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FLEXURAL AND TENSILE BOND STRENGTH, RELATED VIA A STOCHASTIC NUMERICAL APPROACH

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SUMMARY

The flexural strength of masonry parallel to the bed joint depends on the geometry of the cross section, tensile bond strength, fracture energy, stiffness of units and of mortar joints. In experiments, tensile bond strength and fracture energy determined on relatively small specimens, show a large scatter. The influence of their variation upon the variation of the flexural strength of masonry was theoretically investigated in a probabilistic way. This was done by modelling a part of masonry with solids and interface elements and randomly submitting values for tensile bond strength and fracture energy to parts of the joints. In total over 200 non-linear analyses were carried out for 16 different ways of submitting the random strength and fracture energy values using different kind of correlation's between parameters and different values for coefficients of variation.
From the research it was concluded that the characteristic strength of masonry cannot be chosen larger than the characteristic strength of relatively small specimens, allthough it slightly increases with increasing crack length. The characteristic flexural strength of masonry might be take higher in case of cracks that originate in large panels and that are restrained by surrounding uncracked masonry.

Keywords: Flexural Bond Strength, Tensile Bond Strength, Probability, Distribution, Variability, Numerical Analysis.

INTRODUCTION

It is preferable to carry out bond tests on masonry on small specimens to reduce costs of testing. The representativiness of results determined on small specimens for the flexural bond strength parallel to bed joint was investigated in a numerical way, taking into account important parameters that determine the flexural strength: tensile bond strength, post peak

behaviour, stiffness and their coefficients of variation (CV). A similar research was already presented by Van Geel & Van der Pluijm (1994), but at that time only the tensile bond strength was taken into account in a stochastic way.
Parts of masonry with varying length were modelled into a three dimensional finite element (FEM) model. The dimensions of the bond surface varied between 204×98 mm and 1224×98 mm (see Fig. 1).

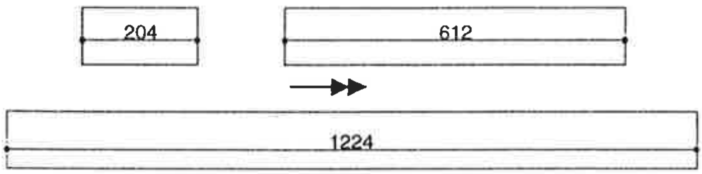

Fig. 1 Overview of the dimensions of the modelled bond surface
with the applied momentvector

Out of distributions of probability, values for tensile bond strength and fracture energy, were drawn and assigned to every part of the bond surface. The assignment of the randomly drawn values to the bond interface was performed in a special way that will be discussed later on. With the assigned properties the ultimate bending moment M_u of the FEM model was determined and a value for the flexural strength was calculated with M_u/Z (with Z being the elastic section modulus). The assignment procedure and the calculation were repeated until a good impression of the mean strength and coefficient of variation (CV) could be derived.
This Monto Carlo type of process was repeated for different assumptions concerning the bond strength, fracture energy and other properties. Before going into these assumptions, the nature of flexural bond strength and the FEM model are discussed into more detail.

FLEXURAL STRENGTH

Although flexural strength is often regarded as a material property, it is the result of the geometry of a cross section, the tensile (bond) strength, stiffness and post peak behaviour when it is considered on the basis of non-linear fracture mechanics developed by Hillerborg et al. (1976). It must therefore be regarded as a structural property.
Masonry under tension is a quasi-brittle material, that shows a descending (softening) branch after the ultimate load has been reached as has been shown in Van der Pluijm (1992) for masonry. When loaded in bending, masonry starts the show non-linear behaviour when the tensile strength is reached in the ultimate fibre. This point is reached at 70-85% of the failure load (Van der Pluijm, Rutten & Vermeltfoort, 1995). The bending moment can still be increased resulting in a shift of the neutral axis towards the compression zone. A schematic view of the internal stress distribution in the cross section at failure is presented in Figure 1 with the bold solid line.

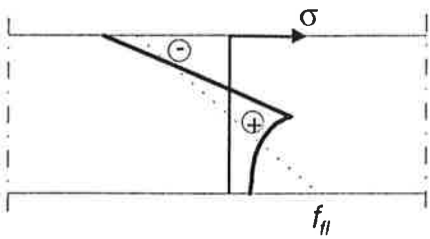

Figure 1 Non-linear stress distribution (solid line) and
the fictitious elastic distribution (dashed line) at maximum load level

The fictitious linear elastic stress distribution that is assumed when the flexural strength is calculated by dividing the ultimate bending moment divided by the elastic section modulus is presented in Figure 1 with a dashed line.
To be able to answer the question posed in the introduction in a numerical way, it is necessary to include the post peak behaviour in the FEM model.

FEM MODEL

The three dimensional FEM models, developed within the DIANA finite element code, were exactly the seem as in Van Geel & Van der Plüijm (1994). An example of a model with a length of 1224 mm is depicted in Fig. 2. In the model, one bed joint is assumed to be decisive for failure, which means that in this joint tensile bond strengths with tension softening diagrams are modelled. The other joint is assumed to behave linear elastically. From a preliminary study, it appeared that a model depth of about two bricks and one joint was necessary to guarantee a correct introduction of stresses (momentum) in the fracturing joint. Brick elements were modelled linear elastically. A mesh refinement was applied near the fracture joint to make a spatial distribution of tensile bond strength values possible.
Brick elements were modelled as quadratic plane stress continuum elements. Bed joints were modelled as quadratic interface elements, representing the combined behaviour of mortar (stiffness) and brick-mortar bond (tensile bond strength and fracture energy). The influence of head joints was completely neglected in this research.
The load was introduced on one side of the model as a pure bending moment by means of a linear altering surface load. The load was increased with a quadratic arc-length method with an automatically adjusted load increment. In combination with an automatic stop criterion for negative load increments, occurring after the peak has been reached, it was possible to complete every calculation without the need of monitoring the calculations.
The opposite side was fully restrained. The decisive joint is located at this side. This restrained side is a symmetry-surface of the model.

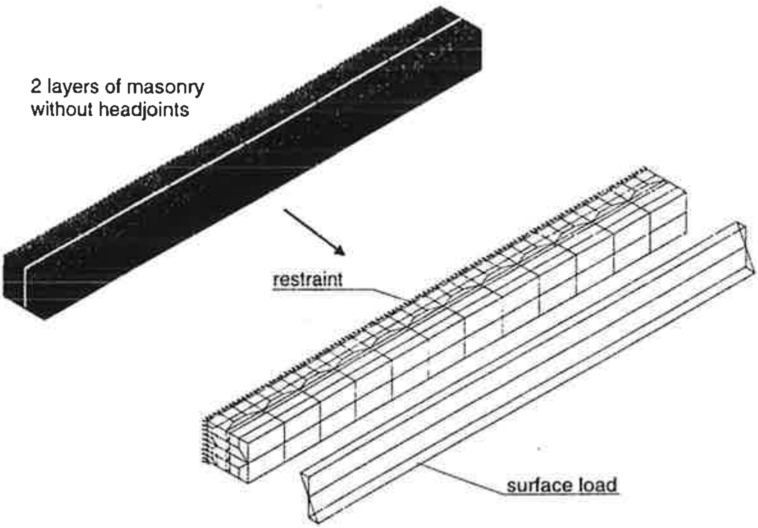

Fig. 2 Example of one of the FEM models with a length of 1224 mm (6 bricks)

As already indicated models with a length of 204 mm, 612 mm en 1224 mm were used (Fig. 1). To study the influence of different assumptions concerning e.g. the CV of the tensile bond strength, the model with a length of 612 mm was mainly used.
As an alternative of applying a surface load, in some series an imposed deformation was used (see Fig. 3).

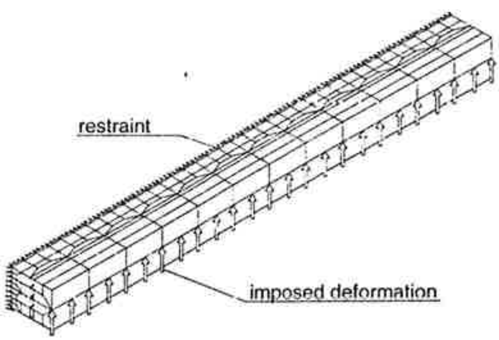

Fig. 3 Load applied using an imposed deformation

Material properties of brick, mortar and interface were based on tests on several types of bricks and mortar, carried out by Van der Pluijm & Vermeltfoort (1991), from which properties were chosen as presented in Table 1. Under tension, a lower Young's modulus is attributed to the mortar elements than under compression. This difference, observed in experiments (see Van der Pluijm & Vermeltfoort, 1991), can be explained by contact areas within the mortar being able to transfer compressive stresses but unable to transfer tensile stresses. In Van der Pluijm (1992), it has been shown that this difference affects the ultimate bending moment only in a very limited way. Tension softening was modelled by means of fracture energies with a formula developed by Hordijk & Reinhardt for plain concrete that can also be applied for masonry (Van der Pluijm, 1992). Because of the fact that the decisive joint is situated a symmetry line of the FEM model only half of the intended value for G_f must be applied to obtain the correct stress-crack-width diagram. The applied value is presented in this paper.

Table 1 Material properties in finite element analysis

Elements	Material property	Value
Bricks	Young's modulus E_b [N/mm^2]	16700
	Poison ratio ν [-]	0.20
All Joints	Compressive Young's modulus $E_{j,t}$ [N/mm^2]	13000
	Tensile Young's modulus $E_{j,t}$ [N/mm^2]	4500
Failing joint	(Mean) tensile bond strength f_{tb} [N/mm^2]	variable
	Fracture energy G_f [N/mm]	variable

DISTRIBUTIONS

The tensile bond strength and fracture energy were the variables that were considered stochastically. Based on many publications e.g. (Lawrence, 1985, Baker, 1981, Baker 1985, Lawrence & Cao, 1988, Van der Pluijm & Vermeltfoort,1995), the statistical distribution of the tensile and flexural bond strength was assumed to be normal. The available data for the fracture energy is too little to determine its distribution. It was assumed to be normal or lognormal. The normal distribution for the fracture energy was replaced by a lognormal distribution when a small mean value led to drawing of negative values.

A pseudo random generator (Merchant, 1981) was used that produces uniformly distributed values between 0 and 1. These values were converted to a standard normal distribution using the central limit theorem and subsequently to the desired normal or lognormal distribution.

ASSIGNMENT OF VALUES

Besides the choice for mean values and standard deviations of the stochastic variables another important question arises: which of the several generated values is attributed to which interface (bed joint) element? In the used model it was assumed that the unit plays an important role in the scatter of bond and mortar properties e.g. due to scatter in suction rate of each unit. In first instance half a unit was chosen as the representative area to assign one value to. Series of calculations were also made with the whole unit as the representative area. The drawn value for the tensile bond strength was taken as the mean of a second distribution with a CV of 10%. Furthermore is was assumed that, due to the drying process of mortar resulting in shrinkage cracks, lower tensile bond strength values occur at the zones of a joint near the outer surface. Therefore values drawn from the second distribution were ordered and the high values were randomly assigned to the elements in central zone and the remaining values randomly to the remaining elements in the outer zones (see Fig. 4).

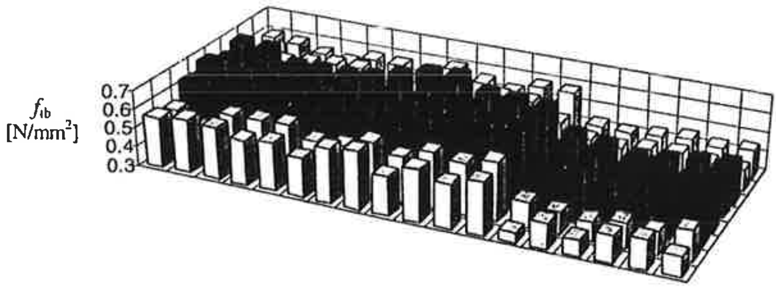

Fig. 4 Example of generated tensile bond strength values of one of the calculations in series B (length = 618 mm) with an assumed representative area equal to one unit

Only one value per representative area was drawn. This resulted in many different combinations of the tensile bond strength and fracture energy, which is in accordance with the experimental data.

CALCULATIONS

Before presenting the results of the calculations in a general way an example will be presented, so aspects mentioned above will become more concrete. The input and output for a calculation series is presented in Table 2 and Fig. 5.

Table 2 Input and output of a calculation series 7

	length = 612 mm	input-values	type of distribution	actual values
Input	mean f_{tb} [N/mm2]	0.50	normal	0.486
	CV of f_{tb} [%]	25.0		29.4
	mean G_f [N/m]	7.0	lognormal	6.62
	CV of G_f [%]	100.0		85.0
Output	mean f_{fl} [N/mm2]	-.-		0.664
	CV of f_{fl} [%]	-.-		8.1

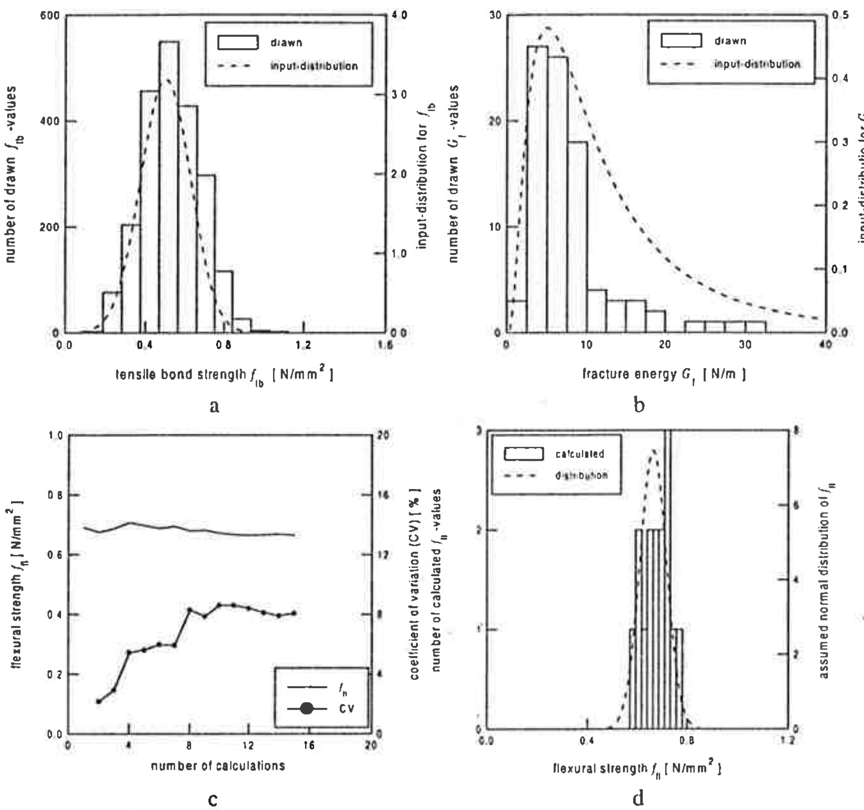

Fig. 5 Input and results of a calculation series 7

For the drawing of values for the tensile bond strength a normal distribution with a mean value of 0.5 N/mm^2 and a CV of 25% was assumed. The drawn values were used as the mean for an area with a length of a half unit. Then values for each element of this area were determined using that value with a CV of 10%. The input-distributions and the drawn values, are also presented in Fig. 5a and b. The values drawn from this distribution that were assigned to the elements resulted in a mean of 0.486 N/mm^2 and a CV of 29.4%. For the fracture energy a lognormal distribution with a mean value of 7 N/m and a CV of 100% was assumed. A normal distribution would lead to drawing of negative values. The values drawn from the lognormal distribution with a mean of 0.07 N/m and a CV of 100% led to a mean of 6.62 N/m and a CV of 85% of the drawn values, that were assigned to the ½ unit area.

After 15 calculations the mean flexural strength was 0.664 N/m^2 with a CV of 8%. A histogram of these results is presented in Fig. 5d. In Fig. 5c the mean and CV are presented as a function of the number of calculations. From this kind of diagrams it was concluded that the number of calculations was sufficient to get a reliable impression of the mean and the CV of a series of calculations.

DISCUSSION OF RESULTS

A summary of the results of the series of interest is presented in Table 3.

Table 3 Overview of calculated series

	model	tensile bond strength f_{tb} [N/mm²]					fracture energy G_f [N/m]						flexural strength f_{fl} [N/mm²]		
series	length	mean	StDev	CV[%]	distr.	Area*)	mean	St.Dev	CV[%]	distr.	Area*)	remark	mean	StDev	CV[%]
1	612	0.492	0.124	25.1	N	½	7.00	-.-	-.-	-	-		0.70	0.027	3.8
5	612	0.489	0.126	25.8	N	½	9.19	5.98	64.8	N	½		0.701	0.077	11.0
6	612	0.489	0.126	25.8	N	½	9.19	5.98	64.8	N	½	$E_{j;t}$ ~ ftb	0.71	0.076	10.7
7	612	0.486	0.143	29.4	N	½	6.62	5.63	84.9	LN	½		0.66	0.053	8.1
8	612	0.491	0.142	28.8	N	½	6.74	2.36	35.0	LN	½		0.70	0.044	6.3
9	612	0.491	0.184	37.4	N	½	6.19	5.25	84.8	LN	½	increased value of CV in representative area	0.64	0.053	8.3
A	612	0.245	0.069	28.1	LN	½	6.56	5.73	87.3	LN	½		0.42	0.032	7.6
B	612	0.508	0.122	24.1	N	1	7.58	7.08	94.3	LN	½		0.67	0.087	12.9
C	1224	0.489	0.130	26.5	N	1	6.95	6.64	95.5	LN	½		0.61	0.098	16.1
D	204	0.544	0.128	23.5	N	1	7.28	6.63	91.1	LN	½		0.77	0.123	16.1
B'	612	0.508	0.122	24.1	N	1	7.58	7.08	94.3	LN	½	as B, deformation controlled	0.71	0.073	10.4
C'	1224	0.489	0.130	26.5	N	1	6.95	6.64	95.5	LN	½	as C, deformation controlled	0.68	0.061	9.1

*) the representative area for submitting bond strength values
½: 102 × 98 mm²; 1: 204 × 98 mm²

In series 1 only the tensile bond strength was treated in a stochastic way. Doing this the CV of the flexural bond strength reduces to 0 with a increasing length of the crack. This series represents the work presented in Van Geel & Van der Pluijm (1994). In series 5 the fracture energy is treated stochasticly by using a normal distribution. One value per ½ brick length area was used. This results in a large increase of CV of the flexural strength from 3.8 to 11 %. However the normal distribution led to drawing of negative values that were not used, resulting in a mean of the fracture energy that was higher than the intended value of 7 N/m. This was corrected in series 7 leading to a lower mean and CV of the flexural strength. The data from the tensile tests showed a correlation between the stiffness of the joint $E_{j;t}$ and the tensile bond strength f_{tb}. This correlation was used in series 6. The influence of this correlation by applying a linear relation between them was, compared with series 5, negligible. In series 8 the assumed VC of the fracture energy was reduced to 40% instead of 100% in the previous series. As expected the results of this series fitted between those of series 1 (VC of G_f is 0 %) and the series with the high values for the CV of the fracture energy. In series 9 the CV of 10% that was used for submitting tensile bond

strength values to the elements of the ½ brick area was increased to 25%. Other assumptions were the same as for series 7. The differences between series 7 and 9 are negligible.
In series A the mean tensile bond strength was lowered to 0.25 N/mm^2 compared with series 7. The CV of the flexural bond strength hardly changes but the ratio between the tensile bond strength and the flexural strength increases from 1.4 to 1.7. This is understandable as the mean fracture energy was the same for both series, resulting in a more ductile behaviour of series A.
In series B the representative area for submitting tensile bond strength values to the elements was increased to 1 brick compared with series 7. As a result the mean flexural strength hardly changed, but the CV of the flexural strength increased from 8 to 13%.
Assumptions for the material-properties in series C and D are the same as in series B, but the model length was different. It could be observed that with an increasing length the mean flexural strength decreases. The influence on the CV is not really unambiguous, but it might be concluded that the CV of the flexural strength is not influenced by an increase of the length of the model for series B, C and D.
Series B and C were repeated using a deformation controlled load (series B' and C'). This resulted in an increase of the mean flexural strength and a decrease of the CV of flexural strength. The ratio between the mean flexural strength and the mean tensile bond strength is presented in Fig. 6 for the series B,C, D and C', D'.

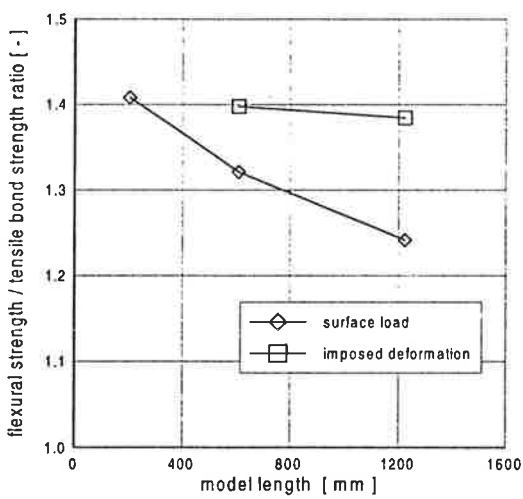

Fig. 6 The ratio between the flexural and tensile bond strength as a function of the length of the model for series B, C, D and B', C'

A decrease of this ratio could be observed with increasing model length. In case of a deformation controlled load, the decrease was small.

CONCLUSIONS

1. To get a reliable impression of the CV of the flexural strength, generally 20 calculations per series were necessary.
2. To get results with a CV of the flexural strength that becomes realistic a CV of 100% for the fracture energy was necessary in combination with a representative equal to one brick .
3. By considering the tensile bond strength and the fracture energy stochastically in combination with a representative area of 1 brick, no influence of the length of the model on the CV of the flexural strength can be found. This is in correspondence with experimental result in Van der Pluijm (1996) where no influence of the specimen width on the CV was found. In Van der Pluijm (1996) an influence between specimens with a length of 1 brick (bond wrench) and wallettes (4-point bending test) was found, but that difference was also caused by differences in test methods. However the found difference was too large to be contributed solely to the difference in test method. The theoretical effect of the decrease of the ratio between flexural and tensile bond strength found here, is another explanation.
4. As the length of the model increased, the chance of the occurrence of low strength values increases. This 'weakest link influence' is limited because partial cracked areas still have moment capacity allowing for redistribution of loads. This is also the main reason for the influence of the type of loading.
5. Using a deformation controlled load, the flexural strength is higher and only decreases in a limited way with an increase of the model length. Now an interesting question arises: which load application is representative for a crack that occurs in practise. Of course a clear answer is not available. In narrow walls the surface load might be representative and for large panels the deformation controlled case, as the cracking part of the masonry is restrained by uncracked parts around it.
6. Based on this research the characteristic strength of masonry cannot be chosen larger than the characteristic flexural strength of relatively small specimens as the outcome of Van Geel & Van der Pluijm (1994) suggested.

ACKNOWLEDGEMENTS

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