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# NON-LINEAR ANALYSIS OF CAVITY WALL SUBJECT TO WIND LOAD

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## ABSTRACT

This paper presents a comprehensive structural analysis of shear connected cavity walls, vertically spanned, subject to wind load. Cavity wall is a masonry assembly comprising two wythes separated by a continuous cavity and tied together, via non-conventional metal connectors. The new shear-connector changes significantly the role and the structural behaviour of traditional cavity walls with flexible ties. Also, the new Standard, CSA CAN3-S304.1 - M94 *Masonry Design for Buildings - Limit States Design*, introduces requirements, such as strength and serviceability, that must be met in design. For both reasons, there is a great need for a rational approach and more realistic prediction of structural performance of the cavity wall.

A computer program has been developed that performs the first order non-linear structural analysis of shear-connected cavity walls taking account of non-linear material properties of the wythes. A rational approach has been employed and the analysis is based on the combination of compatibility method and stiffness method.

In this paper, the proposed method is conceptually founded on the premise that the method of analysis should be independent of the procedure for estimating material properties in order to be valid for current as well as for possible future knowledge of these properties.

The program itself has a twofold purpose: To aid the engineer in the design process of the cavity wall, respecting the limit states design requirements and usage in evaluating the test results of the cavity wall.

## INTRODUCTION

Masonry cavity walls are frequently used to provide superior moisture resistance and energy efficiency for building envelope design. The wall system consists of an air cavity sandwiched between an outer veneer wythe and an inner structural back-up wythe. Traditionally, the back-up wythe has been designed to resist the full lateral imposed load, while the veneer wythe has been regarded as just an architectural facing without any structural importance. By introducing the non-conventional connectors that have the ability to transfer a shear and enable a composite action between two wythes, a contribution of veneer wythe in increasing the stiffness of the system has been achieved (Papanikolas, P.K., Hatzinikolas, M.A., Warwaruk, J. and Elwi, A.E., 1990). Currently, the masonry industry is looking into a method to take advantage of the unused structural potential of the outer wythe by reducing the material and construction costs.

Apparently, any method that ignores the elasto-plastic nature of the cavity wall's component materials cannot provide satisfactory prediction of strength and deflection. The realistic determination of the response of either a plain or reinforced cavity wall demands knowledge of the inelastic behaviour of all constituent parts and the ability to incorporate these into a rational analysis of the real structure. Since a precise analysis is highly complex, this requires a reasonable compromise between reality and the use of simplifying assumptions.

The major difference between the traditional ties and shear connectors is that the Block Shear™ Connector has the ability not only to transfer a lateral load from the veneer to the backup wall, but also to generate shear forces, which in turn produce beneficial positive moments in both wythes. The induced axial compression forces in the veneer wall and the induced axial tension forces in the backup wall are relatively small and for simplicity can be neglected in calculations. Moments created by shear forces are important since they enhance the capacity of both wythes and reduce the crack and deflection (Papanikolas, P.K., Hatzinikolas, M.A. and Warwaruk , J., 1990).

## MATHEMATICAL MODELLING / DECOMPOSITION

Figure 1(a) shows the static system of the cavity wall used in this program. For methodological reasons it is decomposed into three main parts: Veneer wall (I), Shear connector (II) and Backup wall (III). The axial force in the wythes induced by the generated shear force in the connector is neglected. Figure 1(b) depicts the deformation state of these three main parts. The kinematic assumptions and conditions are: the deflection of the veneer and backup wythe at the "supports" is defined by the expression $\delta_i = \delta_{i'} + \Delta L$ and the wythes at the "supports" experience the same rotations. Static equilibrium is maintained for each load level through the iteration process. Flowchart of the program is shown on Fig.2.

Moment-Curvature of the Section

If a member is subjected to flexure, which is the case with a veneer and backup wall due to wind load, the moment-curvature response can be easily determined by using equilibrium and compatibility conditions with the known material stress-strain relationship (Collins, M.P. and Mitchell, D.,1987), see Figure 3. A subroutine calculates M-$\phi$ (moment-curvature) values for brick or block cross-sections . It encompasses the whole range of sections, from plain to fully grouted reinforced sections. The calculation pertains to one metre length of the wall.

It is assumed that the member is subjected to strains in only the axial direction. These strains are uniform over the width of the section, but vary linearly over the depth of the section (i.e. plane section remains plane). Shear strain caused by transverse forces is neglected. The masonry strain distribution can be defined by just two variables: Strain at an outer face($\varepsilon_t$) and strain at the inner face ($\varepsilon_b$ ). The two variables that will be chosen to define the linear strain distribution are the strain at the centroid $\varepsilon_{cen.}$ and the curvature $\phi$. The curvature is equal to the change of the slope per unit length along the member and is also equal to the strain gradient over the depth of the member.

A curve that depicts the distribution of the stresses across the masonry section is approximated by using standard stress-strain curves. The stress-strain compatibility method is based on the hypothesis of the elasto-plastic nature of the body. To make this method applicable for the whole range of loading, besides the compression portion, the tension portion of the stress-strain curve must be also known( Avram, C., Facaoaru, I. Filimon, I. Mirsu, O. and Tertea, I.,1981).

At any section the stresses when integrated over the section must add up to the required sectional forces M and N.

SHEAR CONNECTOR II

A subroutine calculates a relationship between the generated shear-force, which occurs at the location of the hinge, and imposed rotations at the member ends (at the centerline of the veneer and block wall respectively). The maximum shear force (Zmavc R., 1991) is limited by:

$$S_{max} = \frac{M_p}{L_1}, \quad \textit{where} \quad M_p = f_y * z \; ; \qquad z = \frac{d_v^3}{3}, \text{ (for two wires)}$$

The Direct Stiffness method is used in analysing a model beam with rigid end parts.

## VENEER WYTHE I

This subroutine performs the analysis of the veneer wythe. Statically the veneer is assumed to act as a continuous beam. The external effect consists of three parts: the positive wind load, the moments at the "support" joints induced by shear-force and "support" settlements due to flexure of the whole cavity wall.

The methodology of this method (Tichy, M. and Rakosnik, J. 1977) requires the effects of plastic, or more accurately non-linear structural properties, to be separated from linear effects. In the analysis, the mortar joints are assumed to be the only regions where plastic deformation takes place. Plain masonry members subject to out-of-plane bending always collapse through cracking and the formation of a crack always takes place within the tension zone along the joint (Hamid, A.H., Drysdale, R.G., 1988). The segments between the mortar joints are assumed to behave linearly according to principles of elastic analysis. Plastic regions will be categorized into two groups: regions in the vicinity of new introduced hinges *(i)* and regions in other locations *(j)*. Potential plastic regions are shown on Figure 4(a) with help of the moment distribution diagram, which is not known beforehand, but qualitatively could be estimated. A set of *n* simultaneous compatibility equations must then be solved for *n*-number of unknown moments.

The distribution of moments due to external loads is obtained from the system of equations:

$$\delta_{ii}\, x_i + \sum_{i \neq k} \delta_{ik}\, x_k = -\,\delta_{i,w} - \delta_{i,ss} - \delta_{i,sm} \qquad \text{(Eq. 1)}$$

Distribution of the moments due to plastic rotations is computed from:

$$\delta_{ii} X_i + \sum_{k \neq i} \delta_{ik} X_k + \sum_{j} \Psi_j M_{i,\Omega} + \Theta_i = 0 \qquad \text{(Eq. 2)}$$

This system has to be solved only for one plastic rotation, while others are taken as zero. The number of system of equations to be solved corresponds to the number of plastic regions (mortar joints) in the veneer wall.

Total moment at each joint is calculated using the principle of superposition:

$$M_{tot} = M_{el} + \sum_{j} \Psi_j\, M(\Psi_j = 1) + \sum_{i} \Theta_i\, M(\Theta_i = 1) \qquad \text{(Eq. 3)}$$

$$M_{el} = M_o + \sum_{k=1}^{nr} X_k M_k \quad \textit{where,} \quad M_o = M_w + M_{sm}$$

Reactions at all supports are solved via shear forces, from the moment diagram.
The rotations $\Theta_i$ and $\Psi_j$ from Eq. 3 are not known beforehand and an iteration procedure is used as follows:

i. Moment distribution is computed according to elastic theory (Eq.1);

ii. Plastic rotations $\Theta_i$ and $\Psi_j$ are found for each plastic joint from the M-$\phi$ diagram;
iii. Distribution of moments due to imposed unit rotations is computed (Eq.2);
iv. Total moments are obtained from Eq.3;
v. Rotations $\Theta_i$ and $\Psi_j$ are corrected and subsequently the total moments;
vi. Iteration is terminated as soon as the difference in moment values for two subsequent distributions do not exceed a specified value.

## BACKUP WYTHE III

The backup wall is modelled as a simple beam. Loads are: concentrated forces (axial force in the connector) and concentrated moments (due to shear force in the connector). Axial forces, equal to generated shear forces in the connectors are neglected. Initially, the model has a constant stiffness. When $M > M_{cr}$, the program calculates the effective stiffness, following the M-$\phi$ relationship with "tension stiffening" included.

The deflection line of a loaded wythe is more realistic, if irreversible, i.e. plastic component of curvature is taken into account (Horton, R.T., Tadros, M.K. and Bexten, K.A., 1989). At any section where the moment exceeds the elastic limit, plastic deformation takes place. It contributes significantly to the overall deformation performance of the wythe. In the program the method of the conjugate beam is used to compute the deflections of the backup wythe. The conjugate beam is loaded with a fictitious load of intensity numerically equal to $\phi_{tot} = \phi_{el} + \phi_{pl}$ for the actual wythe. The values for curvature can be obtained from the moment-curvature relationship for a finite length of a member (see Figure 5).

So far, only the relation between moment and curvature at a cross-section has been presented and defined. To understand development of curvature along the wall axis and its relation with a moment further explanation is needed. A finite (discrete) length of a reinforced wall is shown on Figure 6, (Radosavljevic, Z., 1988) subject to constant bending moments. It is assumed that the moment is large enough to cause cracking. From the nature of masonry, the cracks always form along the joints, therefore a regular pattern of cracks eventually will form at the joint spacing. At the cracks the masonry has exceeded its tensile strength. No tensile stresses exist in the masonry, and therefore all tensile stresses must be carried by the reinforcement. As for masonry between the joints, it is less stressed and there is no appearance of the primary cracks which are visible at the joints. The bond between the bars and grout will enable some tensile stresses to be transferred from the bars into the adjacent zone. It will cause formation of internal secondary cracks. As a result, a very complex stress-strain state exists between two external cracks. So far, no convenient analytical method can describe and calculate the moment-curvature relation at a section between two cracks. Figure 6 shows qualitatively the stress distribution in the masonry and the bars, and the change of curvature and stiffness along one typical segment of wall:

i. The average stress in the compressive portion of the masonry changes along the height of the wall. The largest average stress occurs at the cracked section, where the neutral line is closest to the compression face. Between the cracks, due to involvement of the tension portion of the masonry, the average compressive stress is much smaller, and the neutral line is close to the centroidal axis;

ii. The masonry cannot carry any tensile stresses at the crack, while between the cracks, below the neutral axis it is in the tension with a very complex stress distribution;

iii. The bars are most stressed at the location of the cracks and the least stressed mid way between two cracks;

iv. Curvature also changes its value, with the highest value at the cracked Section ❷ and the smallest value at the least strained Section ❶;

v. The actual stiffness has a inverse proportional relation with curvature. For practical reason, an average stiffness is used in calculation.

Line A (O-❶-❷-❷'-❸-❹) on Fig.5 represents an idealized moment-curvature response of a reinforced masonry member at a cross-section (Section ❷, Fig. 6) subject to bending with tension failure expected. This type of diagram is used in this study in calculating effective stiffness of a finite height of the wall. The effective stiffness is expressed as a ratio between the applied moment and the average curvature (Line B). As long as the applied moment is less than the cracking moment the average stiffness will be equal to the initial stiffness of Section ❷, which is equal to stiffness of Section ❶( see Fig.6). Up to the limit of elasticity this statement is valid, while for the section ❶-❷, up to the moment of cracking it represents a good approximation. At the cracking moment level, the section experiences significant loss of stiffness and the curvature changes its value ( ❷-❷'). At higher moments up to the moment of yielding, curvature increases linearly at one rate (❷'-❸), and at the final stage up to failure for small moment increment, curvature also increases linearly, but at much faster rate. The effective stiffness will follow the moment-curvature relation expressed by line ❷-❸ (part of line B), instead of line ❷-❷'-❸ (part of line A). The calculated value for stiffness and curvature, taking into account the contribution of masonry in tension between two cracks (hatched area), will be the effective stiffness and the average curvature associated with an applied moment. Once the steel starts yielding (region ❸-❹), the section at the crack is assumed to control the calculation of the stiffness. This is a stage close to failure, and it justifies this kind of approximation. The program includes provisions for balanced and compression failure.

## SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The program computes five points on the moment-curvature diagrams of any plain, partially or fully grouted, with or without reinforcement, brick or block whether standard or non-standard section. It calculates the maximum of the induced shear force as a function of geometric and material properties. It also generates a non-linear load-deflection diagram of a masonry wall, subject to flexure, with the "tension stiffening" factor accounted for. Furthermore, it generates a non-linear load-deflection diagram of the cavity wall due to a lateral load up to the moment of cracking, and has been largely developed to predict behaviour through to failure. The program has been written in FORTRAN.

Due to the recognition of the plastic deformation component in a tension zone of the masonry (mortar joints), two "elastic moments" are introduced: the moment of the limit of elasticity and the moment of cracking. Up to the moment of elasticity only linear distribution of stresses and strains are present in the tension zone. The moment of cracking is a load level when the tension zone is fully plastified and reaches its full capacity. In a reinforced section, an abrupt change of the section stiffness occurs and the bars become effective.

The program facilitates designing the optimal combination of the masonry assembly for a specific application. It is shown that:

i. the larger block unit sizes of reinforced masonry section significantly increase the flexural capacity of the masonry wall;
ii. using the units with higher compressive strength in masonry walls subject to flexure is not economical;
iii. the usage of higher strength type "S" mortar is justified, especially for crack control, since the masonry with type "S" has a 25% greater cracking moment limit compared with the masonry with type "N";
iv. there is no justification for specifying more reinforcement, unless the moment capacity governs the design. On the contrary, it is shown that the increase of the area of the reinforcement decreases the ductility of the section;
v. the number of grouted cores does not increase the ultimate flexural capacity of the reinforced masonry wall section; however, the grout significantly increase the capacity of a non-reinforced section.

The analysis shows that the critical limit state for unreinforced shear connected masonry walls subject to wind load is the tensile failure of the mortar bed of the backup wythe. It occurs at the central portion of the wythe, where the sections are subject to the maximum moments. The cavity walls with reinforced backup wythe are a very effective combination.

The structural analysis of the cavity wall presented should include the effect of the axial load and environmental loads. However, the program itself possesses enough flexibility to be upgraded to accommodate the effect of vertical loads. The computer program should be refined and made more user-friendly.

In order for the performance of the cavity wall to be accurately quantified and verified there is a need for obtaining more accurate information about material properties.

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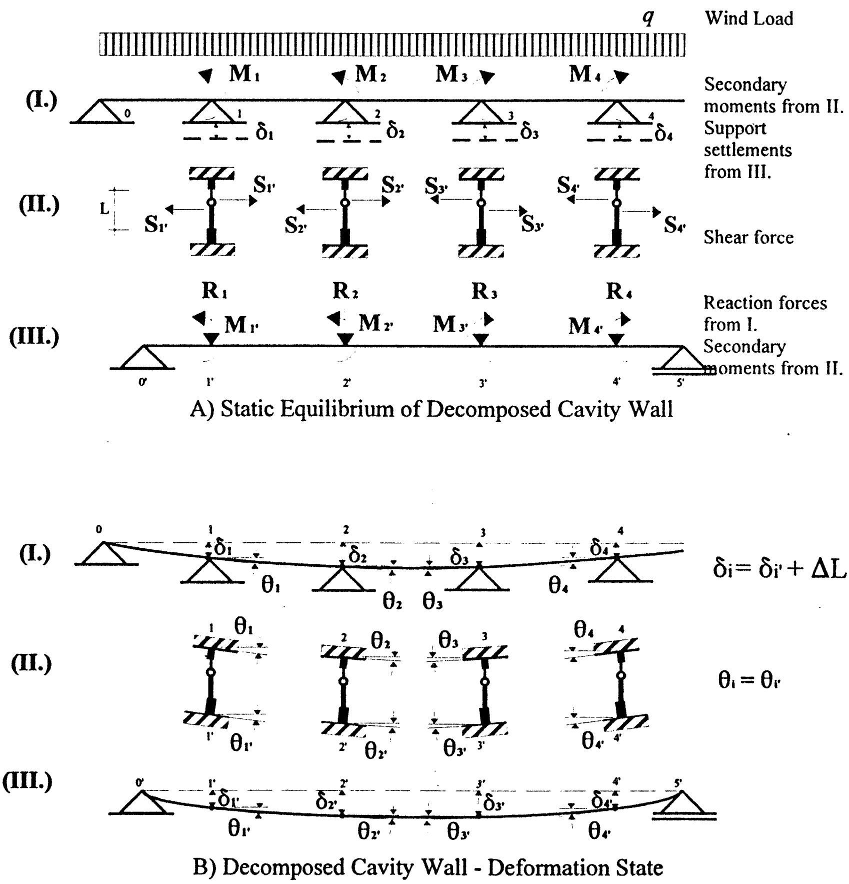


**Figure 1** Decomposition of the Cavity Wall

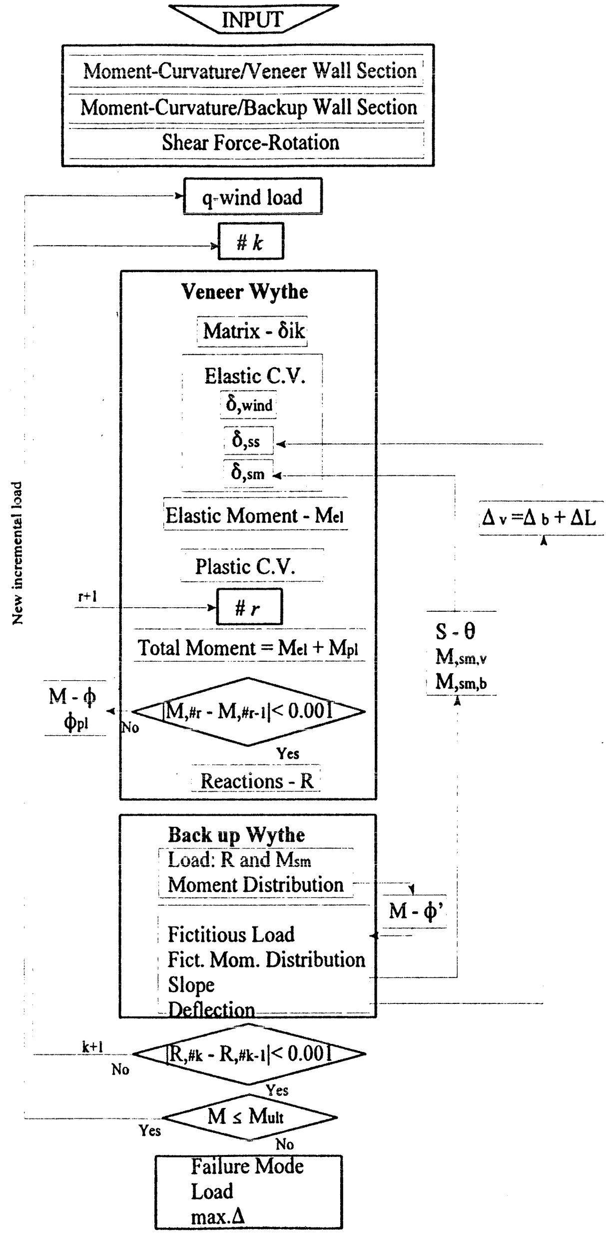


**Figure 2** Flow-Chart

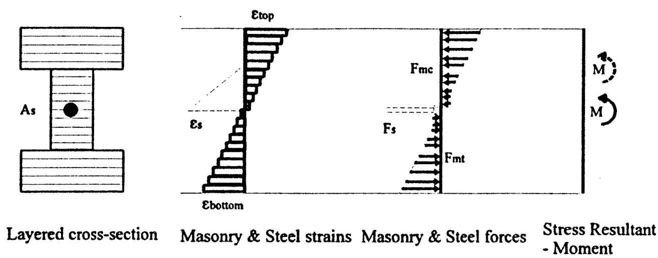


**Figure 3** Calculating Sectional Moment Using Layer Approach

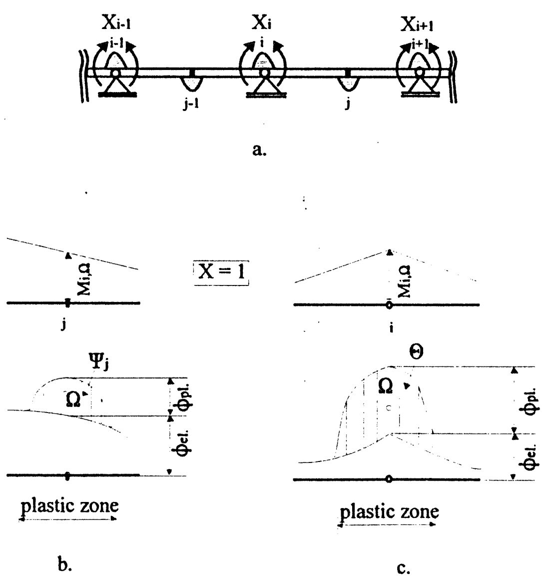


**Figure 4** Plastic Zones in a Masonry Wall

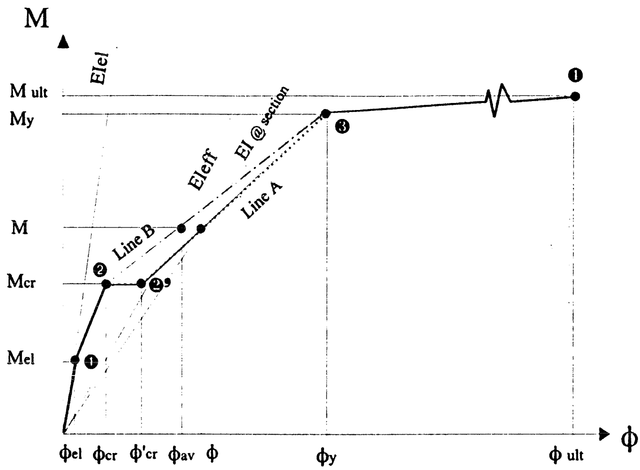


**Figure 5** Moment-Curvature and "Tension Stiffening Effect"

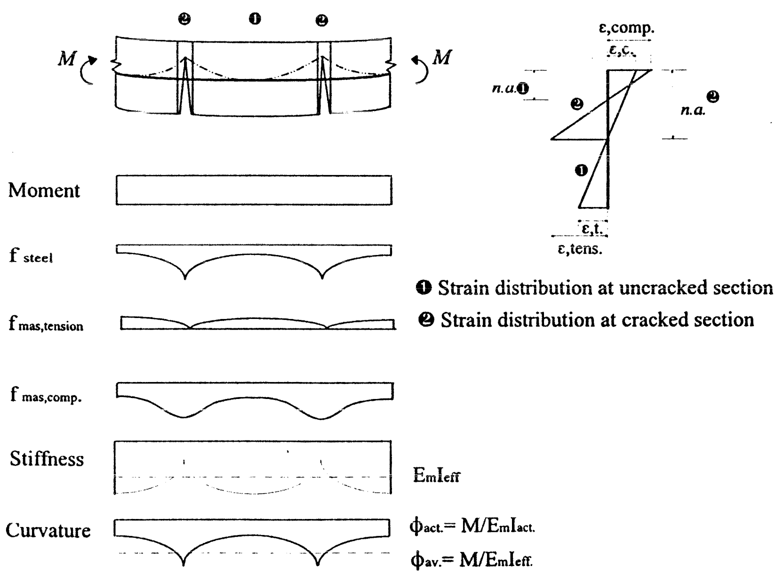


**Figure 6** Stress and Curvature Distribution along a Segment of the Masonry Wall