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DEVELOPMENT OF ANALYSIS TECHNIQUE FOR DOUBLE WYTHE MASONRY WALLS

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ABSTRACT

This paper describes a simplified, but comprehensive, analysis technique for determining load-deflection diagrams for double wythe unreinforced masonry walls. The new method is simple to apply, requiring only minimal data input and modest computer hardware. The method was implemented on an electronic spreadsheet. The complete load-deflection curves for double wythe walls that were tested experimentally were plotted using this method. The results were found to be in good agreement with the experimental data.

BACKGROUND

The determination of the ultimate capacities of axially and laterally loaded masonry walls and reliable estimates of their deflections under various limit loads are essential in limit states design. The accurate prediction of these deflections and loads is a difficult task due to the complex block-mortar interaction behavior, the flexural tensile bond failure at the interfaces between mortar joints and masonry units, and the splitting failure mechanism inherent in concrete blocks under compressive loads.

Masonry wall cross sections and material properties vary along their height even when the walls are uncracked. Spatial variations occur due to property variations and dimensional changes (e.g., tapered face shells) of the concrete blocks, the clay bricks, the grout, and the mortar. Also, the variation in cross sectional properties of concrete blocks in the direction perpendicular to the plane of bending (e.g., staggered web locations) and face shell bedding, induces three dimensional aspects. The sudden change in material properties at the interface between the masonry units and the mortar causes the stresses in this region to be three dimensional and have a high variation.

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An analysis of masonry walls that would consider all the effects mentioned above is almost impossible or impractical. For example, the finite element method (FEM) was used, by the authors, to analyze a masonry wall subjected to an eccentric in-plane vertical load. For fine mesh sizes the computing time was excessive and for coarse meshes the localized stresses were not modeled correctly. A simple, yet accurate, method for generating the load-deflection relationships of masonry walls requiring minimal data input and modest computer hardware is needed. This was the motivation for this work.

INTRODUCTION

A method for the analysis of single wythe masonry walls was developed by the authors (Sakr and Neis, 1994). Formulae for calculating the effective flexural rigidity, EI, at different locations in an unreinforced masonry wall were developed. These formulae were based on the results of two dimensional FEM analyses of block prisms. The analysis method used these formulae in a one dimensional moment-area technique to analyze single wythe concrete block masonry walls. This paper extends the method to unreinforced double wythe masonry walls with an air cavity and metal ties.

According to some available experimental data (Papanikolas et al., 1989; Neis et al., 1991; Neis and Sakr, 1993; and Goyal et al., 1992) lateral deflection curves and ultimate strengths of double wythe masonry walls can be accurately predicted only if the influence of the following factors are considered:

i) eccentricity ratio, e/t
ii) slenderness ratio, h/t
iii) P-Δ effect
iv) properties and location of ties
v) cavity width
vi) nonlinear material properties
vii) masonry prism strength
viii) tensile bond strength of mortar

Not only are all of these effects considered in a rational way in this paper but the complete load-deformation curve for a masonry wall can be determined on a straightforward spreadsheet layout.

The objectives of this paper are:

i) To explain the structural mechanism of double wythe masonry walls,
ii) To develop a simple, yet accurate, analysis technique for double wythe masonry walls, and
iii) To verify the developed analysis technique by comparing its output with some experimental results.

SIMPLIFIED APPROACH TO THE BEHAVIOR OF DOUBLE WYTHE WALLS

The experimental results presented in Sakr (1995) proved that the structural behavior of a double wythe wall depends to a large extent on the type of ties used. The behavior can change from a single wall type behavior for walls with weak ties to a composite wall type behavior for the case of very stiff ties. In the case of metal ties, the ties usually have a limited stiffness since their size and shape should be suitable for a mason to use easily in construction.

It should be noted here that there are two stiffnesses of a tie that affect the structural response of a double wythe wall. These stiffnesses are the axial stiffness and the vertical shear stiffness of the tie.

Metal ties with sufficient axial stiffness should keep the two wythes together laterally such that the wythes have the same lateral deflection and hence the same curvature. Therefore, the following condition applies to the brick and block wythes in a double wythe wall built with many axially stiff ties:

$$\frac{M_{block}}{(EI)_{block}} = \frac{M_{brick}}{(EI)_{brick}} = \frac{1}{\rho_{wall}} \qquad [1]$$

Zmavc et al. (1992) used this approach in the analyses of double wythe walls and stated that the total moment is distributed between the two wythes according to their flexural stiffnesses.

Equation [1] provides a way to calculate the moment in the brick veneer if the curvature of the wall is known. For static equilibrium, the moment in the block wythe should equal to the total applied moment at the section under consideration less the moment in the brick wythe. This is given by [2].

$$M_{block} = M_{total} - M_{brick} \qquad [2]$$

The total applied moment, M_{total}, can be easily determined. In most cases, this will be the moment caused by any eccentric in-plane vertical load plus the moment due to any lateral out-of-plane load, such as wind pressure, w, as shown in Fig. 1.

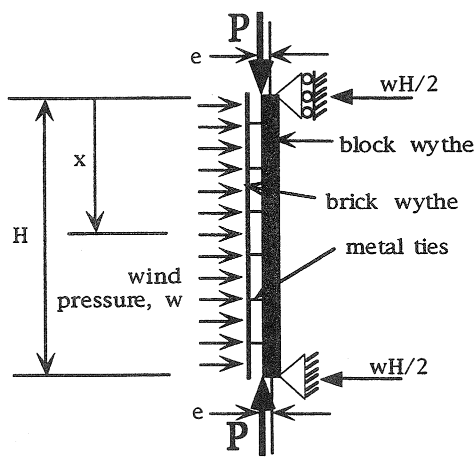

Fig. 1. Typical Applied Loads to a Double Wythe Masonry Wall.

The total moment is given by [3]. Any moment due to self weight could also be included but such terms will usually be negligible.

$$M_{total} = P.e + \frac{wx}{2}[H - x] \quad [3]$$

This moment can be calculated at different locations along the height of the wall. The curvature of the wall can then be calculated using any appropriate analysis technique and then the moment in the brick veneer and the moment in the block wythe can be calculated using [1] and [2], respectively.

The analysis technique for double wythe walls described herein is based on the assumption that the wall ties are uniformly distributed throughout the wall height, are sufficient in number, and have sufficient axial stiffness (and strength) to keep the two wythes together with the same curvature. If the ties cannot satisfy this condition, each wythe should be analyzed as a single wall subjected to the loads acting on it. The effects of the wall slenderness ratio and the load eccentricity ratio are inherent in the calculations of the moments. The following sub-sections explain how the other parameters mentioned in the introduction can be included in the analysis.

Effect of P-Δ

The P-Δ effect can be easily included by modifying [3] to the following form:

$$M_{total} = P(e + y) + \frac{wx}{2}[H - x] \quad [4]$$

The first term in [4] allows for a variation in the applied moment caused by the eccentric vertical load, P, due to the lateral out-of-plane deflection, y. The term (e+y) will be referred to in this paper as the virtual eccentricity, e^*. Since the deflection depends on the moment, and according to [4] the moment depends on the deflection, the solution must be iterative. The effect of the slenderness ratio is also inherent in the calculations of wall lateral deflection.

Effects of Type of Ties and Cavity Width

The shear stiffness of a tie and the cavity width, C_W, control the transfer of the vertical load from the loaded wythe to the unloaded one. As illustrated in Fig. 2(a), the shear forces, V_k, at the edges of a tie create a moment equal to $V_k C_W$. This moment reduces the moment in the loaded wythe.

For the case when the brick wythe is located on the tension side of the block wythe, Fig. 2(b), the axial force in the brick wythe is tension. This increases the axial force in the block wythe. However, since the tie shears cause tension in the brick and the tensile bond strength of mortar is usually small, the shears may well be negligibly small. For this reason, the authors suggest that the shear forces across the ties can be considered zero and ignored for the analysis of cavity walls where the brick wythe is in tension.

The moment created by the shear forces in the ties depends on the cavity width. The moment equals $\Sigma V_k C_W$, where ΣV_k is the sum of the shear forces in all the ties above the cross-section under consideration. Therefore, [2] can be modified to account for the shear stiffness of the ties as follows:

$$M_{block} = M_{total} - M_{brick} - \Sigma V_k C_w \quad [5]$$

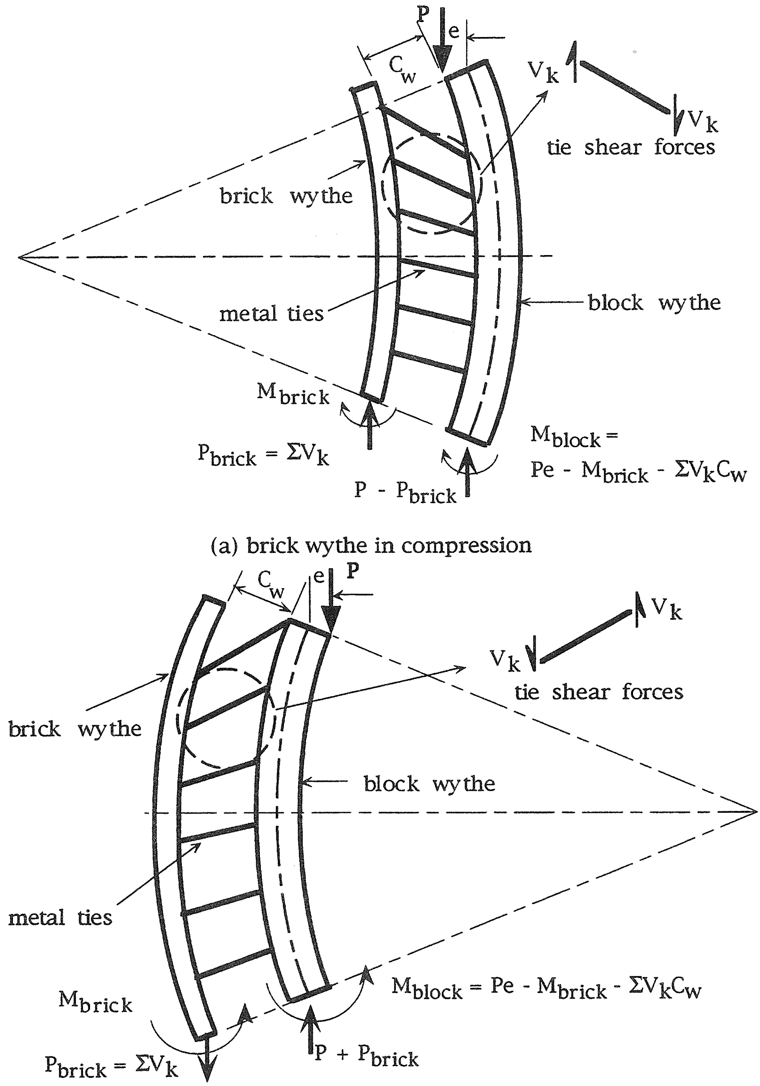

Fig. 2. Distribution of Internal Forces in a Double Wythe Masonry Wall.

The moment in the block wythe in the case where the brick wythe is in compression would be less than its value when the brick wythe is in tension since $\Sigma V_k C_w = 0$ if the brick wythe is on the tension side. Therefore, the ultimate load of a double wythe wall with the brick veneer in the compression side should be greater than the ultimate load of the same wall if the brick veneer is on the tension side. This behavior of double wythe walls was confirmed experimentally by Goyal et al. (1992) by testing double wythe walls subjected to load eccentricities towards to and away from the brick veneer. Equation [5] also explains why the ultimate strength of cavity walls is proportional to the cavity width, C_W, as was concluded by Sakr (1995).

The shear force in a tie depends on the relative vertical deflection between the faces of the two wythes at the tie location. This relative deflection is composed of two terms. The first term is due to the relative axial compression between the two wythes. Axial deformations can be estimated using the expression $\Delta = PL/AE$. The second term is due to the bending deformation effects and this term is difficult to calculate accurately.

As an approximate solution, the total relative vertical deflection between the two wythes can be estimated from simple geometric considerations regarding the deformed lengths of the brick and block wythes. As shown in Fig. 3, the arc length of the center line of the brick wythe is assumed to equal L and the arc length of the block wythe is (L-PL/AE). It is assumed that the ties hold the two wythes together and the wall is considered to be deformed in a circular shape with a radius equal to an average value of the loaded wythe, ρ_{avg}, from the base of the masonry wall up to the tie under consideration. This simplification should not have any significant effect on the calculations of the moments in each wythe.

Using the dimensions and deformed shapes illustrated in Fig. 3, the final deformed length of the two lines defining the adjoining faces of the brick and block wythes can be derived as [6] and [7].

$$h_{brick} = \left[\frac{L}{\rho_{avg} - \frac{t_{bl}}{2} - C_w - \frac{t_{br}}{2}} \right] \left(\rho_{avg} - \frac{t_{bl}}{2} - C_w \right) \qquad [6]$$

$$h_{block} = \left[\frac{L - \frac{PL}{AE}}{\rho_{avg}} \right] \left(\rho_{avg} - \frac{t_{bl}}{2} - C_w \right) \qquad [7]$$

The relative deflection between the two ends of a tie located at height L in the wall is the difference between [7] and [6], namely [8].

$$\Delta_{tie} = \left(\rho_{avg} - \frac{t_{bl}}{2} - C_w \right) \left[\frac{L}{\left(\rho_{avg} - \frac{t_{bl}}{2} - C_w - \frac{t_{br}}{2} \right)} - \frac{L - \frac{PL}{AE}}{\rho_{avg}} \right] \qquad [8]$$

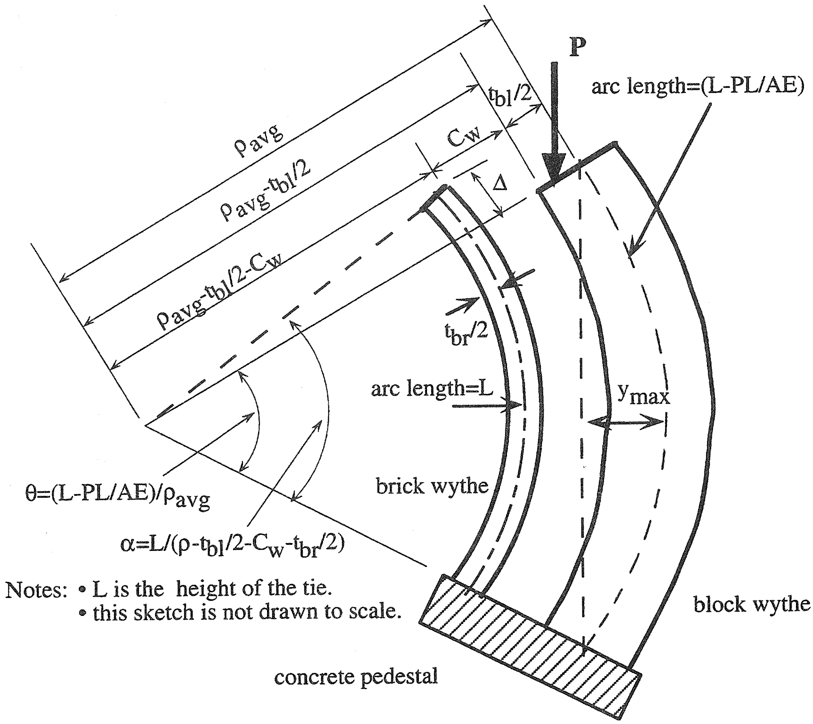

Fig. 3. Approximate Geometry of a Deformed Double Wythe Masonry Wall.

According to [8], the shear deformation of a tie and hence the shear force in a tie is based on: the average curvature of the wall; the location of the tie along the wall height, L; the axial stiffness of the loaded wythe, AE; the cavity width, C_w; and the applied vertical load, P. The effect of the virtual eccentricity, e^*, is inherent in the calculation of the curvature.

The higher the ties above the base, the larger the shear forces in them. At small values of load and virtual eccentricity the vertical deflection and curvature of the loaded wythe are not large enough to create a significant amount of shear force in the ties. As axial and curvature strains increase, shear forces start to develop in the ties with maximum values in the top row of ties. If a tie does not fail by causing crushing of the mortar, the shear force in the tie will reach its limit when the tie yields. The shear force in the tie then remains essentially constant as plastic deformations of the tie take place.

An experimental study on the behavior of four types of ties under shear forces was conducted by Gowrishankar and Neis (1993). Their results provided the shear load-deflection relationships for the ties considered in this study.

The approximate yield shear load per tie and the corresponding deflections for various tie types and cavity spacings are shown in Table 1. It can be noted that the T-Section and the Flat ties have a relatively high shear stiffness in comparison to the PMRI ties (tie types are described in Neis and Sakr, 1993).

Table 1. Estimated Yield Shear Loads and Deflections for Various Tie Types.

Tie Type and Cavity Width	Yield Shear Load, V (kN)	Yield Deflection (mm)
PMRI (76 mm)	0.6	2.5
PMRI (114 mm)	0.6	2.5
T-Section (76 mm)	3.1	4.0
Flat (38 mm)	2.1	5.5
Flat (76 mm) estimate	1.0	22

Note: 25.4 mm = 1 inch, 1 kN = 1/4.448 kips

The values in Table 1 together with [8] can be used to calculate the shear forces in the ties. The shear force in any tie can be taken equal to its yield shear force if the relative deflection between the tie's ends, Δ_{tie}, is greater or equal to the deflection listed in Table 1. Otherwise, the shear force can be estimated from a linear elastic relation using the values given in Table 1.

Effects of Nonlinear Material Properties, Masonry and Bond Strengths
To account for nonlinear material properties, masonry strength, and the tensile bond strength of mortar, a detailed FEM stress analysis was used to develop formulae for the effective flexural rigidity, EI, in various segments of a masonry wall (Sakr and Neis, 1994). A masonry couplet consisting of two half-height hollow concrete blocks with a face shell mortar joint between them was analysed for a range of values of vertical in-plane load, P, acting at a number of values of virtual eccentricity, e*. The results were used to develop empirical formulae for EI for three segments (middle block, end block and mortar region) of the masonry couplet. The formulae in non-dimensional form are as follows:

For the mortar regions:

$$\frac{EI_{effective}}{EI_{gross}} = 11.40 \times 0.00003^{\Psi}, \leq 1.0 \qquad [9.1]$$

For the end-block regions:

$$\frac{EI_{effective}}{EI_{gross}} = 2.14 \times 0.005^{\Psi}, \leq 1.0 \qquad [9.2]$$

For the mid-block regions:

$$\frac{EI_{effective}}{EI_{gross}} = 1.96 \times 0.011^{\Psi}, \leq 1.0 \qquad [9.3]$$

In [9.1], $\Psi = \sqrt{\left(\frac{P/P_o}{0.5}\right)^2 + \left(\frac{e^*/t}{1.3}\right)^2}$ while in [9.2] and [9.3], $\Psi = \sqrt{\left(\frac{P/P_o}{0.5}\right)^2 + \left(\frac{e^*/t}{3.5}\right)^2}$

In the above formulae, EI_{gross} is the flexural rigidity of the block wythe in the plane of bending based on a section through the concrete blocks and P_o is given by $A_m f'_m$, where A_m is the mortar bed area and f'_m is the ultimate strength of a masonry prism. The above formulae were developed based on block thickness of 140 mm (5.5 in.). However, the formulae should be applicable to other thicknesses.

DEVELOPMENT OF THE ANALYSIS TECHNIQUE

The analysis accounts for the eight factors mentioned in the introduction and uses the effective EI relationships in a moment-area technique to find the curvature of the wall and the moments in each wythe. The solution method must be iterative if the P-Δ effects and nonlinear material properties are to be considered. The analysis procedure can be summarized as follows:

a) Divide the wall into segments from top to bottom with nodes at the end of each segment. Divide each block into four segments and each mortar joint into one segment. This is the one dimensional discretization of the wall. In subsequent steps the subscript "i" will refer to a node number. The distance of each node from the top of the wall is x_i.
b) Start with the effective EI equations [9.1] to [9.3] as developed from a two or three dimensional finite element analysis. This includes geometric and nonlinear material effects.
c) Assume initial lateral deflections, y_i (say $y_i = 0$) at each node.
d) Calculate the total applied moment at the end of each segment using [3].
e) For each tie level, at a height L_k measured from the bottom of the wall, calculate the tie shear deflection, Δ_{tie_k}, using [8].
f) Use the value of Δ_{tie_k} to estimate the shear forces in the ties, V_k, using Table 1. Accumulate the shear force for each tie level from top to bottom.
g) Calculate the moment in each brick wythe segment, M_{brick_i}, using [1]. To allow for the effect of the mortar tensile bond strength in the brick wythe, limit the maximum moment in the brick wythe, M_{brick}, to $Z_{br} f_t$ (the brick section modulus times the tensile bond strength of the mortar).
h) Calculate the moment in the block wythe segment, M_{block_i}, using [5].
i) Calculate the ratio $EI_{effective}/EI_{gross}$ for in each segment using [9.1] to [9.3] and then calculate the effective flexural rigidity.
j) Calculate the wall curvature for each segment, ρ_i using [10].

$$\rho_i = \frac{(EI)_{block(i)}}{\left[\frac{M_{block(i)} + M_{block(i-1)}}{2}\right]} \qquad [10]$$

k) Calculate the area of the elastic moment diagram for each segment, ω_i, by multiplying $1/\rho_i$ by the length of the segment, as follows:

$$\omega_i = \frac{\left(M_{block_i} + M_{block_{i-1}}\right)}{2EI_{block_i}}\left(x_i - x_{i-1}\right) \quad [11]$$

l) Using the areas of the elastic moment diagram, ω_i, calculate the slope of the wall at each node, θ_i.

$$\theta_i = \theta_{i-1} - \omega_i \quad [12]$$

m) Using the slopes, calculate the modified lateral deflections at each node, y_{mod_i}, using the relation:

$$y_{mod_i} = y_{mod_{i-1}} + \frac{\theta_{i-1} + \theta_i}{2}\left(x_i - x_{i-1}\right) \quad [13]$$

n) Use these lateral deflections as the new deflections for the next iteration. When P is smaller than the wall ultimate load the solution should converge after only a few iterations.
o) Incrementally increase the load P and repeat steps "c" to "n" to produce a complete load-deflection curve. Wall failure is implied when the deflections diverge as happens in a buckling failure. The maximum compressive stress in the block wythe should be monitored since it can limit the ultimate load if a compression failure should occur.

IMPLEMENTATION AND APPLICATIONS

The calculations that are required in the one dimensional analysis technique described above were automated into a spreadsheet computer program.

To verify the validity of this analysis technique, the unreinforced double wythe masonry walls tested by Sakr (1995) were analyzed. Figures 4 and 5 show the load-deflection curves for two different wall configurations. The experimental data shown in each figure are for three identical walls. The numerical results showed a good agreement with the experimental data.

SUMMARY AND CONCLUSIONS

A one dimensional analysis technique was developed for unreinforced masonry walls. The analysis technique accounted for many factors including material nonlinearities, mortar bond strength, type of ties, cavity width, and P-Δ effects. The nonlinear material properties were included by developing formulae for the effective flexural rigidities along the wall. The required input data and the computer hardware required were minimal which make this analysis method suitable for use in a design office to predict the ultimate load of a masonry wall and its maximum deflection under service loads. The method can be extended to include reinforced masonry walls.

The structural analysis technique presented herein is not an exact technique but it is based on rational, justifiable assumptions. The output of the analysis of masonry walls using the developed technique compared favorably with the experimental results. The analysis method is accurate and extendible in predicting the ultimate load and the load deflection relationship of masonry walls. The spreadsheet procedure is simple and easy.

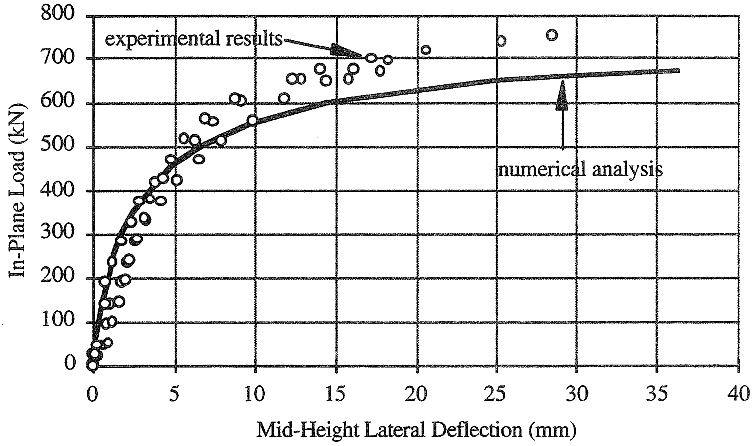

Fig. 4. Comparison between the Numerical and Experimental Results for Double Wythe Walls (h/t=20, e/t=1/6, C_W=76 mm, and PMRI ties).

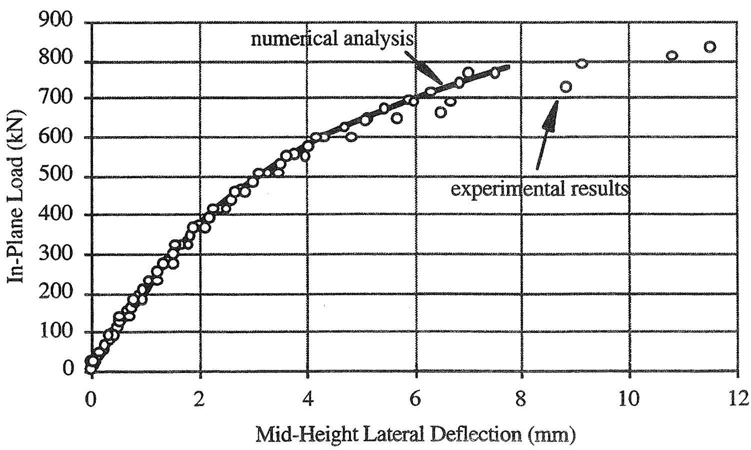

Fig. 5. Comparison between the Numerical and Experimental Results for Double Wythe Walls (h/t=20, e/t=1/3, C_W=76 mm, and T-Sec. ties).

Note: 25.4 mm = 1 inch, 1 kN = 4.448 kips.

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