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MATERIAL PARAMETER IDENTIFICATION FOR MASONRY CONSTITUTIVE MODELS

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ABSTRACT

Many masonry structures resist applied loads and the effects of environmental change by complex interaction between the various structural elements and the supporting ground. Consequently, in order to obtain a realistic assessment of the in-service behaviour or strength of an existing masonry structure, it is generally necessary to use relatively sophisticated numerical methods of analysis. Any numerical or analytical model, however sophisticated, requires some form of constitutive model which contains a number of parameters (or phenomenological coefficients) to be found for each specific material.

Conventionally, the material parameters for a masonry constitutive models are determined directly from the results of tests on small assemblages or material samples in which it is usually assumed that the stress and strain fields in the test specimens are uniform. This assumption cannot be justified in the case of masonry which is an intrinsically inhomogeneous material. An alternative method of material parameter identification is proposed that better reflects the complex nature of masonry and the range of stress state types that exists in most masonry structures.

The method is based on the results obtained from tests on a series of larger specimens in which there is a variety of stress-state types. Assumed material parameters are initially used in a finite element simulation of the large-scale experiments. The differences between the results obtained experimentally and those predicted numerically are then minimized by adjusting the material parameters in the constitutive model using an advanced optimization technique.

INTRODUCTION

As a first step in the assessment of the strength or likely future in-service performance of an existing structure, most engineers make a series of simplifying assumptions. Typically, structures are idealised as a series of interconnected elements of construction. For example, each house in a row of three or four-storey terraced properties, a very common form of construction in many parts of Europe, may be considered as separate roof cladding, roof members, floors, masonry facade walls, masonry party walls, basement walls, foundations and the supporting ground. In practice, however, the interaction between all these elements of construction and the ground is highly complex. Indeed, it is the static indeterminacy and inherent robustness of buildings of this type that help them to accommodate thermal and moisture movements, excessive wind loading, small ground movements and other design effects that are difficult to predict accurately.

Similarly, when attempting to estimate the deflection of a masonry arch bridge or the degree of cracking under service loading it is common practice to consider the arch ring, spandrel walls, parapets, wingwalls, abutments, fill material, foundations and the existing ground as more or less separate entities. Research has shown, however, that the in-service performance of masonry arch bridges is highly dependent on the interaction and relative stiffness of the aforementioned elements. There is little doubt that such behaviour has been largely responsible for the longevity of many brick or stone masonry arch bridges throughout the world.

Simplified, idealised models of real structures have been used by engineers for many years to design new buildings consisting of interconnected beams, slabs and columns. There is some logic, therefore, in using similar models when later assessing the strength and in-service performance of such buildings. Indeed, such an approach can help engineers to carry out strength checks and, where necessary, the design of simple strengthening or repair measures with a minimum of delay. Generally, provided that the overall stability and robustness of the structure are taken into account, this relatively unsophisticated approach to strength assessment and repair is likely to be sufficiently accurate and economical.

However, such an approach is not warranted with many existing masonry structures such as those where:-

a) The form of construction is very complex, e.g. large structures such as viaducts, arch bridges, cathedrals, castles, fortifications, etc. with complex walls, vaults, buttresses, arches and other similar forms of masonry construction.

b) Where the loading is complex, e.g. various forms of seismic loading, blast or impact loading, complex variations in the ground conditions, etc.

c) Where there is considerable reliance on soil-structure interaction, as is the case with most of the above structures.

In such cases, it is usually necessary to develop a more sophisticated model to help gain a better appreciation of the real behaviour of the structure under complex loading conditions.

Even the briefest review of the literature published in the last 10 years concerning the design or performance of masonry structures indicates that increasing numbers of engineers are developing or using more sophisticated models as aids to predicting the in-service behaviour or strength of existing masonry structures. Although other models have been developed, most tend to be based on the finite element method.

All numerical and analytical models require some form of constitutive model. This paper examines the methods commonly used to determine the material parameters used in masonry constitutive models and proposes an alternative approach that better reflects the complex nature of masonry and the range of stress state types that exists in most masonry structures.

CONVENTIONAL IDENTIFICATION OF MATERIAL PARAMETERS

In order to simulate numerically the mechanical behaviour of masonry structures under various conditions in service with sufficient accuracy and computational efficiency, it is necessary to develop appropriate constitutive models. All such models contain a number of material parameters or phenomenological coefficients which are usually determined from the results of compression, tensile bond and shear tests on small assemblages of masonry or, in some cases, separate tests carried out on the masonry units and small mortar specimens. The main advantages of testing small masonry assemblages or brick, block, stone or mortar samples are:-

a) The tests are simple and relatively inexpensive to perform.

b) No specialist testing equipment is required.

c) A statistically representative sample of masonry specimens can be tested.

The anisotropic characteristics of masonry can usually be accounted for by testing small specimens in orthogonal directions or by adopting a discrete element-based finite element model. However, the simple conditions under which the small specimens are tested in the laboratory do not reflect the more complex boundary conditions and combinations of stress state types that exist in the masonry in a real structure.

PROPOSED ALTERNATIVE METHOD OF MATERIAL PARAMETER IDENTIFICATION

To overcome any limitations of experimentation with specimens under a homogeneous stress state, the use of a series of *"non-trivial"* experiments has been proposed (Toropov and Van der Giessen, 1993). Such experiments are designed to create a range of inhomogeneous and multi-axial stress and strain fields and usually involve large or whole

structural elements as test specimens. Unlike the conventional approach, described previously, when using non-trivial experiments the values of the material parameters cannot be determined directly from the experiments. Instead, it is necessary to carry out a numerical analysis to simulate each non-trivial experiment. Clearly, when performing such an analysis, it is necessary to assume initial values for the material parameters. The material parameter identification problem can then be considered as an optimization problem in which the function to be minimised is an error function that expresses the difference between the displacements measured from the non-trivial experiments and those obtained from the numerical analysis.

It is proposed to use a similar approach to determine the material parameters for masonry constitutive models. Engineers and researchers have used a number of different constitutive models for masonry in the past. Although the authors consider that the proposed method of identifying the material parameters can be used, in principle, for any constitutive model, it cannot account for any fundamental inaccuracies. Clearly, if the constitutive model does not describe the mechanical behaviour of masonry with sufficient accuracy, it should not be used.

It is suggested that the non-trivial experiments used for each combination of masonry unit and mortar could include some or all of the following:-

a) Wall panels subjected to in-plane flexure.

b) Wall panels subjected to out-of-plane lateral loading.

c) Wall panels subjected to in-plane "racking shear" with different levels of axial compression.

d) Wall panels subjected to concentric and eccentric loading.

e) Wall panels subjected to concentrated compressive loading.

f) Columns subjected to concentric and eccentric compressive loading.

g) Beams subjected to torsion.

To account for the inherent variations in the materials and unavoidable variations in workmanship, it is suggested that, for each of the non-trivial experiments noted above, at least three specimens are tested. As part of the experimental phase of the work, it is also recommended that material parameters are determined directly from the testing of masonry assemblages and masonry unit and mortar specimens so that any differences obtained using the new method described in this paper can be identified.

The proposed method is summarised in Figure 1; a more detailed explanation of the general approach to material parameter identification considered as an optimization problem, is described below.

FORMULATION OF THE MATERIAL PARAMETER IDENTIFICATION PROBLEM AS AN OPTIMIZATION PROBLEM

Consider the phenomenological coefficients of the constitutive model to be identified as components of the vector $x \in R^N$. Then the optimization problem can be formulated as follows:-

Find the vector x that minimizes the objective function,

$$F(x) = \sum_{\alpha=1}^{M} \theta^{\alpha} F^{\alpha}(x), \quad A_i \le x_i \le B_i \quad (i = 1,\ldots,N) \tag{1}$$

where

$F^{\alpha}(x)$ is the dimensionless function,

$$F^{\alpha}(x) = \{ \sum_{s=1}^{S_{\alpha}} [R_s^{\alpha} - R^{\alpha}(x, \tau_s^{\alpha})]^2 \} / \{ \sum_{s=1}^{S_{\alpha}} [R_s^{\alpha}]^2 \}, \tag{2}$$

$F^{\alpha}(x)$ is a measure of the deviation between the computed α -th individual response and that observed from the experiment;

M is the total number of individual specific response quantities (denoted by α, which can be measured in the course of experiments and then obtained as a result of the numerical simulation. When carrying out the non-trivial experiments described above, displacements will be measured at different increments of applied loading.

τ^{α} is a parameter which defines the history of the process in the course of the experiment (e.g. the time or the loading parameter);

The values τ_s^{α} (α =1,..., M, s = 1,..., S_{α}) define the discrete set of S_{α} data points;

R_s^{α} is the value of the α-th measured response quantity corresponding to the value of the experiment history parameter τ_s^{α};

$R^{\alpha}(x, \tau_s^{\alpha})$ is the value of the same response quantity (i.e. displacement) obtained from the numerical simulation;

θ^{α} is the weight coefficient which determines the relative contribution of information yielded by the α-th set of experimental data;

A_i , B_i are lower and upper limits on the values of material parameters stipulated by physical considerations.

MULTIPOINT APPROXIMATION TECHNIQUE

The optimization problem, (1), has the following characteristic features:-

i) the objective function is an implicit function of parameters $\boldsymbol{x}$.

ii) to calculate values of this function for the specific set of parameters $\boldsymbol{x}$ requires the use of a non-linear numerical (e.g. finite element) simulation of the process under consideration, which usually involves a large amount of computer time.

iii) function values may present some level of numerical noise, i.e. they can only be estimated with finite accuracy.

The direct implementation of any conventional non-linear mathematical programming techniques would involve an excessive amount of computer time. Furthermore, the convergence of a method cannot be guaranteed due to the possible presence of numerically induced noise in the objective function values and/or its derivatives. The iterative multi-point approximation concept (Toropov et al., 1993) has been developed to deal with these complications, and has been successfully used to solve various structural optimization and identification problems.

The technique is based on the iterative approximation of computationally expensive and noisy functions $F^{\alpha}(\boldsymbol{x})$, *(*$\alpha$ = 1,..., *M)* by simplified noiseless functions. The initial optimization problem, (1), is replaced then with the succession of simpler mathematical programming sub-problems as follows:-

Find the vector $\boldsymbol{x}_k^*$ that minimizes the objective function

$$\widetilde{F}_k(\boldsymbol{x}) = \sum_{\alpha=1}^{M} \theta^{\alpha} \widetilde{F}_k^{\alpha}(\boldsymbol{x}), \quad A_i^k \le x_i \le B_i^k,\ A_i^k \ge A_i,\ B_i^k \le B_i \ (i = 1,...,N) \quad (3)$$

where:-

k is the iteration number;

The move limits A_i^k and B_i^k define a sub-region of the optimization parameter space where the simplified functions $\widetilde{F}_k^{\alpha}(\boldsymbol{x})$ are considered as current approximations of the original implicit functions $F^{\alpha}(\boldsymbol{x})$. To estimate their accuracy, the error parameter $r_k = \left| [F(\boldsymbol{x}_k^*) - \widetilde{F}_k(\boldsymbol{x}_k^*)] / F(\boldsymbol{x}_k^*) \right|$ is evaluated. This gives a measure of the discrepancy between the values of initial functions and the simplified ones at a point corresponding to the solution of the current optimization sub-problem. Any conventional optimization

technique can be used to solve (3) because the functions involved in its formulation are simple and noiseless.

Methods of regression analysis are used to construct the simplified expression for the objective function $\tilde{F}_k^{\alpha}(x)$ in (3), the aim being to obtain an analytical expression that reflects the behaviour of an object considered as a function of its parameters, based on a discrete set of experimental results. Here and in the remainder of this section, an *experiment* means a *computational experiment* using the numerical model of the process under consideration. It is essential to note that it is not the intention to construct simplified expressions that are adequate in the whole of the search region determined by side constraints A_i and B_i in (1) because it takes too large a number of numerical experiments in the case of a real-life multi-parameter problem. Therefore, it is proposed to construct such expressions iteratively only for separate search sub-regions determined by move limits at each step of the iterative process. Thus, the simplified functions $\tilde{F}_k^{\alpha}(x)$ give piece-wise approximations of the initial functions $F^{\alpha}(x)$.

Now consider the problem of formulating the simplified functions, $\tilde{F}_k^{\alpha}(x)$. It is assumed that a simplified function can be expressed in the following general form:-

$$\tilde{F}_k^{\alpha}(x) = \tilde{F}_k^{\alpha}(x, a^{\alpha}). \qquad (4)$$

The vector $a^{\alpha} = (a_1^{\alpha}, \ldots, a_L^{\alpha})$ in (4) consists of so-called *tuning parameters*, i.e. free parameters the value of which is determined on the basis of numerical experiments at points located in the optimization variable space R^N in accordance with some plan of experiments. The tuning parameters are found using the weighted least-squares method as the solution of the following minimization problem:-

$$\min G_k^{\alpha}(a^{\alpha}), \quad G_k^{\alpha}(a^{\alpha}) = \sum_{p=1}^{P} w_p^{\alpha} [F^{\alpha}(x_p) - \tilde{F}_k^{\alpha}(x_p, a^{\alpha})]^2 \qquad (5)$$

where

P is the total number of points in the plan of experiments;

x_p is the vector of optimization variables that defines the current point;

w_p^{α} is the weight coefficient that characterises the relative contribution of the p-th experiment's information.

The solution of the optimization problem (5) is the vector a^{α} that makes up the simplified function (4). A proper choice of the weight coefficients w_p^{α} $(p = 1, \ldots, P)$ can substantially

improve the properties of simplified functions $\tilde{F}_k^{\alpha}(x,a^{\alpha})$ (Toropov and Van der Giessen 1993).

After formulation of the functions $\tilde{F}_k^{\alpha}(x,a^{\alpha})$, the current optimization problem (3) is solved and the error parameter r_k for the obtained point x_k^* is estimated. Next, the move limits A_i^k and B_i^k are to be determined the for the next iteration. First, the condition is checked of whether the error parameter r_k was sufficiently small, i.e. the simplified models of a current iteration were accurate enough. If this requirement is not satisfied, then the size of the search sub-region of the next step must be reduced. If the simplified models were accurate (i.e., r_k was sufficiently small), the decision is to be made on how to move the search sub-region.

If the point obtained, x_k^*, is located inside the k-th search sub-region, i.e. none of the move limits is active, then that point can be considered as the current approximation of the solution x^*. In that case, the next search sub-region should be reduced and the other conditions of the search termination should be checked. Otherwise, if some of the side constraints in (3) are satisfied as equalities, the search must be continued. This means that the search sub-region must be moved further in the direction $(x_k^* - x_{k-1}^*)$ of the previous step. Depending on the accuracy of approximations, either a new plan of experiments in the next search sub-region should be chosen or the approximations can be used once again in the new search sub-region. The search process is terminated when (*i*) the error parameter r_k is sufficiently small, (*ii*) none of move limits is active and (*iii*) the sub-region has reached a required small size.

Consider the problem of formulating the simplified expressions $\tilde{F}_k^{\alpha}(x,a^{\alpha})$. The efficiency of the optimization technique depends greatly on their accuracy. Note that properly chosen simplified expressions in our case of the optimization problem (1) with no behavioural constraints imposed, must allow for an internal minimum point inside the search region defined by side constraints A_i and B_i, otherwise the convergence of the method is likely to be slow; i.e., the number of calls for the evaluation of the functions $F^{\alpha}(x)$ would be large. The simplest form of an expression which satisfies the above requirement, is a full quadratic polynomial in x (linear in a). If there are no other considerations available, then it can be considered as a reasonable choice. However, it requires at least $N(N+1)/2 + N + 1$ calls for the evaluation of the functions $F^{\alpha}(x)$ in (2), which can be a large number in the case of a multi-parameter identification problem where N can easily be of the order of 10.

There is an alternative approach based on so-called *mechanistic models*, which finds increasing application in empirical model-building (Box and Draper, 1987). The parameter estimation of such models requires the implementation of the most general non-linear form of the least-squares method. These models are constructed on the basis of physical considerations, which can sometimes provide clues to the nature of a phenomenon under consideration. The designer of such a mechanistic model can typically use *a priori*

information, such as analytical solutions for a simplified geometrical shape, loading or boundary conditions of the specimen under consideration. Clearly, in this approach, the researcher's experience and engineering judgement is essential to create high-quality approximations. Typically, the available information presents the description of the process under consideration as a function of the experiment history parameter τ^{α}. Such information can be used for the formulation of the simplified model in the following form:-

$$\tilde{F}_k^{\alpha}(x,a^{\alpha}) = \{\sum_{s=1}^{S_{\alpha}}[R_s^{\alpha} - \tilde{R}_k^{\alpha}(x,a^{\alpha},\tau_s^{\alpha})]^2\} / \{\sum_{s=1}^{S_{\alpha}}[R_s^{\alpha}]^2\}, \tag{6}$$

where

$\tilde{R}_k^{\alpha}(x,a^{\alpha},\tau^{\alpha})$ is a simplified model of the process under consideration.

This approach has been used successfully in the problem of material parameter identification for a large deformation plasticity model from the experimental data obtained using a solid steel bar in torsion (Toropov and Van der Giessen, 1993). In order to construct a mechanistic model, i.e. physically motivated high quality approximations, analytical and simplified numerical solutions for a tubular specimen have been used. A drawback of this approach, its lack of generality, can be mentioned because every new problem involves a considerable amount of analytical work to obtain high quality mechanistic approximations.

A more general approach can be beneficial in cases when the numerical noise affects mostly the accuracy of derivatives of the error function (1) and to a lesser extent the function values. Then it can be easier to use an algorithm of sequential quadratic programming (SQP) which builds up an approximation of the inverse of the Hessian matrix using the objective function values and its derivatives. Again, the multi-point approximations of the objective functions should be used iteratively instead of original function values.

It should be noted that in problems of parameter identification for non-linear constitutive models, the traditional use of finite differences for the evaluation of derivatives would almost certainly spoil the convergence of the optimizer because the accuracy of derivatives is severely affected by the noise. As the simplified functions are used to evaluate the derivatives only, the requirement for simplified functions to allow for an internal minimum can be waived. A similar approach was used by Marti (1991) who replaced the gradients of stochastic functions by gradients of the polynomial response surface models. As the approximations are constructed in each iteration to evaluate the derivatives only, fairly simple approximation types can be used, e.g. quadratic polynomials without cross-terms:

$$\tilde{F}_k^{\alpha}(x,a^{\alpha}) = a_0^{\alpha} + \sum_{i=1}^{N}(a_{2(i-1)+1}^{\alpha}x_i + a_{2(i-1)+2}^{\alpha}x_i^2) \tag{7}$$

which contains $2N+1$ tuning parameters to be found in each iteration by the least-squares surface fitting.

APPLICATION TO MODELS FOR LARGE ELASTOPLASTIC DEFORMATIONS

The above procedure proposed for masonry constitutive models has previously been used to identify material parameters in constitutive models for large clasto-plastic deformations which include deformation-induced anisotropy, as it occurs in engineering metals under large strains. The inherent physical non-linearity of such models is further complicated by the complex history of the deformation-induced anisotropy. The simplest possible experiment, uni-axial tension, poses two major difficulties for the parameter identification in large deformation plasticity models, namely:-

a) for models that include deformation-induced anisotropy, the uni-axial response need not depend on all material parameters;

b) at large strains, necking intervenes and leads to a complicated three-dimensional stress and strain state.

Clearly, the experiments used to provide the data for the identification of the material parameters must be free from such instabilities and provide a means to identify all pertinent parameters for the considered class of models. Examples, demonstrating the use of the proposed technique for large deformation plasticity models include (i) test data for a solid bar in torsion (Toropov and Van der Giessen 1993) and (ii) test data for the cyclic bending of thin metal sheets, (Toropov et al. 1997, Yoshida et al. 1998).

CONCLUSIONS

Many existing masonry structures are of complex construction, are subjected to complex loading or rely on complex composite action with the supporting ground for their strength and stability. As a result, engineers often need to use relatively sophisticated numerical models when attempting to assess the strength and in-service performance of such structures. The material parameters used for masonry in the numerical models are usually based on the results of simple tests that do not reflect the more complex boundary conditions and combinations of stress-state types that exist in a real masonry structure..

A more appropriate method of determining the material parameters for masonry constitutive models is proposed. The method allows for the determination of the material parameters by minimizing the difference between displacements measured from a series of non-trivial laboratory experiments and those obtained by numerical simulation. In principle, it is applicable to any non-linear constitutive model and an almost unlimited variety of non-trivial experiments can be used including structural components, to account for inhomogeneous material response.

REFERENCES

Box, G.E.P. and Draper, N.R. 1987. Empirical model-building and response surfaces. Wiley.

Marti, K. 1991. Stochastic programming: numerical solution techniques by semi-stochastic approximation methods. In Slowinski, R. and Teghem, J. (eds.), Stochastic Versus Fuzzy Approaches to Multi-objective Mathematical Programming under Uncertainty. Kluwer, pp. 23-43.

Toropov, V.V., Filatov, A.A. and Polynkin A.A. 1993. Multiparameter structural optimization using FEM and multipoint explicit approximations. Structural Optimization, **6**, 7-14.

Toropov, V.V. and Van der Giessen, E. 1993. Parameter identification for non-linear constitutive models: finite element simulation - Optimization - Nontrivial experiments. In Pedersen, P. (ed.), Optimal Design with Advanced Materials, Elsevier, pp. 113-130.

Toropov, V.V., van der Giessen, E. and Yoshida, F. 1997. Material parameter identification for large deformation plasticity models. Proceedings of Euromech 357 Colloquium, Kerkrade, The Netherlands, Kluwer, pp. 81-92

Yoshida, F., Urabe, M. and Toropov, V.V. 1998. Identification of material parameters in constitutive model for sheet metals from cyclic bending tests. International Journal of Mechanical Sciences, vol. 40, No 2-3, pp. 237-249.

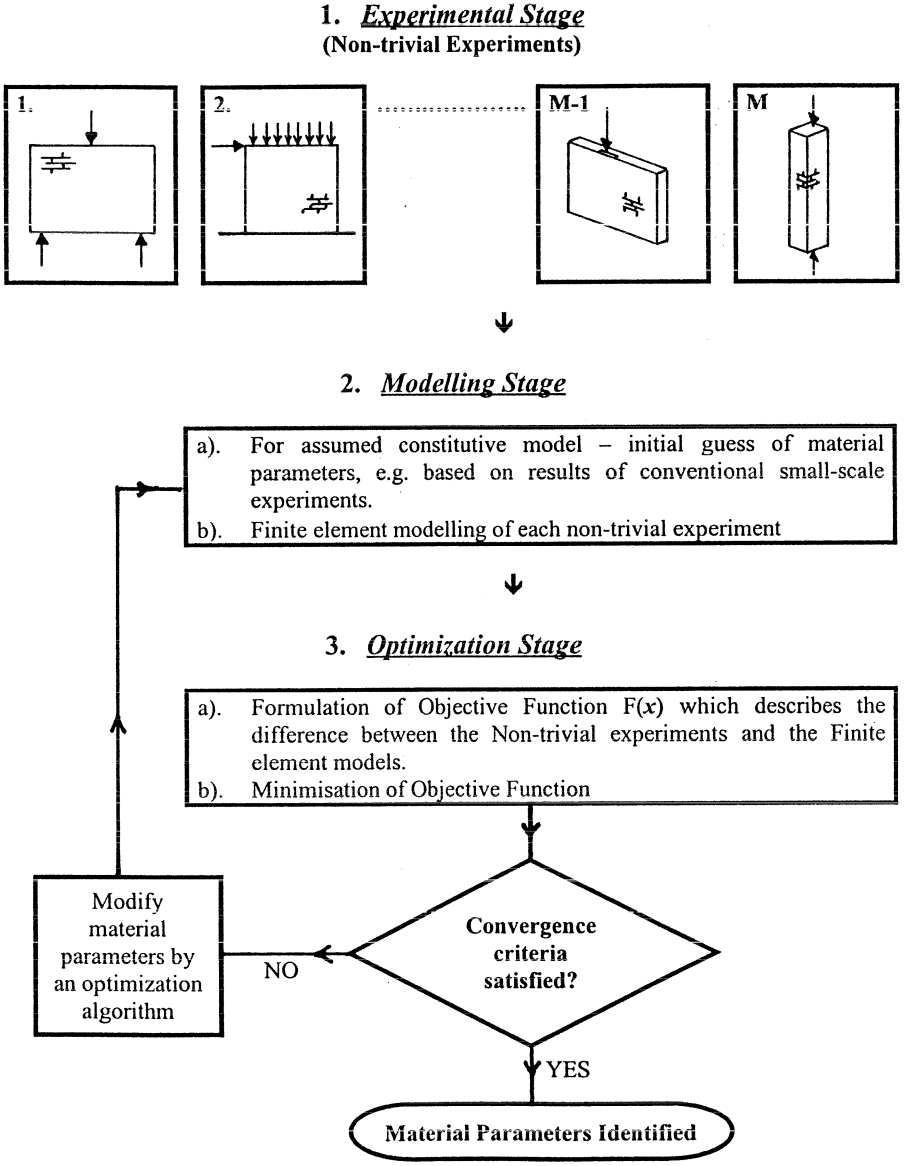

Figure 1. Summary of proposed method of material parameter identification