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INFLUENCE OF SPATIALLY VARIABLE MATERIAL PROPERTIES ON THE IN-PLANE SHEAR BEHAVIOUR OF UNREINFORCED MASONRY WALLS - STOCHASTIC NUMERICAL ANALYSES

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ABSTRACT

A detailed and accurate stochastic analysis of a structure allows for greater insight into the variability of potential failure mechanisms than simplified analytical or deterministic finite element models. Furthermore, this technique facilitates an estimation of the reliability of the type of structure under examination. In order to perform an accurate stochastic numerical analysis of unreinforced masonry walls, the finite element model must be calibrated via experimentally obtained material properties as well as a baseline structural response. This facilitates the greatest accuracy for the applied numerical simulations and allows for an estimation of the model error of future simulations not compared to an explicit experimental counterpart. Considering an existing, detailed experimental study, a modelling strategy for examining unreinforced masonry walls with spatially variable material properties was developed. These analyses were able to estimate the mean load resistance of the examined walls with a greater accuracy than a deterministic model, as well as capturing the variability of shear capacity and the observed damage to the experimentally tested walls. As the tested specimens failed almost exclusively in a rocking mode, a failure mechanism highly dependent upon the structures' geometry, the variability of the peak strength was minimal. However, the observed damage and presence of some local sliding and stepped cracking indicates that the proposed methodology is likely to capture more variable and unstable failure modes in shear walls with a smaller height-to-length ratio or those subject to greater precompressions.

KEYWORDS: *arched brick wall, monte-carlo simulation, shear wall, spatial variability, stochastic finite element analysis, unreinforced masonry*

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INTRODUCTION

Unreinforced masonry (URM) structures are highly susceptible to damage and collapse when subjected to unfavourable loading conditions. Perhaps the most adverse of these loading conditions is that induced under laterally aligned forces, such as earthquakes or wind events. This vulnerability is due in part to the high mass and initial stiffness of masonry which results in the attraction of greater inertial loading, particularly under earthquake conditions. The effects of this high load attraction are exacerbated by the fact that URM structures have a low tensile resistance and so are at risk of failure and collapse when subjected to high in-plane shear forces, and relatively low out-of-plane loading.

Structures intended to resist lateral loading often require the design and construction of stiff shear walls. These walls are typically intended to resist both vertical gravity loads as well as any lateral loading through in-plane shear which are subsequently transferred between storeys and into the structure's foundation. As such, the behaviour, reliability, and method of assessment of masonry shear walls are of key importance in the mitigation of collapse for URM buildings. Due to the high variability of masonry properties, the application of finite element analyses, supplemented by experimental modelling, provides a powerful tool for the investigation of URM shear walls and facilitates the refinement of design procedures and considerations of risk and reliability.

Recent developments in the study of URM structures with spatially variable material properties, in conjunction with the application of computational methods of estimating these structures' load resistance, such as in [1, 2, 3], have led to an improved understanding of the structural reliability of these structures. For example, the application of stochastically variable loading and material properties by [4] resulted in a 66% increase in the compressive design capacity of structural masonry due to the revision of the capacity reduction factor for walls concentrically loaded in compression in the subsequent Australian Standard for masonry design [5]. However, few studies have considered the effect of variable material properties of structures subjected to more complex loading, such as in-plane shear.

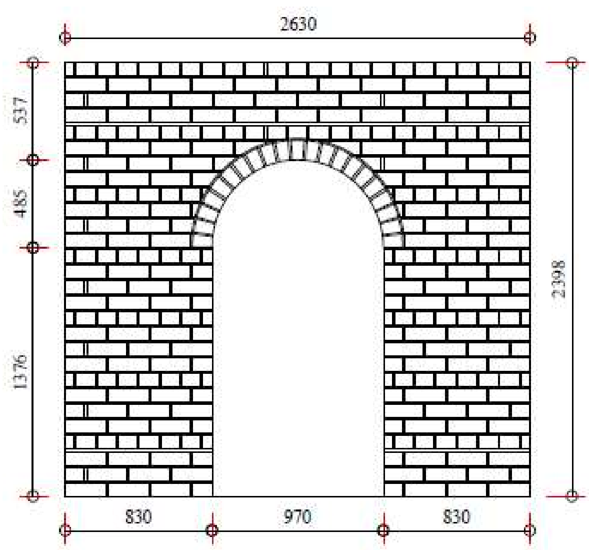
The reliability of URM structures subjected to complex loading mechanisms may be investigated through the application of repeatable experimental models, which may be supplemented by stochastic finite element analyses (SFEAs). One such numerical modelling approach that may be applied in the assessment of URM shear walls is the simplified, two-dimensional, micro-model proposed by [6]. This modelling strategy has been used extensively in masonry research [7, 8, 9] and allows for the consideration of the nonlinearity of URM structures, with a focus on the response on the unit-mortar interface [6].

DESCRIPTION OF EXPERIMENTAL TESTING

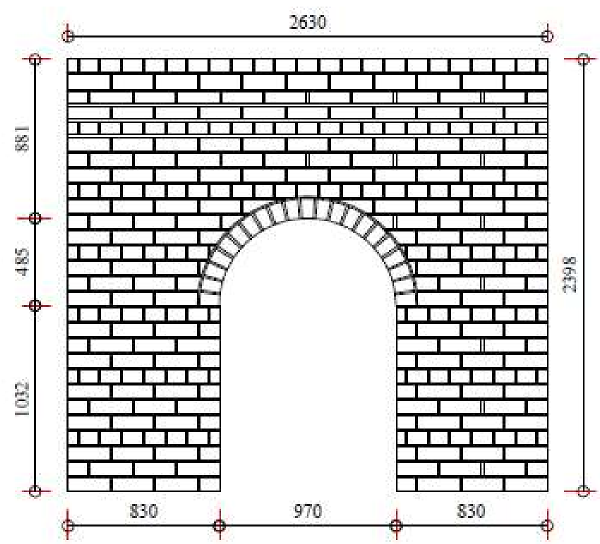
The current study is based upon the work undertaken by [9], specifically, the laboratory testing and FE modelling of a series of perforated (arched) URM walls. The masonry wall specimens considered in this study are presented in Figure 1. These walls were subjected to a progressively increasing, cyclic, in-plane shear displacement, up to a peak in-plane drift of 48 mm (2.0%) [9], consistent with the recommendations of [10]. The envelopes of the hysteresis loops of the lateral in-plane load versus in-plane drift for each of these test specimens were then simulated through the application of a monotonic displacement up to this ultimate displacement. The asymmetric pier tests undertaken in the previous study have been excluded from this paper, and as such the wall configurations considered in this study, with their relevant naming conventions, are as follows where pre-compression refers to the average compression stress in each pier resulting from the constant vertical force applied during testing.

- WS_0.2 Shallow spandrel under 0.2 MPa of pre-compression,
- WS_0.5 Shallow spandrel under 0.5 MPa of pre-compression,
- WD_0.2 Deep spandrel under 0.2 MPa of pre-compression, and
- WD_0.5 Deep spandrel under 0.5 MPa of pre-compression.

These specimens, comprised of two wythes, are 230 mm in thickness, utilise common bond coursing and contain a header row of units every fourth course. Furthermore, all these structures were constructed using solid clay brick units of nominal dimensions: 230 mm long, 110 mm wide and 76 mm high, with 10 mm thick mortar joints and utilising the same mortar mix ratio of 1:2:9 (cement: lime: sand) measured by volume [9].



(a) Shallow spandrel



(b) Deep spandrel

Figure 1: Wall specimens tested by [9] (all dimensions are in millimetres)

The in-plane lateral loading was applied to the test specimens using a horizontally aligned hydraulic jack in contact with a steel loading H-beam; a 200UC 46.2, and transferred into the walls via two composite beams bonded to the structure, as shown in Figure 2. Furthermore, vertical pre-compression was also applied via a hydraulic jack, through a spreader beam and into the composite beams, which were aligned with the piers of the test walls.

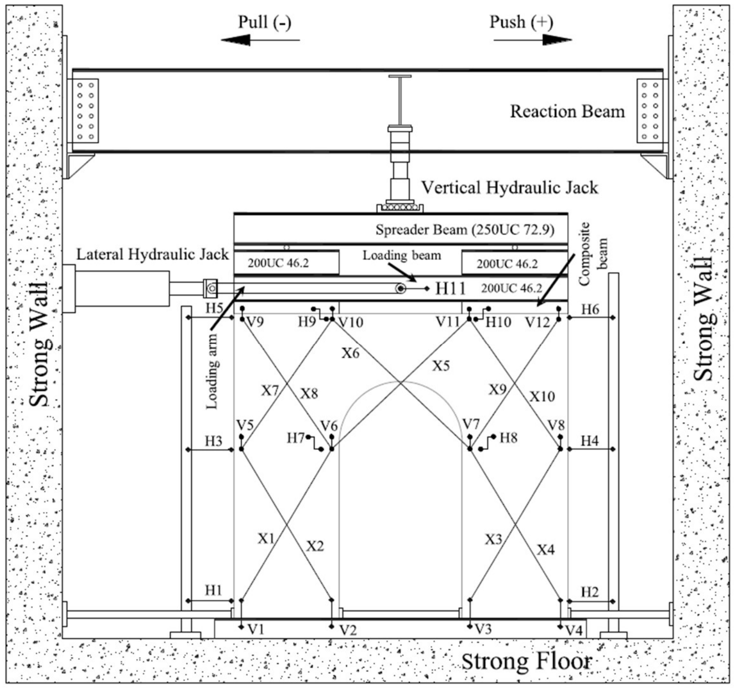


Figure 2: Elevation of experimental wall test setup and instrumentation [9]

NUMERICAL MODELLING

The numerical models representing the full-scale perforated URM walls tested by [9] were created with the finite element (FE) software package DIANA 10.3, utilising the simplified micro-modelling approach described by [6]. This modelling strategy was adopted as the results presented by [9] indicate that the more simplistic macro-modelling approach, such as was used by [11], over-estimated the peak load resistance of the structure. Furthermore, a detailed micro-modelling approach was not utilised due to the significantly higher computational expense of each simulation; an attribute that would be greatly exacerbated by the use of Monte-Carlo simulations.

In the simplified, two-dimensional micro-modelling approach, the clay brick units were modelled using eight noded, plane stress elements (CQ16M as in [12]) with a membrane thickness of 230 mm. While the simplifying assumption of a membrane thickness, and thus uniform cross-sectional properties, does not allow for an accurate representation of the two wythes interlocked with header rows, as shown in Figure 1, as only the in-plane behaviour is examined in the current study, this simplification will not greatly affect the modelled response of the URM walls.

As shown in Figure 3, the mortar in the masonry structures is not explicitly modelled. In order to represent the geometry of the masonry, each masonry unit is expanded so as to incorporate adjacent bed and perpend joints. The interface between the units and mortar joints is represented with quadratic one-dimensional (CL12I as in [12]), combined cracking-shearing-crushing interface elements. These unit-mortar interface elements, along with discrete cracking interface elements discussed in [12] which were used to represent a local failure of masonry units, comprised the FE components capable of capturing the non-linear response of the examined structures.

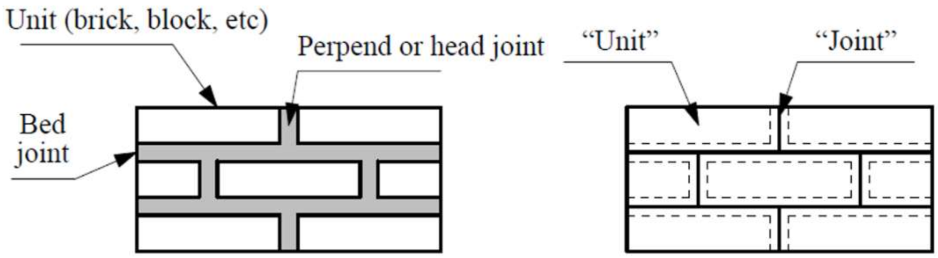


Figure 3: Expanded geometry of masonry units utilised in the simplified micro-modelling approach [13]

Adoption of a monotonic loading scheme
A key simplification made in this study is the adoption of a monotonic application of load; in this case, a prescribed displacement, in the finite element analyses (FEAs), as opposed to the cyclic displacement applied during the laboratory testing by [9]. This simplification was made for several reasons. Firstly, as the focus of this study is to examine the variability of the peak load resistance of URM shear walls, it was only necessary to capture the envelope or ‘back-bone’ of the hysteresis loops presented by [9]. It has been shown in this previous study that this envelope may be accurately approximated using a monotonic loading scheme.

Furthermore, with the clear alternative to a monotonic loading scheme being the application of the same cyclic displacements to an FE model, the applicability of such a loading scheme was examined in this study. It was found that this modelling procedure is much more computationally expensive, with model runtimes increasing by as much as a factor of 50. As this study requires the application of Monte-Carlo simulations, minimising the computational expense of each simulation was essential.

Material properties
In addition to the full-scale laboratory tests undertaken by [9], a range of material characterisation tests were performed using the same type of masonry unit and mortar. The masonry prism compression test [14, 15], triplet shear test [16] and bond wrench test [15] were performed in order to characterise the material properties of the unit-mortar interface. The modulus of rupture test [17] was conducted to define the flexural tensile strength of masonry units. The material properties determined from these laboratory tests have been supplemented by recommended or assumed values which describe those characteristics whose values are difficult to accurately determine through the reasonably simple tests performed by [9], such as the tensile and shear fracture energies.

Several conversion or assumptions have been made of the material properties determined from these laboratory tests in order to apply these values to the FE models. Firstly, as for both the discrete unit cracking and unit-mortar interface models utilised by [12], the tensile strength is defined using the direct tensile strength rather than the flexural tensile strengths determined from the modulus of rupture and bond wrench tests. As such, these experimentally determined values have been divided by 1.5 to represent this material parameter [9]. The conversion of the modulus of rupture to a direct tensile strength is not prevalent in literature, however, a similar ratio has been used for the flexural and direct tensile strengths of plain concrete [18], and has been observed in previous studies of URM structures to produce satisfactory results. For the unit-mortar interface model, this conversion is based upon the findings of [19] and [18].

STOCHASTIC MATERIAL MODELS

The material parameters of greatest significance to the strength of the shear walls considered in the current study are the tensile and shear bond strengths of the unit-mortar interfaces, as well as the tensile strength of the masonry units. Consideration of the spatial variability of these parameters will allow for a representative range of peak strengths to be captured as critical points within a given simulation are made weaker or stronger.

Additionally, the stiffness of the unit-mortar interface elements is also of interest, as a high strength bond with a low stiffness may not attract enough load to induce failure until a sufficient amount of cracking and load redistribution has taken place. This may lead to an overall reduction in the peak strength of the structure. Application of a variable joint stiffness was achieved by considering a spatially variable bed and perpend joint thicknesses. While the adopted modelling strategy does not explicitly model the mortar within a structure, the linear normal and shear stiffnesses of these interfaces are inversely proportional to the assumed thickness of a joint [6]. However, as the thickness of neither the bed nor perpend joints of the experimental walls tested have been measured, the stochastic parameters of these thicknesses have been estimated from [20] where the thickness of more than 1700 bed and perpend joints were measured.

In previous studies of stochastically variable material behaviour, the determination of an appropriate probabilistic distribution has been made through the application of the Kolmogorov-Smirnov (K-S) test, as well as the examination of the goodness-of-fit of the Inverse Cumulative Distribution Function (CDF^{-1}) [4, 7, 21]. This methodology readily allows for the elimination of unacceptable PDFs and facilitates the adoption of the most suitable distribution to be made by considering the key areas of the CDF. The adopted stochastic material models derived from the methodology are summarised in Table 1, below.

Table 1: Stochastic material models adopted for SFEAs.

Property	Mean	COV	Unit	Distribution
Unit modulus of rupture	1.067	23.2%	N/mm^2	Discrete
Flexural tensile bond strength	0.156	31.2%	N/mm^2	Discrete
Shear bond strength	0.149	44.4%	N/mm^2	Discrete
Bed joint thickness	8.87	34.0%	mm	Lognormal
Perpend joint thickness	12.08	29.3%	mm	Lognormal
Ratio of flexural to direct tensile strength of clay brick masonry units	1.53	10.2%	-	Lognormal
Ratio of flexural to direct tensile strength of unit-mortar interface	1.50	13.2%	-	Lognormal

As noted in Table 1, a discrete random variable distribution has been adopted for the stochastic generation of the material strength parameters. This was done due to the limited sample size of experimental data undertaken by [9] which resulted in no suitably representative distribution being derived to fit these data sets. Thus, stochastic modelling of these parameters is achieved by randomly sampling directly from the set of laboratory results provided by [9].

SPATIAL VARIABILITY AND CORRELATIONS

A key component of this study is the effect of considering spatial variability of material properties. The application of spatial variability to each of the SFEAs undertaken in this study was achieved by prescribing each interface element in an FE model a unique material designation. These unique properties were defined as the bed and perpend joints of a given unit, as well as the potential cracks of a masonry unit, and were allocated the stochastically generated parameters of each interface during the construction of each model. In addition to the probability distributions presented in Table 1, the spatial correlations of these parameters were also be defined. This has been done through the consideration of the physical nature of these correlations, as well as the review of similar relationships in literature.

Correlation of interface strengths

The spatial variability of the chosen stochastic strength properties has been discussed by [22, 23]. An experimental study by [7] found that the average unit correlation coefficient for flexural tensile bond strength was low: between 0.22 and 0.50, for courses within a wall, and recommended a value of 0.40. However, it is also noted by [7] that a correlation coefficient of 0.40 represents a weak relationship between variables, and suggests that the spatial correlation of the flexural tensile bond strengths within a course of a wall is not significantly distinct from statistically independent variables [2, 7]. Furthermore, as the current study considers the tensile and shear bond strengths of the unit-mortar interface to be discrete random variables, the application of partial correlations is less significant than for a continuous PDF. This diminished significance is due to the fact that the discrete nature of these CDFs will still permit large differences in adjacent material properties for even strongly correlated variables, as well as increasing the likelihood of adjacent interfaces comprising the same material properties for weak correlations.

Correlation of joint thickness

Similar to that of interface strength parameters, it was found that the spatial correlations of bed and perpend join thickness was not distinct from statistically independent variables. This was observed through the estimation of a correlation coefficient for each joint type: 0.13 and 0.10 for bed and perpend joints respectively. It is suggested by [24, 25] that statistical dependence may be reasonably considered for correlation coefficients outside of the limits estimated from Equation 1.

$$\rho = \pm 2\sqrt{1 / N} \tag{1}$$

Where, ρ is the significance limits for an estimated correlation coefficient, and N is the number of observations. As this limit is approximately ±0.59 for both the bed and perpend joints, this study will assume statistical independence of the thickness of adjacent mortar joints.

DETERMINISTIC AND SFEA RESULTS

Applying the various stochastic material properties discussed above, supplemented by the assumed deterministic parameters presented by [9], Monte-Carlo simulations were run for the four wall configurations considered in this study. These were compared to deterministic models consisting of the mean experimental and assumed material properties. The load versus displacement behaviour of each simulation was extracted, as well as the predicted cracking patterns. These results allowed for an estimation of the mean and COV of peak strength to be made, as well as facilitating an investigation into the non-linear response and failure mechanism of each simulation.

Peak in-plane shear force

Prior to the application of any SFEAs, the deterministic FE models were analysed for each unique wall configuration to verify the accuracy of the adopted modelling strategy. The results of this comparison are presented in Table 2. It was observed that a deterministic model produced a good estimation of the peak strengths observed experimentally, with a maximum difference between the two estimates of 10.9%. It is also noted that the data sample of the wall configuration maintaining this largest difference consists of only a single test, and thus two values for use in estimating the mean peak load. As such this difference may change when supplemented by more experimental data. Furthermore, it may be observed that in three of the four cases, a conservative estimate of the peak load resistance has been achieved through the application of a deterministic FEA.

Once the accuracy of the numerical modelling strategy utilised in this study was verified, four sets of Monte-Carlo simulations were run. As with the deterministic models, the load-displacement behaviour and cracking patterns were extracted from each Monte-Carlo simulation. These results, presented in Table 2, allowed the mean and COV of the peak load resistance to be estimated for each wall configuration. Furthermore, the absolute difference between the SFEA and experimental mean was calculated. It was observed that, in all four cases, the SFEA models produced a more accurate estimation of the peak load resistance than the deterministic models. In particular, the difference observed for the deep spandrel models was reduced by as much as 30%.

Table 2: Experimental and FEA estimations of the mean and COV of peak load resistance

Wall configuration	Mean			COV	
	Experimental mean (kN)	SFEA peak load (kN)*	Deterministic (kN)*	Experimental	SFEA
WS_0.2	41.3	39.6 (4.1%)	39.5 (4.4%)	2.4%	2.0%
WS_0.5	71.5	71.0 (1.0%)	70.8 (1.0%)	3.9%	1.2%
WD_0.2	48.7	45.0 (7.6%)	43.4 (10.9%)	5.6%	2.0%
WD_0.5	74.6	80.2 (7.5%)	82.2 (10.2%)	4.3%	1.4%

* Bracketed values refer to percentage difference from experimental mean.

While the results of the SFEAs indicate that the examined specimens maintain a very low variability, this does not contradict what was observed experimentally. In both cases, a small COV has been estimated, the largest being 5.6% in the case of WD_0.2. This apparent low variability of load resistance is likely caused by the geometric configuration of the structures which has resulted in an exclusively rocking-type failure mechanism, an inherently invariable failure mode. Furthermore, it may be observed that the COV of SFEAs is notably different to that of the laboratory specimens, with the largest difference being more than 50%. However, the COVs for the experimental specimens shown in Table 2 do not consider the effect of variations in the testing procedure, nor in the specimens themselves. Consideration of these values would result in a significant reduction of the COV of the experimental peak load, as outlined by [26].

Sensitivity analysis of spatial variability

A key component of the work undertaken in this study is the consideration of the influence of spatial variability of material properties within the examined shear walls. The effect of spatial variability was examined by comparing the results discussed above with those obtained from nonspatial SFEAs. The latter involved the application of Monte-Carlo simulations using randomly generated material inputs, however, in this case, material properties were applied uniformly across the height and length of the wall rather than varying from joint to joint.

It was found that consideration of spatial variability did not greatly affect the estimated mean peak strength when compared to the nonspatial simulations. However, it did have a significant effect on the variability, greatly decreasing the COV of peak load resistance in some cases, as shown in Table 3.

Table 3: Effect of spatial variability on peak in-plane shear resistance

Wall configuration	Mean			COV	
	Deterministic (kN)	Nonspatial (kN)	Spatial (kN)	Nonspatial	Spatial
WS_0.2	39.5	39.9	39.6 (0.8%)	3.8%	2.0%
WS_0.5	70.8	72.4	71.0 (2.0%)	9.2%	1.2%
WD_0.2	43.4	44.7	45.0 (0.7%)	2.4%	1.9%
WD_0.5	82.2	81.9	80.2 (2.1%)	5.6%	1.4%

* Bracketed values refer to percentage difference between spatial and nonspatial models.

This observation may be explained by the fact that when failure initiates at a weaker than average joint or unit within the wall, the presence of stronger surrounding material in a spatially variable simulation allows for more effective stress redistribution (load sharing) compared to a nonspatial model in which all the surrounding material has the same strength. This averaging effect which occurs in a spatially variable system results in much lower variability in the system response. Furthermore, the slightly lower mean peak wall strength observed in three of the four cases investigated can be attributed to the higher probability of encountering a weak joint or unit in each spatial simulation, compared to the nonspatial simulation. A similar trend of high COVs of wall strength for nonspatial SFEAs was observed for URM walls subjected to out-of-plane bending by [23].

CONCLUSIONS

An FE modelling strategy was developed for URM walls subjected to in-plane lateral loading. Utilising this approach, the suitability of a monotonic loading scheme in the modelling of a cyclically loaded URM shear wall was examined. It was found that the monotonic approach produced a good fit for the load-displacement envelope of the cyclically loaded experimental tests.

The influence of spatial variability of material properties within these URM walls was examined through the application of Monte-Carlo simulations. It was observed that the SFEAs produced a better estimation of the experimental mean strength than any of the applied deterministic models. While it was noted that the variability of this peak strength was low in all cases, this observation was consistent with the results obtained experimentally. It may be concluded that this was caused by the observed failure modes being highly dependent upon the wall geometries and boundary conditions, more so than on the material strengths. It is expected that structures that are more susceptible to alternate failure modes; specifically, structures that have a smaller height-to-length ratio or are more highly confined will exhibit greater variability in their peak strength.

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