 = -1 indicates reverse curvature bending. For combined axial and lateral loading, the effect of slenderness ratios, with values of h/t = 6, 12, 18, 24, 30, and 36 was investigated. A wall with nominal cross-section thickness of 150 mm and length of 1 000 mm was used and pinned support conditions for both top and bottom of the wall were assumed. The total reinforcement area was assumed to be $250mm^2$. The compressive strength for masonry was taken as 15 MPa. The load increment was 10 kN and the tolerance of the convergence was set as 0.001 on the elements of successive load vectors. The number of iterations within the non-linear analysis was set at 200

RESULTS AND DISCUSSION

Figures 2 and 3 illustrate the relationships between EI, e/t, and h/t with e_1/e_2 =1 for single-layer reinforced and plain walls under eccentric compressive loading, respectively. For comparison, maximum and minimum values of 0.05 and 0.25 for EI_{eff}/EI_0, and the transition between these values, shown as a broken line, as recommended by S304.1-M94 (1994), for reinforced walls (h/t taken as 6 in the analysis) and a constant value of 0.4, as recommended for plain walls, are also indicated in Figure 2. In general, as e/t increases, EI values decrease precipitously between e/t values of 0.1 and 0.3 and less so thereafter. The reduction of EI may be attributed to the presence of both material nonlinearity and crack development. As e/t approaches larger values, the reduction of EI is predominantly caused by tensile cracking resulting in a reduced moment of inertia. At a fixed value of e/t, EI_{eff}/EI_0 increases with increasing slenderness reflecting lower stress and lower crack levels at ultimate loads. A similar trend of variation of EI_{eff}/EI_0 was observed for e_1/e_2 = 0 and –1.

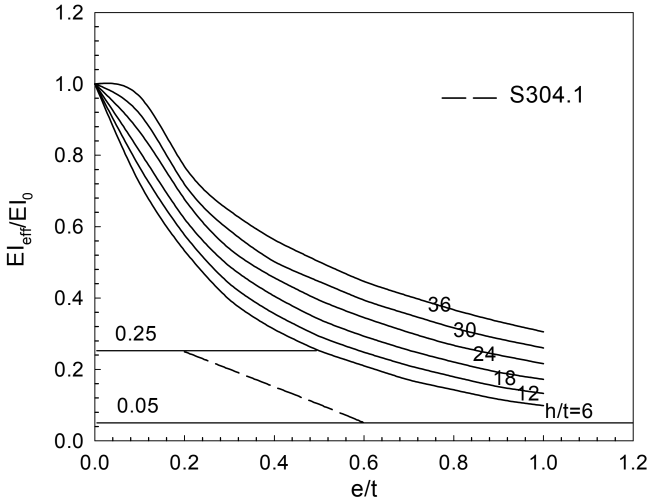

Figure 2: EI_{eff}/EI_0 vs. e/t for Single-Layer Reinforced Walls under Eccentric Compressive Loading

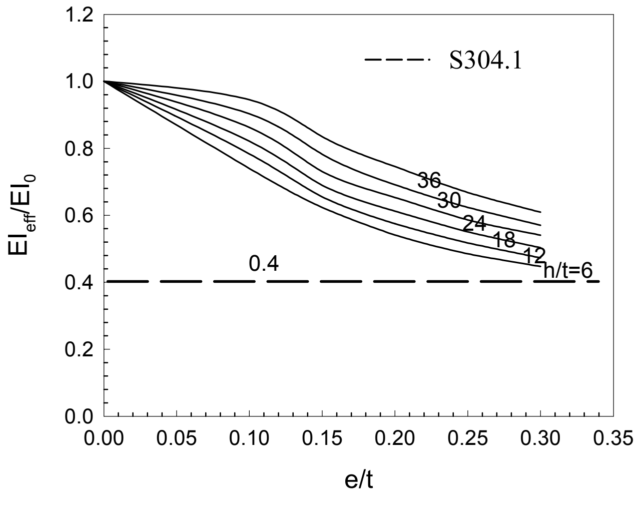

Figure 3: EI_{eff}/EI_0 vs. e/t for Plain Walls under Eccentric Compressive Loading

For walls subjected to lateral load combined with axial load, the relationships between EI, e/t, and h/t are illustrated in Figures 4 and 5 for single-layer reinforced and plain walls, respectively. In this case, e is defined as the equivalent eccentricity calculated as the ratio of maximum primary moment to applied axial load.

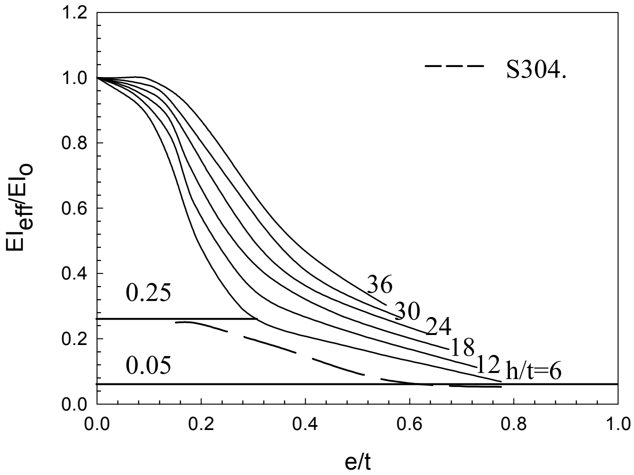

Figure 4: EI_{eff}/EI_0 vs. e/t for Single-Layer Reinforced Walls under Combined Loading

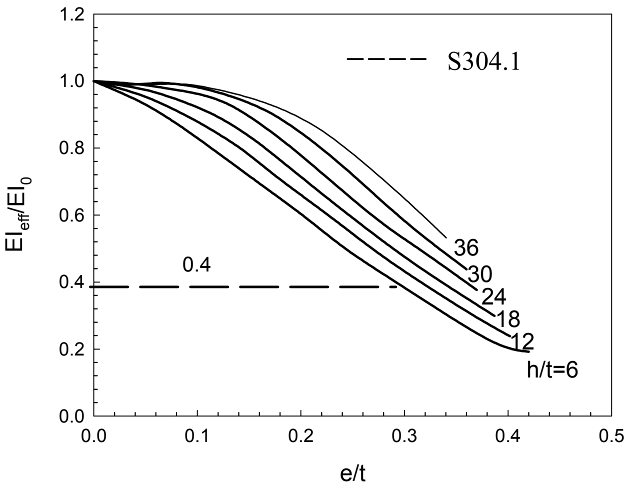

Figure 5: EI_{eff}/EI_0 vs. e/t for Plain Walls under Combined Loading

Overall, analytically determined values of EI_{eff} are higher than values recommended by S304.1-M94. This underestimation is most significant for walls failing at low eccentricities and high slenderness ratios.

The effects of various e_1/e_2 ratios on the relationship between EI_{eff} and e/t for h/t=18, are compared in Figure 6. For a given value of e/t, EI_{eff} increases as e_1/e_2 changes from 1.0 to 0.0 and then to –1.0. For walls with single curvature (e_1/e_2=1.0), deflections and therefore crack depths, are larger while the corresponding axial loads are smaller than

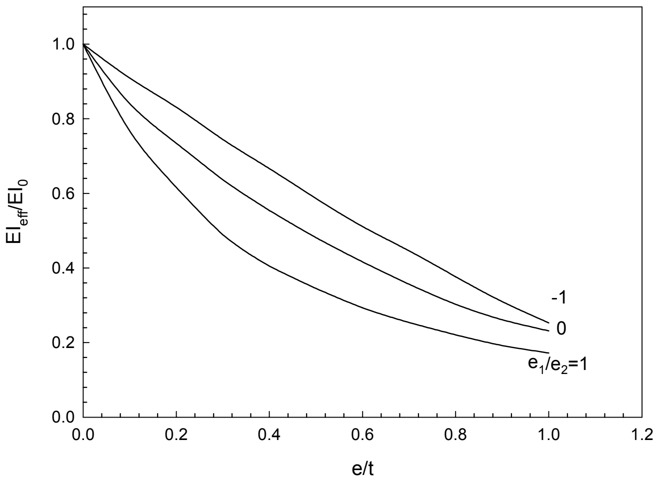

Figure 6: EI_{eff}/EI_0 vs. e/t for Single-Layer Reinforced Walls with varying e_1/e_2 ratios

those for walls with double curvature (e_1/e_2= -1.0). Thus EI_{eff} is reduced primarily due to cracking rather than by non-linear stress-strain behaviour for walls bent in single curvature. For walls bent in double curvature, on the other hand, deflections are less and axial loads are higher and consequently reduction in EI_{eff} as e/t increases is primarily due to non-linear stress-strain effects. The result is that the reduction in EI_{eff} with increasing e/t values is less dramatic for walls bent in double curvature. Although Figure 6 applies only to walls with h/t=18, similar trends were noted for other values of h/t between 6 and 36 and for plain walls.

Analytical results of EI_{eff}/EI_0 vs. e/t obtained for both reinforced and plain walls under both eccentric compressive loading and combined axial and lateral loading for various slenderness ratios are plotted in Figure 7. In each case, a lower bound curve representing minimum values of EI_{eff}/EI_0 is shown.

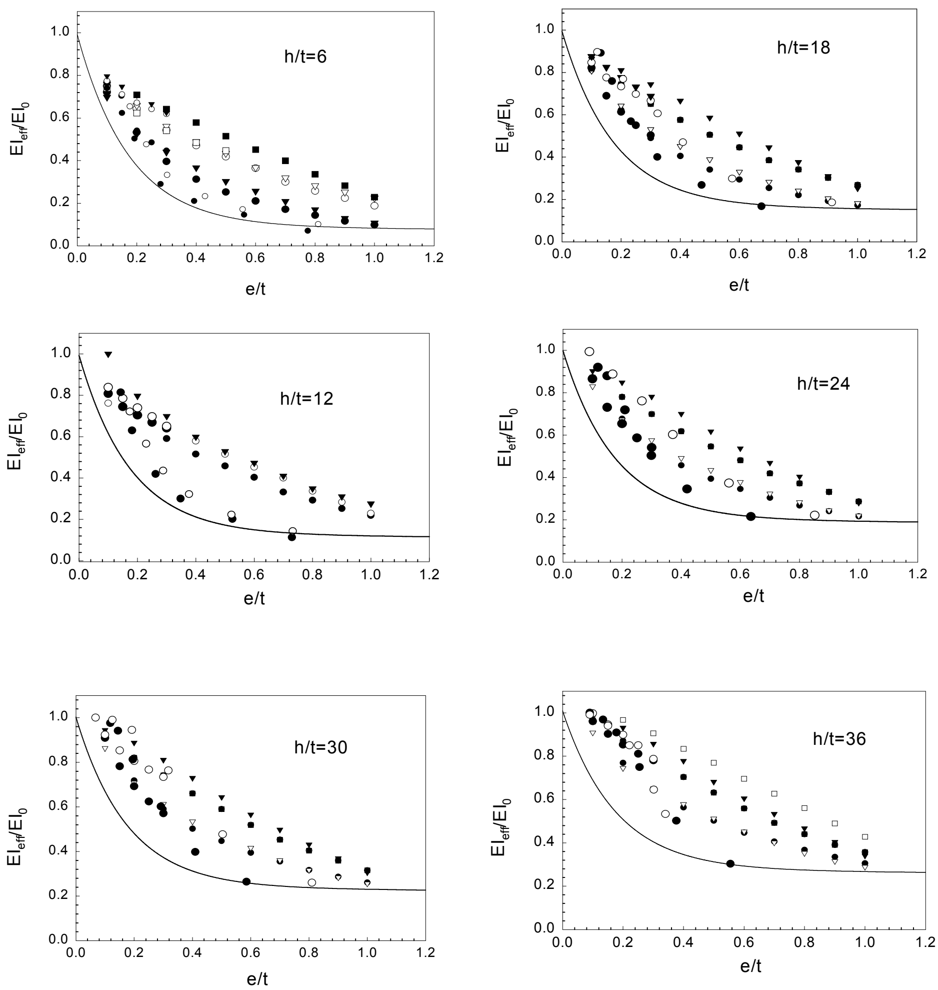

Figure 7: EI_{eff}/EI_0 vs. e/t for Various Slenderness Ratios

A regression analysis was performed and an overall lower bound curve for the curves shown in Fig. 7 was determined as follows:

$$EI_{eff} / EI_0 = 0.05 + 0.007h/t + 0.95\exp^{(-\alpha)} \qquad (5)$$

where, $\alpha = 6e/t + 0.007h/t$.

Eqn. (5) also agrees well with analytical results for double-layer reinforced walls. It should be noted that Eqn. 5 has been demonstrated to accurately account for changes in rigidity due to stress level and crack depth variations along the height of a member. Therefore, it is believed to provide a realistically conservative design value of the effective rigidity.

CONCLUSIONS

A study has been conducted to investigate the effect of various parameters on the flexural rigidity of reinforced and plain masonry walls. The interrelationships among slenderness ratio, eccentricity ratio, moment gradients and flexural rigidities of masonry walls under both eccentric compressive loading and combined axial and lateral loading were extensively examined. Compared with values determined in this study, S304.1 tends to underestimate EI_{eff} values leading to conservative predictions for wall capacities. This underestimation is most significant for walls failing predominantly by compression. Based on analytical results, an equation is presented to calculate EI_{eff} values for various parameters investigated herein. Additional research is being carried out to include a wider range of parameters. It is expected that this research may yield a more realistic evaluation of EI_{eff} than that presently recommended for use in Canadian limit states design of masonry load bearing walls.

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