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RELIABILITY ANALYSIS OF REINFORCED MASONRY WALLS

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ABSTRACT

A new methodology is presented here to estimate the systems reliability of reinforced masonry walls subjected to out of plane normal loading. The concept of yield line theory combined with linear programming is implemented to investigate the collapse patterns of a wall panel modeled as a plastic slab. The safety margins related to the most dominant failure modes are then entered into systems reliability analysis and an overall reliability index is computed.

Key words: Masonry wall; Yield Line Theory; Finite Element; FOSM Reliability; Series Reliability.

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INTRODUCTION

A reinforced masonry wall subjected to out of plane normal forces may be idealized as a plastic slab that behaves in a manner similar to that of a reinforced concrete slab. The principles of classical yield line theory may therefore be applied to estimate ultimate bending strength of this panel. In the past, the yield line technique has been used to predict the lateral, out of plane strength of masonry walls, under differing load patterns and varying boundary conditions with relative levels of success [1], [8] and [7]. However, it should be realized that the yield line method basically produces an upper bound solution for an assumed failure pattern and consequently may provide an unsafe evaluation of strength of a reinforced masonry wall.

A rational approach may then be to search for exhaustive collapse patterns and in sequel, in concert with load and the material strength variability involved, be able to predict safe or near safe load bearing capacity of a reinforced masonry wall under the assumptions made. The answer to this problem could be provided by a blend of finite element yield line analysis and the structural reliability theory. The present paper attempts to address how this can be accomplished. The procedure follows by first discretizing the masonry wall into finite elements and then by using FEYLA [6], a finite element analysis yield line computer program, different collapse patterns for the masonry wall under consideration are found. The most dominant failure modes from among these failure modes are then entered into reliability analysis to estimate a system reliability index. The following sections are relevant to the present study under consideration.

MOMENT CAPACITY OF MASONRY PANEL

The search for the most dominant modes using FEYLA [6] requires the moment capacity of a panel along the two principal orthogonal axes as input. Moment capacity of a panel is computed on the basis of ultimate strength design principles as follows.

$$M = A_s f_y \left(d - \frac{A_s f_y}{1.7 f_m^{'} b} \right) \tag{1}$$

where M is the moment capacity; and depending upon the value of A_s in x and y directions, M assumes the values of M_x and M_y. For more details about the terms in expression (1), refer to [2].

FEYLA PROGRAM: LINEAR PROGRAMMING FORMULATION FOR OPTIMAL COLLAPSE PATTERNS

Based on the theoretical models suggested by [5] and [4], FEYLA [6] computer program can be used to predict the optimal collapse load parameters and associated failure patterns for a plastic slab. In this formulation the basic finite elements are triangles and a linear program is used in the analysis. This formulation is presented below.

Find $\lambda^c, \theta_j^+, \theta_j^-, \delta_i^+, \delta_i^-$ for j = 1, 2, …, N_{yl} and i = 1, 2, …, n such that,

$$\lambda^c = \sum_{j=1}^{N_{yl}} r_j^+ l_j \theta_j^+ + \sum_{j=1}^{N_{yl}} r_j^- l_j \theta_j^- = \min \tag{2}$$

subject to,

$$\left(\theta_j^+ - \theta_j^-\right) - \sum_{i=1}^{n} \left(\delta_i^+ - \delta_i^-\right) a_{ij} = 0 \qquad j = 1, 2, \dots, N_{yl}$$

and,

$$\sum_{i=1}^{n} \left(\delta_i^+ - \delta_i^-\right) e_i = 1$$

where $\theta_j^+ \geq 0, \theta_j^- \geq 0, \delta_i^+ \geq 0, \delta_i^- \geq 0, r_j^+ \geq 0, r_j^- \geq 0$ and N_{yl} is total number of potential lines, n is the number of mechanisms, e_i is the concentrated nodal force at the nodal point i and λ^c is the load parameter at collapse in the wall panel and a_{ij} is the influence coefficient. The LP formulation can be set in the standard simplex format. For more details of this formulation and programming strategy refer to FEYLA.

RELIABILITY THEORY

In modern reliability techniques for evaluation of safety of the structures the variables connected with uncertainties like load, resistance and the model, are modeled by stochastic variables and processes. Through the principles of structural reliability it may be possible to estimate the reliability of specific mode, as well as overall structural systems reliability. This may be performed, for example, by FORM (first order reliability method) techniques. For more information on this subject see [9] and [3].

In FORM, a transformation $\overline{T}$ of the generally correlated and non-normally distributed variables $\overline{X}$ (x_1, x_2, …) into standardized and normally distributed $\overline{U}(U_1, U_2 ..., U_n)$ is defined. Let $\overline{X} = \overline{T}(\overline{U})$. In the $\overline{U}$-space the reliability index β is defined as

$$\beta = \min_{g\left(\overline{T}\left(\overline{u}\right)\right)=0} \left(\overline{u}^T \overline{u}\right) \tag{3}$$

Eqn.(3) is a general nonlinear optimization problem with one constraint. The solution point $\bar{u}^*$ of the optimization problem is closest to the origin in $\bar{u}$ -space and is called the design point.

Each of the collapse patterns represents a potential failure mode and therefore the reliability analysis, including all relevant dominant failure modes, forms the basis of the series system. Let the number of failure modes be m. Assuming linearized safety margins, the systems reliability index, β_S ,can be estimated from,

$$\beta_S = -\phi^{-1}\left(1 - \phi_m\left(\bar{\beta}; \bar{\bar{\rho}}\right)\right) \tag{4}$$

where $\phi_m(.)$ is the m-dimensional normal distribution function. $\beta_1,\ldots,\beta_m$ are the reliability indices of the failure elements determined by the FORM analysis and $\bar{\bar{\rho}}$ is correlation matrix.

To estimate the series system reliability calculation in Eqn.(4) a number of methods can be used. The following are used here:

- Hohenbichler approximation
- Simple bounds
- Ditlevesen bounds.

EXAMPLE

Consider a square wall panel reinforced isotropically and supported on two adjacent edges as shown in Figure 1. It is loaded with a uniformly distributed normal load modeled by a stochastic variable P~ N(3.3, 0.33) and has bending moment capacities modeled as stochastic variables M^+~ N(1.0, 0.1) and M^-~ N(0.47, 0.047) where N(.) denotes normal distribution showing values of mean and the standard deviation.

Shown in Figure 2, the wall panel is discretized into five candidate yield lines (+ve or -ve) and three active mechanisms generated respectively by nodes 1, 2 and 3. Referring to Figure 2, the rotation influence matrix corresponding to unit downward deflections at nodal points 1, 2 and 3 is:

$$a_{ij} = \begin{bmatrix} 2.943 & 0.0 & 3.378 & 2.432 & 2.943 \\ 0.0 & 6.112 & -1.964 & 0.0 & -6.112 \\ 0.0 & -4.4 & 0.0 & -1.414 & 4.4 \end{bmatrix}$$

where, ι = 1, 2, 3 is the number of active mechanisms and j = 1, … 5, is the number of potential yield lines.

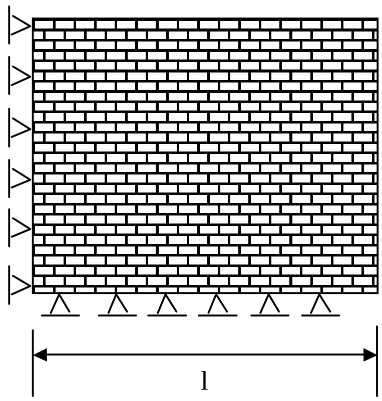

Figure 1

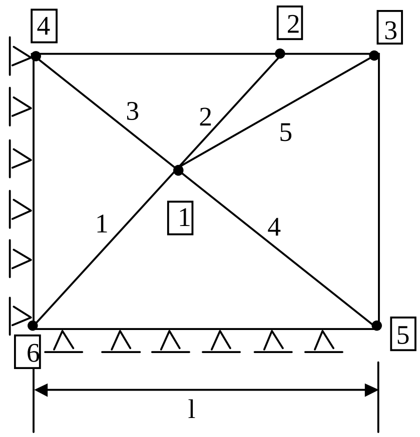

Figure 2

The external work done by each node going through a unit deflection is given as follows

$$p\begin{bmatrix}0.333\\0.0698\\0.116\end{bmatrix}$$

where p is the uniformly distributed normal pressure.

Using the computer program FEYLA [6], the linear combinations of the various nodal displacements result in the following statically admissible yield patterns:

Combination No. 1

The nodal displacements for this combination are δ_1 = 1.0 δ_2 = 1.72 δ_3 = 1.72 and the failure pattern for this displacement set is shown in Figure 3.

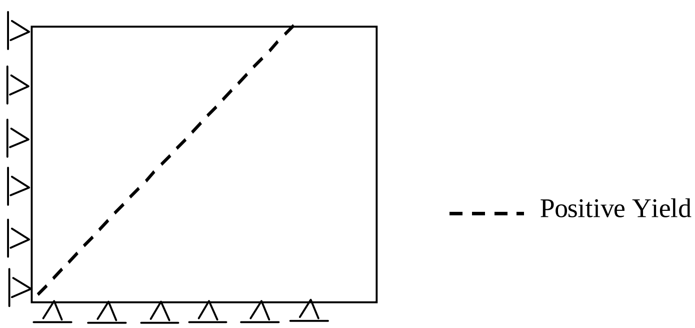

Figure 3

The safety margin and reliability index for this case are:

$$M_1 = 2.108\ M^{+} + 0.0\ M^{-} - 0.38P \Rightarrow \beta = 3.48 \qquad (5)$$

Combination No. 2

The nodal displacements for this combination are $\delta_1 = 0.0$, $\delta_2 = 0.72, \delta_3 = 1.0$ and the failure pattern for this displacement set is shown in Figure 4.

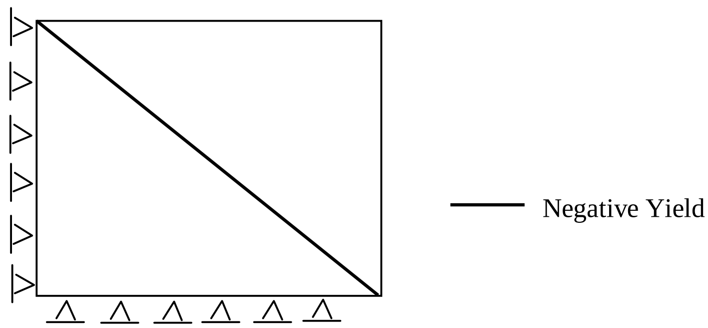

Figure 4

The safety margin and reliability index for this case is;

$$M_2 = 0.0\ M^{+} + 2.0\ M^{-} - 0.1667P \Rightarrow \beta = 3.58 \qquad (6)$$

Combination No. 3

The nodal displacements for this combination are $\delta_1 = 1.0 \quad \delta_2 = 3.216 \quad \delta_3 = 3.798$ and the failure pattern for this displacement set is shown in Figure 5.

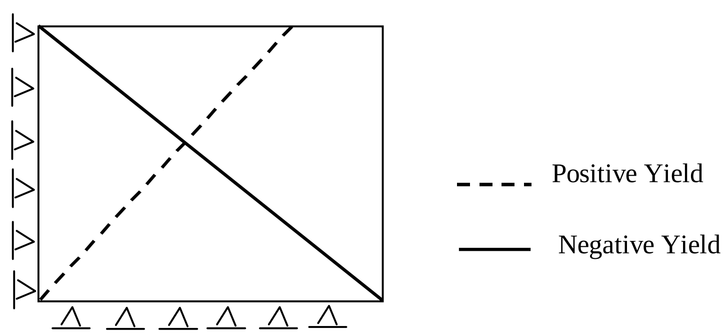

Figure 5

The safety margin and reliability index for this case are:

$$M_3 = 3.627\ M^+ + 4.15\ M^- - 1.0P \ \Rightarrow\ \beta = 3.84 \qquad (7)$$

Since the reliability index obtained in combination No.3 is comparatively larger than for the other two cases, only the first two cases will be included in the systems reliability analysis. The systems reliability analysis is essentially at series level, and the safety margins considered are Eqns. (5) and (6). The correlation coefficient between safety margins M_1 and M_2 is given by $\rho\left[M1, M2\right] = 0.63$ and the systems reliability index is $\beta_S = 3.34$.

CONCLUSION

- A simple procedure for the reliability analysis of reinforced masonry panels, using concepts of yield line theory, has been demonstrated.

- In addition to moment capacities modeled as stochastic variables, other important variables such as modulus of elasticity of wall panels in two orthogonal directions can also be included in the analysis.

- Since the finite element yield analysis program FEYLA is capable of including non-isotropic reinforcement patterns, arbitrary shapes and boundary conditions of the wall panels, and variable thickness, the methodology may be extended to include all general cases.

- Since the failure patterns are generated deterministically and the safety margins obtained from these failure patterns are treated stochastically, there may be a possibility of using Monte Carlo Simulation and FEYLA to generate stochastically based failure patterns for the reinforced masonry walls through directional simulation.

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