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A NUMERICAL METHOD FOR THE STRENGTH ASSESSMENT OF MASONRY ARCH BRIDGES

Ashraf F.Ashour and Stephen W.Garrity,
Department of Civil & Environmental Engineering,
University of Bradford, Bradford, West Yorkshire, England. BD7 1DP.

ABSTRACT

A numerical method of estimating the strength of masonry arches is presented. The method is based on a mechanism analysis of the arch at collapse with the failure mode idealised as an assemblage of rigid blocks separated by zones of displacement discontinuity; the masonry is assumed to be rigid-perfectly plastic. The shape and position of the fracture lines and the displacement of the rigid blocks of masonry are the variables in the energy equation. Minimisation of the predicted collapse load produces the optimum shape and position of the fracture lines. Comparisons of the predicted collapse loads and crack patterns at failure show good agreement with the results obtained from model tests in the laboratory.

INTRODUCTION

The brick or stone masonry arch was used by many ancient civilisations as a structural form and, by the Middle Ages, the requisite construction techniques and experience had spread to many countries. Since then, many thousands of masonry arches have been built. In particular, numerous bridges and viaducts were constructed to aid the development of strategically important settlements and communities into what have become the towns and cities of the modern world.

By the early part of this century, the use of brick and stone masonry for bridge construction declined in favour of reinforced concrete, cast iron, wrought iron and steel construction. In spite of this decline, there are still many thousands of brick or stone masonry arch bridges, tunnel linings and viaducts to be found throughout Europe. In the UK alone, it is estimated that there are in the order of 70,000 masonry arch spans almost all of which are at least 100 years old. Furthermore, many of these structures are still in use and form the heart of the nation's transportation infrastructure as well as being an important part of the UK's industrial heritage.

Strength Assessment and Strengthening of Masonry Arch Bridges

As is the case in many other countries, most rail, highway and canal bridges in the UK are subjected to regular visual inspections and more thorough principal inspections as part of a bridge management strategy. Although many masonry arch bridges have been found to be in extremely good condition, given their age, it is not surprising that a variety of defects have been identified. These include:- arch ring separation; diagonal cracking and hinging of the arch ring; cracks in the abutments and piers; leaning and bulging wingwalls and spandrel walls; spandrel wall separation; dropped voussoirs; leached mortar joints and frost damage of the masonry units (Sowden 1990, Page et al. 1991, Page 1993). To complicate matters further, many arch bridges have been widened or strengthened using materials or forms of construction that are different from those used in the original construction.

At present, standard public highway live loading in the UK comprises a maximum axle loading of 38 tonnes. As part of the gradual harmonisation of the European Union member states, a European directive has been issued to increase the maximum public highway live loading to 40 tonnes. There is a possibility that this upper limit may be increased to 44 tonnes in the future. As a result, in recent years, many engineers in the UK have been engaged in a major programme of bridge strength assessment and repair. As part of this work, a number of strength assessment methods have been developed and used for masonry arch bridges; these are considered in more detail, later. In addition a variety of repair methods have been used including grouting, pressure pointing, partial reconstruction, retro-reinforcement, saddling, lining of the intrados or soffit, stitching and the use of spandrel wall ties (Sowden 1990, Welch 1995, Garrity 1995a).

Design of New Masonry Arch Bridges

A critical review of existing highway structures in the UK (Garrity 1995b) indicated that clay brick masonry has considerable potential as a durable material for new highway structures including short span arch bridges. This view is gaining support from an increasing number of bridge engineers in the UK, particularly those engaged in the management of the nation's bridge stock who have found that the maintenance costs of stone and brick masonry arch bridges are, in general, significantly less than modern steel and concrete alternatives (Halsall et al 1994). This experience led, in part, to the construction in 1993 of Kimbolton Butts Bridge which is thought to be the first all-new clay brick arch bridge built in the UK since the turn of the century (Garrity et al 1995). The growing interest in the use of masonry arches for new bridge construction is reflected in the commissioning of a new design guide and advice note for the design of new unreinforced masonry or concrete arch bridges by the UK Department of Transport (Mair 1995) and the publication of design guidance by the UK Brick Development Association (Cox et al 1996).

METHODS OF ANALYSIS OF MASONRY ARCHES

Basic Requirements

Numerous tests on model and full-scale arches in the laboratory (Royles et al 1991, Melbourne et al 1995, to name a few) and redundant arches in the field (Page 1993) have contributed a great deal to the understanding of the behaviour of masonry arch bridges. A number of different modes of failure have been identified. Of these, the 4-hinge collapse mechanism and snap-through failures are the most common. In addition, it is now well documented that the load carrying capacity of a masonry arch bridge is influenced by the degree of continuity between the spandrel walls and the arch ring; the restraining effect of wingwalls; the interaction between the arch ring, the spandrel walls and the fill material and the structural integrity of the arch ring. Consequently, it is evident that defects such as spandrel wall separation, ring separation and cracking of the arch ring, are also likely to influence the strength of an arch bridge.

Hence, for maximum economy, the methods of analysis used for strength assessment or for the design of a new arch bridge or the repairs for an existing structure, should take into account any likely interaction between the different elements of construction and, where appropriate, any defects. Failure to model the behaviour with sufficient realism could, in some cases, lead to repair works that will not only be costly but might also impair the long term performance of the bridge (Melbourne 1991). Worse still, a very conservative method of strength assessment that yields a considerable underestimate of the strength of a bridge, could lead to the specification of very substantial, unsightly repairs or even demolition.

When designing new masonry arch bridges, the use of a very conservative method of analysis could result in the specification of forms of construction that would be prohibitively expensive to build. Consequently, it is very unlikely that many engineers will choose to design new masonry arch bridges even though the high initial costs of construction will be offset in the long-term by low maintenance costs.

Review of Methods of Analysis

A number of different methods of analysis have been developed for masonry arch bridges (Hughes et al 1997). Of these, the "*cracking-elastic*" and the "*mechanism*" methods (Heyman 1982, Gilbert et al. 1994, Boothby 1995) are the most versatile. Consider the cracking-elastic method first; this is generally based on a finite element approach. Experience shows that the method becomes computationally very complex and expensive if attempting to model anything other than just the undamaged arch ring and the fill material of a single span bridge. It is, however, likely to be useful for modelling the in-service behaviour of a structure up to the onset of first cracking provided that a realistic constitutive model is used.

The mechanism or rigid-block method of analysis is likely to be the better of the two methods of analysis for strength assessment as it offers greater versatility with less computational complexity and expense. In particular, it has been used to model arch ring separation (Melbourne et al. 1995) and it should also be possible to model multi-span construction, the effects of spandrel walls and wingwalls, a range of defects and different forms of repair. This paper introduces a numerical technique for assessing the strength of a single-span masonry arch based on a rigid-block method of analysis.

MATERIAL MODELLING

The narrow failure zones of masonry arches can reasonably be modelled to be in a state of plane stress. Modified Coulomb failure criteria (May et al. 1986) with tension cut-off, as shown in Fig. 1, will be considered as the failure criteria for masonry.

The behaviour of masonry is characterised by strain softening not by the yield plateau that would normally be required to apply the plastic theory. To account for this behaviour, the masonry strength is reduced by applying an effectiveness factor. Hence, the effective plastic compressive strength σ_{pc} is given by:

$$\sigma_{pc} = \nu_m f_c \qquad (1)$$

where f_c is the brickwork compressive strength and ν_m is the effectiveness factor for compression strength. Similarly, the plastic tensile strength σ_{pt} of masonry is given by:-

$$\sigma_{pt} = \rho_m f_t \qquad (2)$$

where f_t is the tensile strength of masonry ($\cong$ {0.0-0.1} f_c) and ρ_m is the effectiveness factor for tensile strength. As the ductility of masonry in tension is very limited, ρ_m will be very small. The reason for introducing these factors is to account for the limited ductility of masonry and to obtain reasonable quantitative predictions. In practice, the effectiveness factor is calculated for a structure by calibrating the theoretical calculated capacity against experimental results.

PROPOSED METHOD OF ANALYSIS

Mechanism of Failure

The experimental tests referred to earlier show that, at collapse, the arch ring will generally fail in a hinged mode as shown in Fig. 2. This mode may be idealised as an assemblage of five rigid blocks separated by four yield lines. Rigid blocks *I* and *V*, representing the abutments of a single span bridge, are fixed. The other three rigid blocks, *II*, *III* and *IV*, rotate about instantaneous centres.

Internal Energy Dissipation

At collapse, a "yield" line or plastic hinge will form between each two rigid blocks. In the general case, all the deformations are located in the yield line, that is, an idealisation of a narrow discontinuity zone with many criss-crossing cracks and crushing zones. The shape of the yield line and the displacements of the rigid blocks are the unknowns and will be obtained by minimising the collapse load of the arch ring predicted from the energy equation. The numerical technique presented in this paper is based on that developed to predict the load capacity of reinforced concrete deep beams (Ashour et al. 1994) with modifications to accommodate the material properties of masonry.

The internal energy dissipated by masonry along a yield line l separating the two rigid blocks I and II, as shown in Fig. 3(a), is to be calculated. The displacement components of rigid block I referred to the origin O of the global axes are U_I (horizontal), V_I (vertical) and Ω_I (rotational). Similarly, the corresponding displacement components of rigid block II are U_{II}, V_{II} and Ω_{II}. The relative displacements across the yield line l referred to the origin O are:-

$$\begin{aligned} u_l &= U_{II} - U_I \\ v_l &= V_{II} - V_I \\ \eta_l &= \Omega_{II} - \Omega_I \end{aligned} \tag{3}$$

The position of the instantaneous centre of relative rotation O' is given by:-

$$\begin{aligned} x_l &= \frac{v_l}{\eta_l} \\ y_l &= -\frac{u_l}{\eta_l} \end{aligned} \tag{4}$$

The yield line l is divided into N segments as shown in Fig. 3(a). The radial co-ordinates of different stations along the yield line are fixed, and the angular co-ordinates θ_i of different stations are varied and will be obtained after minimising the collapse load predicted by the energy equation of the arch ring. The relative displacement δ_i of the segment "i" is:-

$$\delta_i = r_i \eta_i \tag{5}$$

where r_i is the distance between the instantaneous centre of relative rotation O' and the midpoint of the segment "i" as shown in Fig. 3(b). If the local co-ordinate system shown in Fig. 4(a) is considered, the corresponding plastic strain rates are:-

$$\begin{aligned} \varepsilon_n^i &= \frac{\delta_i}{\Delta}\cos\gamma \\ \varepsilon_t^i &= 0 \\ \varepsilon_{nt}^i &= \frac{\delta_i}{2\Delta}\sin\gamma \end{aligned} \tag{6}$$

where Δ is the width of the narrow yielded zone between the two rigid blocks *I* and *II* and γ is the angle between the relative displacement δ_i and the yield line normal *n* as shown in Fig. 4(b). The principal strains are:-

$$\varepsilon_1^i = \frac{\delta_i}{2\Delta}(1+\cos\gamma)$$
$$\varepsilon_2^i = -\frac{\delta_i}{2\Delta}(1-\cos\gamma) \qquad (7)$$

The energy dissipation *W* per unit volume is given by:-

$$W = \sigma_1\varepsilon_1^i + \sigma_2\varepsilon_2^i \qquad (8)$$

and the dissipation W_i per unit length measured in the direction of the t-axis along the yield line is:-

$$W_i = \Delta b W$$
$$= \Delta b(\sigma_1\varepsilon_1^i + \sigma_2\varepsilon_2^i) \qquad (9)$$

where *b* is the dimension perpendicular to the (*n-t*) plane. The significant possible position of stresses on the yield surface is at the corner *A*, see Fig. 1. For point *A*, substituting $\sigma_1 = \sigma_{pt}$ and $\sigma_2 = \sigma_{pc}$, the dissipation is given by:-

$$W_i = \sigma_{pt}\frac{b\delta_i}{2\Delta}(1+\cos\gamma) + \sigma_{pc}\frac{b\delta_i}{2\Delta}(1-\cos\gamma)$$
$$= \frac{b\delta_i}{2\Delta}\left[\sigma_{pt}(1+\cos\gamma) + \sigma_{pc}(1-\cos\gamma)\right] \qquad (10)$$

The above equation (10) for energy dissipation is valid for both shear-tension ($-\pi/2 \leq \gamma \leq \pi/2$) and shear-compression ($\pi/2 \leq \gamma \leq 3\pi/2$) states. The energy dissipated W_c^i by the masonry in segment *i* of the yield line is given by:-

$$W_c^i = W_i l_i \qquad (11)$$

where l_i is the length of the segment *i*. The total energy dissipated W_c in the masonry over the entire length of the yield line is:-

$$W_c = \sum_{i=1}^{N} W_c^i$$

$$= F(\theta_i, U_m, V_m, \Omega_m) \qquad (12)$$

If the number of yield lines is m, then the total energy dissipated W_I is:-

$$W_I = \sum_{l=1}^{m} W_c \qquad (13)$$

External Work Done

The external work done by the external loads is calculated from:-

$$WE = \lambda \sum_{m=1}^{NB} \sum_{j=1}^{NPL} (P_{jv} V_{mj} + P_{ju} U_{mj}) \qquad (14)$$

where P_{jv} and P_{ju} are the vertical and horizontal components of the plane external load at joint j, V_{mj} and U_{mj} are the vertical and horizontal displacements of joint j in the rigid block m, NB is the number of rigid blocks, NPL is the total number of point loads on each block, and λ is the load factor. It is assumed that all loads acting on the arch increase in proportion to the load factor. The above summation accounts for all external loads on every rigid block in the system.

Optimum Solution

Equating the internal energy dissipated in the arch by the masonry to the external work done by the external loads, the capacity λ of the arch may be obtained:-

$$\lambda = f(U_m, V_m, \Omega_m, \theta_i) \qquad (15)$$

where the implicit function f is derived from equations (13) and (14).

Minimisation of the above energy equation to obtain the least upper bound could be done by numerical optimisation methods. The minimisation has been achieved using the Quasi-Newton algorithm routine which is built in the NAG library subroutines (NAG Ltd 1990). This routine requires the user to provide bounds on different variables and a starting point for each variable. The output after minimisation includes different rigid block motions and angular co-ordinates for each segment of the yield line which can be used to plot the shape of the yield line as the radial co-ordinates are fixed.

COMPARISON WITH PREVIOUS TEST RESULTS

The proposed technique has been used to analyse two brick arches that had been previously tested to collapse in the laboratory (Garrity 1995a). The dimensions, material properties and collapse loads of the two arches are given in Fig. 5. The four hinged mechanism shown in

Fig. 4 is adopted. A total of 10 segments equally distributed across the arch depth is considered. The effectiveness factors for masonry in compression and tension are assumed to be 0.5 and 0.005, respectively. The failure loads obtained from the present analysis are 2.13 kN and 3.35kN. These compare with experimental collapse loads of 1.6kN and 4.0kN, respectively.

CONCLUSIONS

Masonry arch bridges are highly complex structures. When assessing the strength of an existing damaged bridge and designing any strengthening measures, it is essential to use a method of analysis that can model the interaction of all the principal structural elements and can accommodate a range of potential defects. Failure to do so could lead to costly or unnecessary repairs. The mechanism or rigid block method of analysis is likely to be the least computationally expensive method currently available for masonry arch bridges that is also capable of accounting for a variety of defects and construction details. A new numerical method of rigid-block analysis has been presented. Reasonably accurate predictions of the strength of two model arches tested in the laboratory were obtained.

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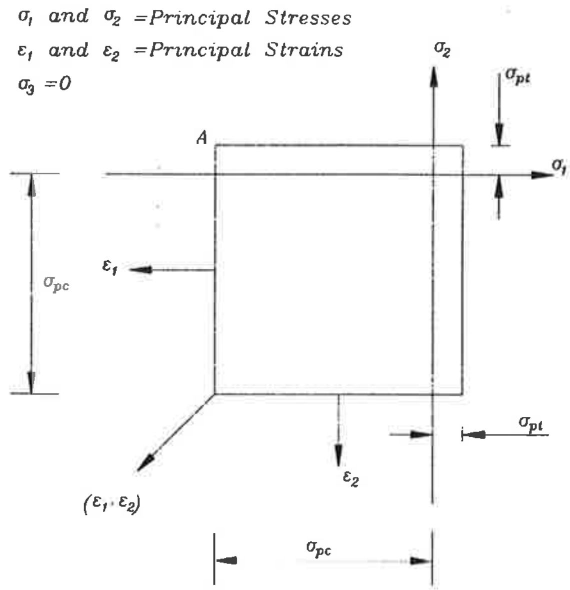


Fig. 1 Square Yield Surface for Masonry in Principal Axes

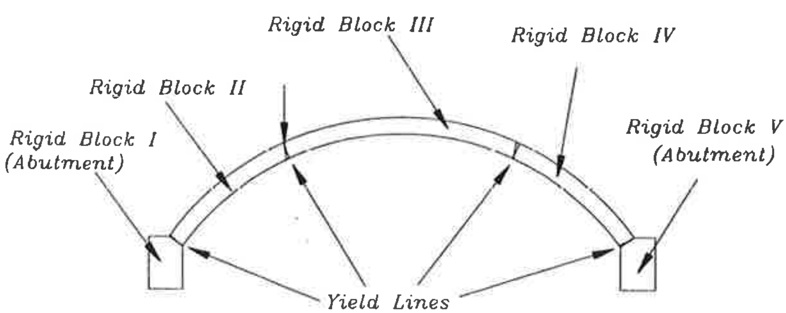
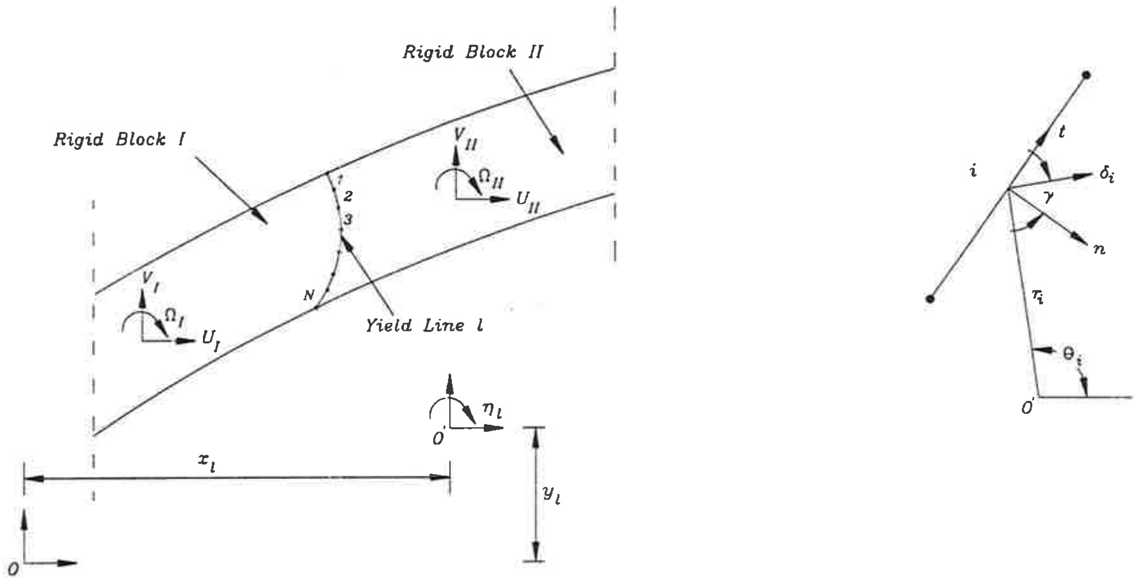


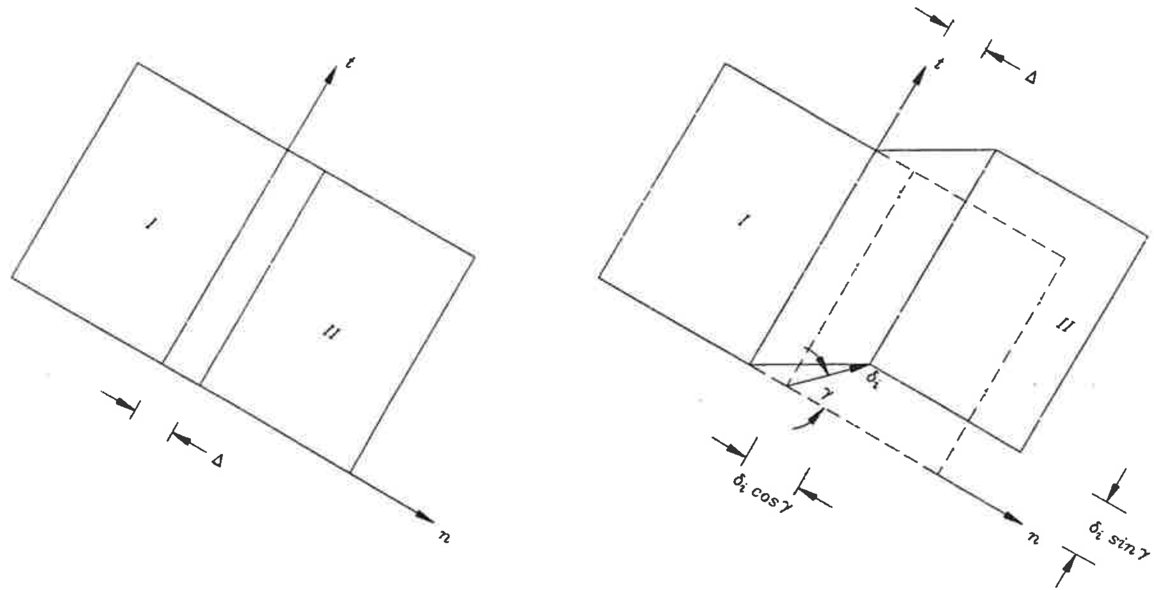
Fig. 2 Mechanism of Failure for Masonry Arch



(a) Yield line between Two Rigid Blocks

(b) Segment i of the Yield Line

Fig. 3 Calculation of Energy Dissipation along the Yield Line



(a) Idealised Discontinuity Zone at Segment i

(b) Relative Displacement Components

Fig. 4 Displacement Discontinuity Zone between Two Rigid Blocks

	Arch 1	Arch 2
f_c (N/mm^2)	4.2	6.6
f_t (N/mm^2)	0.21	0.33
ν_m	0.5	0.5
ρ_m	0.005	0.005
Capacity (kN)	1.6	4.0

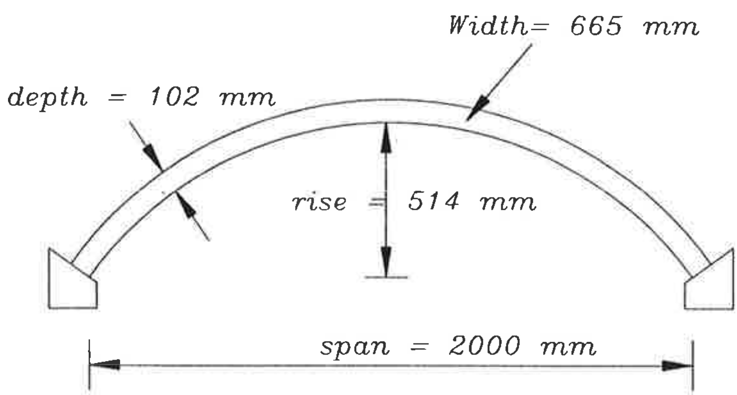


Fig. 5 Test Arch Details (Garrity 1995a)