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Deflection of Reinforced Masonry Wall Under Out-of-Plane Loads

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ABSTRACT

Although deformation is an important criterion in strength design procedures, only a limited number of experimental studies have been performed to evaluate the deformation of reinforced masonry walls under out-of-plane loads. The accuracy of the currently proposed methods for predicting out-of-plane deformation of masonry are shown to be unsatisfactory when compared to the various experimental data available in the literature. This paper presents a new and practical method for predicting lateral deformation of reinforced masonry walls under out-of-plane loads. The proposed method offers a good compromise of behavioral accuracy and simplicity of design. Reduction in stiffness due to cracking under service loads and tension stiffening effect between cracks have been incorporated in the proposed method. Based on the comparison between the proposed method and all other methods available for predicting the out-of-plane deformation of masonry walls, the proposed method of predicting masonry deformation was within a +/- 20 % of the experimental data.

INTRODUCTION

With the widespread acceptance of the strength design method for reinforced masonry

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structures and the realistic recognition of the additional strength of masonry in compression according to the nonlinear relationship between stress and strain, sections could be made smaller and members become more slender. Such members deflect a greater amount than those designed under the working stress method. Therefore, a well-defined criteria for evaluating the deflection of reinforced masonry structures designed by the strength method is a viable need for serviceability requirements of masonry structures where deformations do not adversely affect their use or appearance.

Numerous experimental programs have been performed to study the effect of various parameters on the design strength of reinforced masonry elements (ACI-SEASC Task Committee 1982, Abboud 1987, Abboud et al. 1987 and 1990, Hamid et al. 1989 and 1990, Horton et al. 1990). However, comparatively few of these experimental programs have evaluated the deflection of reinforced masonry walls under out-of-plane loads. Three proposed methods currently are available in the literature for calculating the short-term deflection of masonry walls under out-of-plane loads (ACI-SEASC Task Committee 1982, Abboud 1987, Horton et al. 1990). The ACI-SEASC Task Committee Method was adopted by the Uniform Building Code, 1985. The accuracy of the currently proposed methods are evaluated in reference (Abboud et al. 1993 and Lu 1993) and are shown to be unsatisfactory when compared to the various experimental data available in the literature. This paper presents a new and more practical method for predicting lateral deformation of reinforced masonry walls under out-of-plane loads. The test data, obtained in the Task 3.2(a) Program of the Joint U.S.-Japan Technical Coordinating Committee on Masonry Research (TCCMAR) (Hamid et al. 1989 and Abboud et al.1994), is used to develop the mathematical model of the proposed method. The proposed method, together with the other three methods, are qualitatively and quantitatively compared with TCCMAR test data

TEST DATA

The test data, obtained in the Task 3.2(a) Program of TCCMAR, is used herein to develop the mathematical model for the proposed method in calculating the lateral deflecting of reinforced masonry walls under out-of-plane loads. Task 3.2(a) Program (Hamid et al. 1989) is one of the research projects of the TCCMAR aimed at studying the behavior of reinforced block masonry walls under out-of-plane monotonic and cyclic loads. In total, fourteen full-scale simply supported reinforced concrete block masonry walls were constructed and tested. The behavior of vertically spanning reinforced block masonry walls were evaluated experimentally. The effects of different parameters on deflection, flexural strength, ductility and failure modes were investigated. The out-of-plane deflection profile of each wall was documented, The tested wall panels were 1.22 m (4 ft.) wide by 3 m (9.8 ft.) long by 0.15 m (0.5 ft.) thick, and centrally reinforced. The wall panels were tested as a simply supported element at the top and bottom under out-of-plane loading. Two equal line loads were applied to the face of the wall panel at the third points to provide a pure bending zone in the middle portion of the span.

Data from wall panels, that experienced shear failure, were eliminated from this study. Only those wall panels tested under monotonic load and which had flexural failures were chosen. Relevant properties of the wall panels and terminology are listed in Table 1. The first two columns in Table 1 provide identification data for the wall panels.

Table 1 Test Matrix

Wall No.	Wall Designation(b)	Vertical Steel		Grouting	Rebar Location(c)	E_m Ksi
		Amount	Percent(a)			
W1	6PLFG5M	2#5	0.23	Fully	C	2080
W2	6PLFG7M	2#7	0.44	Fully	C	1940
W3	6PLPG5M	2#5	0.23	Partially	C	1550
W4	6MCPG6M	2#5	0.23	Partially	C	1345
W5	6PLFG3M	6#3	0.24	Fully	S	1940
W6	4.5PLFG4M	2#4	0.19	Fully	C	1440

(a) Vertical steel percent = A_s / bt, where b is the actual wall width and t is the actual wall thickness
(b) The definition of 6PLFG5M or 6MCPG5M is:
6 = nominal wall thickness in inch;
PL = Portland cement;
MC = Masonry cement;
FG = Fully grouted;
PG = Partially grouted;
5 = Vertical reinforcing bar size (i.e., #5); and
M = Monotonic loading
(c) S = Double reinforcement (Staggered); and
C = Centrally reinforced

DEVELOPMENT OF PROPOSED METHOD FOR MASONRY

Assumptions for the Proposed Method

Elastic deflections can be expressed in the general form:

$$\Delta = \frac{\beta}{EI} \qquad [1]$$

where EI is the flexural rigidity and β is a coefficient based on load, span and support arrangement. The particular problem in reinforced masonry structures is, therefore, the determination of the appropriate flexural rigidity EI for a member consisting of masonry units, grout, mortar and reinforcement, with properties and behavior as widely different as the individual material.

The basic assumptions made for the theoretical development of the flexural stiffness of a reinforced masonry wall, subjected to out-of-plane load, are associated with:

1] Tension Stiffening: An idealized moment-curvature relationship of reinforced masonry member is similar in shape and boundary conditions to that of reinforced concrete and is shown in Fig. 1. The slope of the curve represents the flexural rigidity of the section. If the maximum moment is sufficiently small that the stress in the masonry does not exceed the modulus of rupture, no flexural tension cracks will occur and the slope (EI) is based on an uncracked section (Fig. 1). When the load causes the stresses to exceed the flexural tensile strength of the masonry and assuming that there is no tension strength for the masonry in the tension zone, the slope abruptly reduces to the fully cracked section slope, $(EI)_{cr}$. However, as indicated by various researchers (Abboud 19987 and 1993, and Hamid et al. 1989 and 1990, Horton 1990), the slope of the moment-curvature curve is continuous and there is no abrupt change. The smooth transition of the moment-curvature from uncracked section to cracked section is attributed to the existence of tensile stiffening in reinforced masonry. The effect of tension stiffening is considered in the proposed method by developing an empirical interpolation between curvatures (or deflections) using the boundary values of noncracked and cracked section properties.

2] The Contribution of the Mortared Area in the Tensile Zone to the Section Moment of Inertia is Negligible: Various researchers have shown in their flexural element tests that the cracks are usually initiated at the bed joints under very low loads and have depth approximately equal to the depth of the face shell. A similar observation can be drawn from TCCMAR test data. Therefore, the tensile force required to resist bending in uncracked sections is provided primarily by the grout. The moment of inertia of the critical section, therefore, is calculated on the basis of section properties of the wall with only one face shell. The modified moment of inertia is noted as I_g^f.

3] Variable Modulus of Elasticity: The flexural stiffness variation of the TCCMAR wall test panels with the bending moment is idealized in Fig. 2. The flexural stiffness of reinforced masonry walls decreases from the value of the uncracked section, $(EI)_g^f$, to that of totally cracked section, $(EI)_{cr}$. Since for these two specific limits, the moment of inertias are known as I_g^f and I_{cr}, respectively, the corresponding values of the modulus of elasticity of masonry, defined as $(E_m)_g^f$ and $(E_m)_{cr}$, can be obtained.

Before the first grout crack occurs, the value of the modulus of elasticity of the gross cross section, $(E_m)_g^f$ can be obtained from known test data, M_{cr} / Δ_{cr}, as:

$$(E_m)_g^f = \frac{\beta M_{cr} L^2}{\Delta_{cr} I_g^f} \qquad [2]$$

The parameters on the right side of the above equation are all known for each wall panel.

When the section is totally cracked, the modulus of elasticity decreases to a value of:

$$(E_m)_{cr} = \frac{\beta M_u L^2}{\Delta_u I_{cr}} = \alpha (E_m)_g^f \qquad [3]$$

where α is a reduction factor and is expressed as the ratio of the two extreme values of E:

$$\alpha = \frac{\beta M_u L^2}{\Delta_u I_{cr} (E_m)_g^f} = \frac{M_u I_g^f \Delta_{cr}}{M_{cr} I_{cr} \Delta_u} \qquad [4]$$

The reduction factor α has been determined for the TCCMAR wall panels and summarized in Table 2. The reduction factor is significantly effected by the extent of grouting and unit size, whereas reinforcement ratio has no effect (Table 2).

Table 2 List of Reduction Factors (α)

Wall No.	P_{cr}, lb	P_u, lb	Δ_{cr} in.	Δ_u in.	I_g in^4	I_g^f in^4	I_{cr} in^4	α
W1	1820	2977	0.19	5.85	705	343	39	0.48
W2	1630	4371	0.19	6.07	705	343	57	0.51
W3	704	2715	0.08	6.57	899	343	63	0.26
W4	651	2660	0.14	1.92	899	343	58	0.38
W5	1334	3545	0.25	6.48	705	343	54	0.52
W6	1045	1431	0.14	4.81	367	169	26	0.25

Mathematical Model

The basis for the development of the new deflection method is the incorporation of a variable flexural stiffness factor rather than a variable moment of inertia, as the primary parameter for predicting wall deflection. Prior to the initial cracks, prediction of reinforced masonry deflection is defined by the gross member stiffness, $(EI)_g^f$ properties. After the section cracks, the deflection is controlled by an effective stiffness, $(EI)_{eff}$. When the masonry reaches its moment strength M_n, the stiffness reaches $(EI)_{cr}$. Because the modulus of elasticity (E_m) for masonry is not constant as well as the moment of inertia (I) changes as the section undergoes cracking, these two factors can not be separated out from the flexural stiffness (EI). Therefore, the new method uses effective stiffness $(EI)_{eff}$ instead of using a single effective modulus of elasticity $(E_m)_{eff}$ or an effective moment of inertia $(I)_{eff}$ to estimate lateral deflectioñ of reinforced masonry.

A trilinear moment-curvature model with three identification points (cracking moment, yielding moment, and ultimate moment) has been widely used to represent the moment-curvature relationship of flexural members. Although this trilinear model is numerically simple to formulate, a sudden change in the slope or stiffness introduces numerical difficulty.

Fig. 2 clearly shows that when stiffness changes from the upper bound value, $(EI)_g^f$, to the lower bound value, $(EI)_{cr}$, the slope of the curve changes continuously without sudden change. The newly proposed deflection method, which utilizes a variable flexural stiffness (EI) after the section cracks, will obtain the value EI by interpolating between the upper and lower bounds. The equation for effective stiffness $(EI)_{eff}$ after section cracking is given as:

$$(EI)_{eff} = (EI)_g^f \, R + (EI)_{cr} \, (1 - R) \qquad [5]$$

where:

$(EI)_g^f$ = $(E_m)_g I_g^f$; modified gross sectional stiffness
$(EI)_{CR}$ = $\alpha \, (E_m)_g \, I_{cr}$; cracked sectional stiffness
R = f (X); stiffness intrepolation factor
X = $(M_n - M_a) / (M_n - M_{cr})$; moment ratio
M_n = section nominal moment strength
M_a = service load moment acting at the condition under which deflection is computed
M_{cr} = cracked service moment
E_m = modulus of elasticity o masonry in compression

The above equation is a weighted average stiffness between the two bounded theoretical stiffness values of $(EI)_g^f$ and $(EI)_{cr}$. The equation can be used for the full range of service load level, from the cracking moment, M_{cr}, to the section nominal moment strength, M_n. This overcomes the numerical difficulty generated by a sudden change in the slope of the trilinear model.

As for the interpolation factor, R, from Equation 5, can be expressed :

$$R = \frac{(EI)_{eff} - (EI)_{cr}}{(EI)_g^f - (EI)_{cr}} \qquad [6]$$

All of the components on the right side of equation 6 are known from the test data and as shown by the equations below:

$$(EI)_{eff} = \frac{\beta M L^2}{\Delta} \qquad [7]$$

$$(EI)_g^f = \frac{\beta M_{cr} L^2}{\Delta_{cr}} \qquad [8]$$

$$(EI)_{cr} = \frac{\beta M_n L^2}{\Delta_u} \qquad [9]$$

where:

Δ	= test data, midheight lateral deflection
M	= test data, midheight lateral bending moment under applied load
L	= height of tested masonry wall.

From TCCMAR test data, a regression model of the stiffness interpolation factor (R) vs. moment ratio, (X), is idealized as shown in Fig. 3. It is clearly seen that when $M = M_{cr}$ and before the section cracks, R = 1 and $(EI)_{eff} = (EI)_g^f$; while as M increases from M_{cr} to M_n and the grout starts cracking, R decreases to 0 and $(EI)_{eff}$ is approximated by $(EI)_{cr}$. This R vs. X plot can be approximated by the shape of a polynomial curve and the mathematical model for R can be established as :

$$R = \alpha_1 X + \alpha_4 X^4 \qquad [10]$$

The fourth order expression is used to increase the accuracy in estimating stiffness. Based on regression analysis of least squares method on the TCCMAR test panels, the average value for coefficients α_1 and α_4 are 0.4 and 0.6, respectively.

Proposed Method

The lateral deflection of reinforced masonry members is obtained by utilizing the concept of a variable effective flexural stiffness. The proposed effective flexural stiffness is bounded between uncracked and cracked flexural stiffness. Therefore, the proposed deflection procedure is as follows:

Before the reinforced masonry wall cracks, the lateral deflection is:

$$\Delta = \frac{\beta M L^2}{(EI)_g^f} \qquad M \leq M_{cr} \qquad [11]$$

After the reinforced masonry wall starts to crack, the lateral deflection is:

$$\Delta = \frac{\beta M L^2}{(EI)_{eff}} \qquad M_{cr} < M \leq M_n \qquad [12]$$

where

$(EI)_{eff}$	$= (EI)_g^f R + (EI)_{cr} (1 - R)$; effective flexural stiffness
$(EI)_{cr}$	$= \alpha (E_m)_g I_{cr}$; cracked section stiffness
α	= modulus of elasticity reduction factor
	= 0.5 for fully grouted section

= 0.32 for partially grouted section

R = Stiffness interpolation factor

= 0.4X + 0.6X

X = (M_n - M_a) / (M_n - M_{cr}), moment ratio

M_n = nominal moment strength

M_a = service load moment acting at the condition under which deflection is computed

M_{cr} = cracked service moment

E_m = modulus of elasticity of masonry in compression

I_g^f = modified gross section moment of inertia with one face shell only

I_{cr} = cracked section moment of inertia.

COMPARISON OF METHODS

For comparison purposes the three currently available methods in the literature for calculating the short-term deflection of reinforced masonry walls under out-of-plane loads, are summarized below:

The ACI-SEASC Task Committee Method: The deflection prediction procedure presented by the Task Committee is based on a bilinear load-deflection relationships. The gross moment of inertia was used to compute the uncracked section deflection, and additional deflection beyond the cracking load was computed based on the cracked moment of inertia. The proposed mid-height deflection formula for a simply supported wall subjected to uniform pressure is:

$$\Delta = \frac{5 M_a L^2}{48 E_m I_g} + \frac{5 (M_a - M_{cr}) L^2}{48 E_m I_{cr}} \qquad M_{cr} < M_a < M_n \quad [13]$$

where:

M_a = service bending moment at the midheight cross-section, including P-Δ effects.

M_{cr} = cracking moment strength of the masonry wall.

M_n = moment strength of the masonry wall.

E_m = modulus of elasticity of masonry.

I_g = gross moment of inertia of the wall cross-section.

I_{cr} = cracked moment of inertia of the wall cross-section.

L = height of the wall.

Abboud Method: An expression for an effective moment of inertia for reinforced concrete block masonry walls, I_{em}, was obtained by curve fitting and is shown below.

$$I_{em} = (I_g - 2I_{cr}) \left(\frac{M_{cr}}{M_a} \right)^4 \qquad M_{cr} < M_a < M_y \qquad [14]$$

where:

M_y = yield moment strength of the masonry wall.
M_a = service bending moment at the condition under which deflection is computed.
M_{cr} = cracking moment strength of the masonry wall.
I_g = gross moment of inertia of the wall cross-section.
I_{cr} = cracked moment of inertia of the transformed wall cross-section.

Horton Method: Interpolation was performed between two bounded deflection values, which were the uncracked and cracked member deflections. Equations were presented for the flexural deformation of the reinforced masonry walls subjected to out-of-plane loads. The equations are as follows.

$$\Delta = \Delta_{cr} (1 - \alpha) \qquad [15a]$$

$$\alpha = \left(\frac{M_{cr}}{M_a} \right)^2 \left(1 - \frac{I_{cr}}{I_g} \right) \left(2 - \frac{M_{cr}}{M_a} \right) \qquad [15b]$$

where:

Δ = maximum deflection
Δ_{cr} = maximum deflection with cracked properties.
α = tension stiffening coefficient.
M_a = bending moment due to all applied loads.
M_{cr} = bending moment at first cracking.
I_g = gross section moment of inertia with one face shell only.
I_{cr} = cracked section moment of inertia.

The above three methods along with the proposed method for calculating the lateral deflection of reinforced masonry are compared using the TCCMAR test data presented in Table 1. The actual load-deflection curve versus that computed by the proposed methods are plotted in figure 4 through 9 in order to provide a qualitative comparison. A 20 percent offset was drawn to illustrate a reasonable bound for the deflection prediction.

Since the properties of the wall section before cracking are well defined by the gross cross section, the deflection calculation by all four methods begins at the initiation of cracking as shown in figures 4 through 9. For this starting point of the comparison, the load at first cracking was obtained from the test data, while the modulus of elasticity of masonry, E_m, was obtained from prism testing.

From Figures 4 through 9, it can be observed that none of the currently proposed methods (The ACI-SEASC Task Committee, Abboud, and Horton methods) possessed the ability to predict lateral deflection of the reinforced masonry walls under out-of-plane loads within a reasonable degree of accuracy. All of these methods underestimated the deflection beyond the 20 percent offset. The ACI-SEASC Method which was generated from the concept of superposition of the uncracked deflection and cracked section deflection, underestimated the deflection of the wall after cracking by 30 to 50 percent. As cracking and deflection of the walls increased, ACI-SEASC underestimated the wall deflection by more than 70 percent. This increasingly inaccurate prediction capability be attributed to the method's assumption of a constant modulus of elasticity for the cracked section and to the direct superposition of the uncracked section deflection component onto the cracked section components in the bilinear model for lateral deflection.

The Abboud method, which was developed from curve fitting of test data on small scale masonry wall panels under out-of-plane load, showed a tendency to replicate the shape of the load-deflection curves. However, this method underestimates the deflection by more than 30 percent after cracking of the wall, and then overestimates the deflection as cracking and deformation of the wall increased. This is due to the fact that the proposed equation for predicting the effective moment of inertia of the section does not utilize the lower bound value for a cracked section. Further, the method is very sensitive to the ratio of cracked moment to ultimate moment, but did show a reasonable prediction of the wall deflection where this ratio was higher than 0.6.

Horton's method was developed from the bilinear model of deflection, showed similar behavior to the ACI-SEASC Method. The Horton Method has indirectly incorporated the effect of tension stiffening to the section stiffness. The underestimation of the deflection prediction by this method can be attributed mainly to the assumption of a constant modulus of elasticity after the section is cracked and to the characteristic behavior of the bilinear model for deflection.

The new proposed method presents the most reasonable lateral deflection prediction of the three methods being compared. The polynomial curve model demonstrated the advantage of the proposed over the bilinear models of the ACI-SEASC and Horton Methods.

CONCLUSION

A new practical method is proposed in this paper for predicting lateral deflection of reinforced masonry walls under out-of-plane loads. The proposed method offers a good compromise of behavioral accuracy and simplicity of design. Reduction in stiffness due to cracking under service loads and tension stiffening effect between cracks have been incorporated in the proposed method. Based on the comparison between the proposed method and all the other methods available for predicting out-of-plane deformation of masonry walls, the proposed method of predicting masonry deformation was within +/- 20 % of the experimental data.

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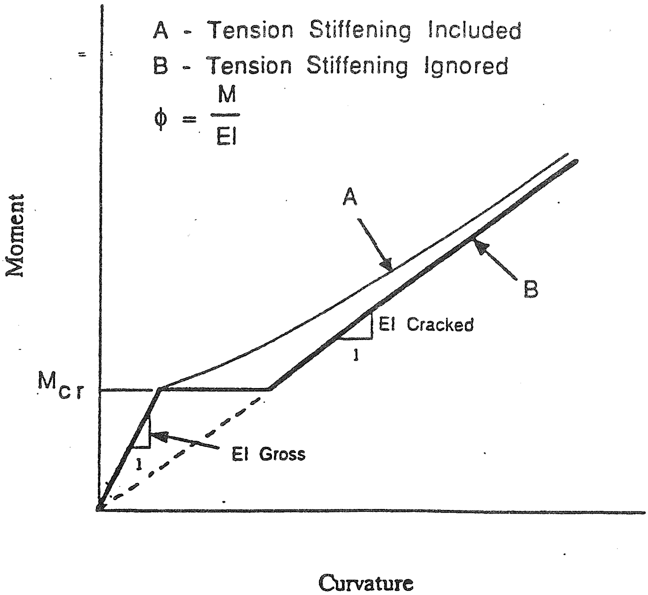

Fig. 1 Idealized Moment-Curvature Diagram.

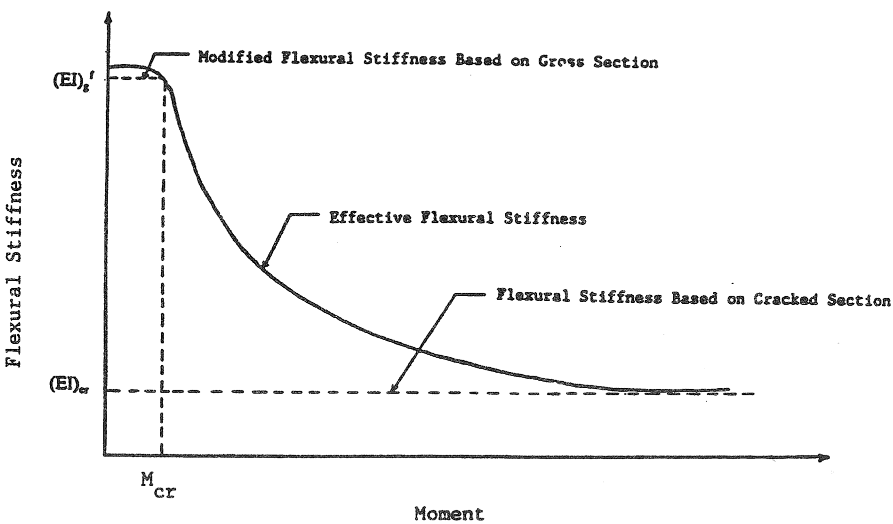

Fig. 2 Idealized Flexural Stiffness vs. Moment Curve for TCCMAR Wall Test Panels.

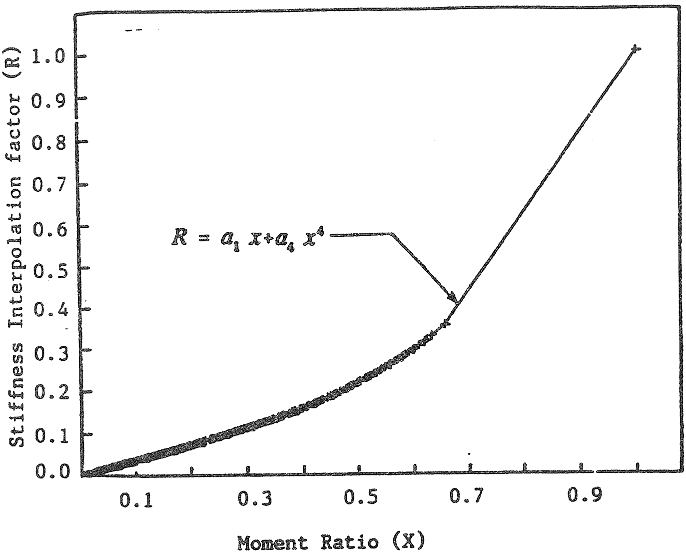

Fig. 3 Idealized Regression Model of Stiffness Interpolation Factor vs. Moment Ratio.

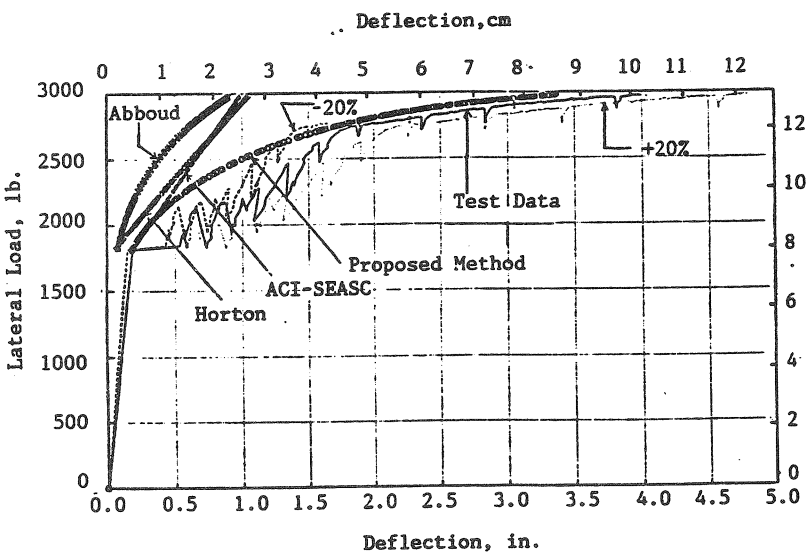

Fig. 4 Comparison of Methods with TCCMAR Test Panel Wall W1

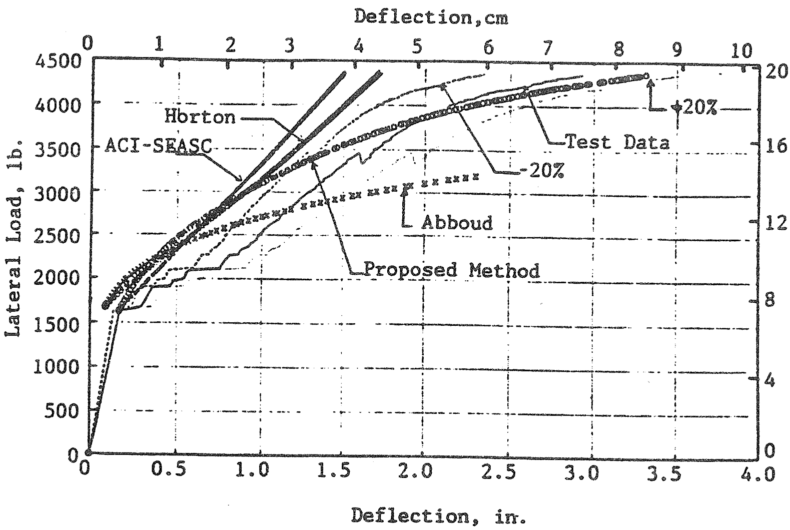

Fig. 5 Comparison of Methods with TCCMAR Test Panel Wall W2.

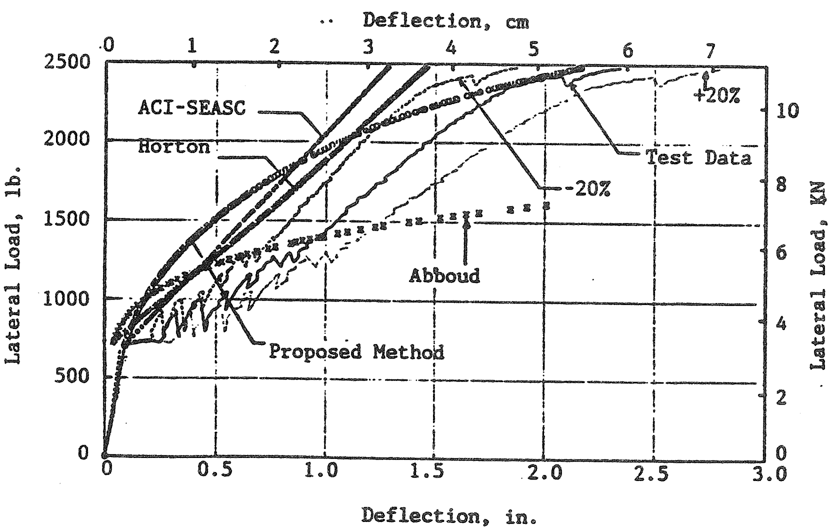

Fig. 6 Comparison of Methods with TCCMAR Test Panel Wall W3.

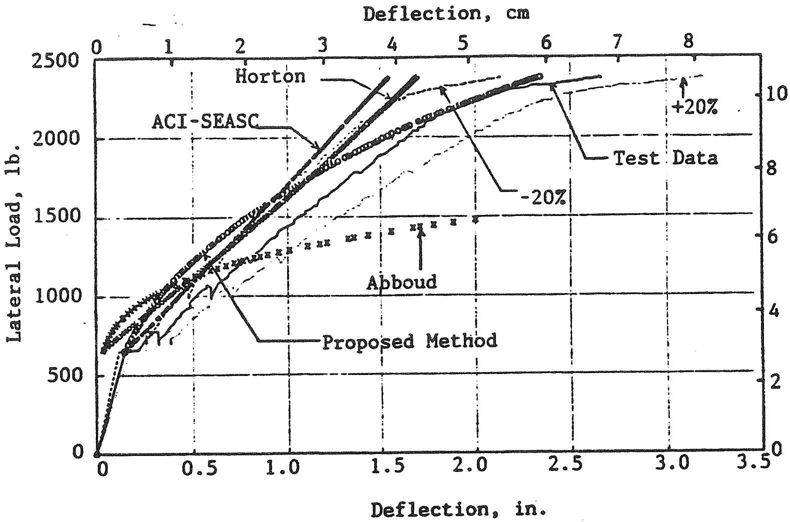

Fig. 7 Comparison of Methods with TCCMAR Test Panel Wall W4.

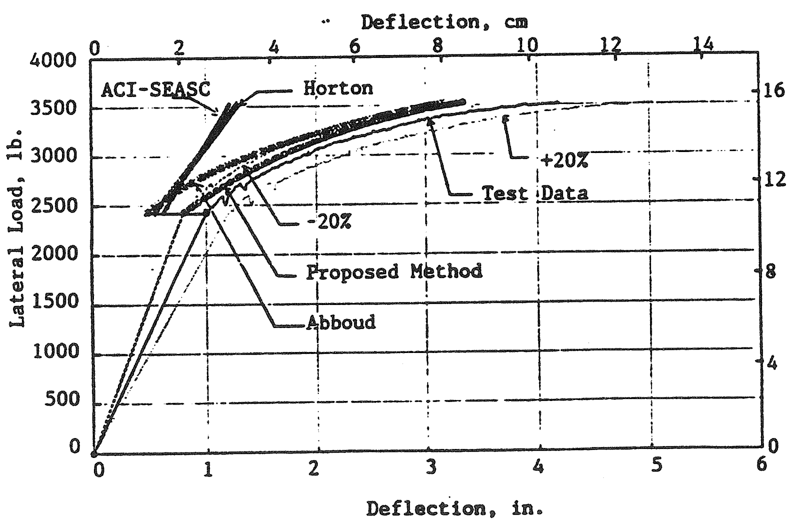

Fig. 8 Comparison of Methods with TCCMAR Test Panel Wall W5.

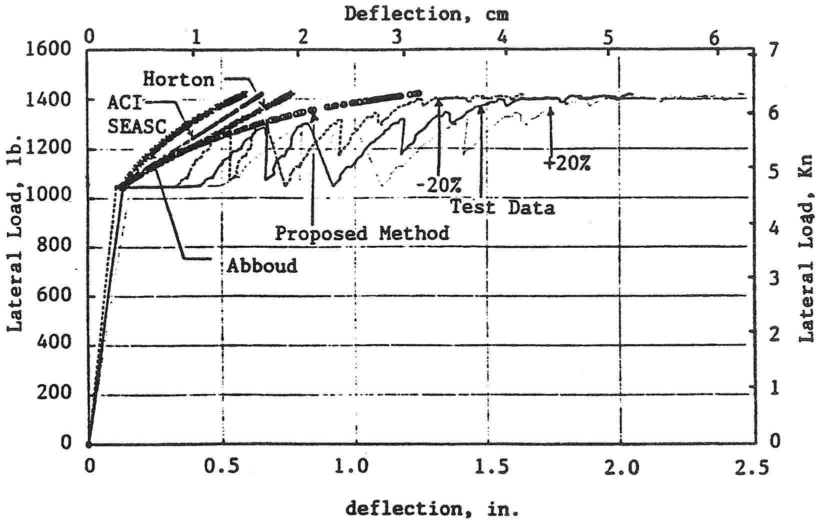

Fig. 9 Comparison of Methods with TCCMAR Test Panel Wall W6.