14TH CANADIAN MASONRY SYMPOSIUM
MONTREAL, CANADA
MAY 16TH – MAY 20TH, 2021

# BEHAVIOUR OF MASONRY ARCH SYSTEM UNDER VARIOUS TYPES OF LOADS: STATE OF THE ART AND MICRO MODELLING

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## ABSTRACT

Arch structures were the main form of construction before the advent of reinforced concrete. Arch structure resists load in compression if the thrust line lies inside the kern. The majority of old arch structures were constructed using stones and masonry material. In India, arch construction was widely used in bridges built in the pre-independence era. Capacity assessment of these old arch structures is needed due to deterioration over the years and increased load demands. Over the years, a number of methods have been developed to analyze masonry arch structures under different type of loads. Initially, elastic methods were developed using the thrust line concept, but later, the limit analysis methods have gained acceptance. Lower bound and upper bound approach was developed to calculate the collapse load for masonry arch structures. Majority of these studies were focused on the longitudinal behaviour of arches, i.e., treating the arch as a two-dimensional structure. Less attention has been paid to the three-dimensional behaviour of such masonry arch systems. Theories related to the dynamic load behaviour of masonry arch structures were also developed treating the whole system as SDOF system. Various experimental studies confirmed the dynamic behaviour of masonry arch system. The present study is focused on the state of the art of analysis of masonry arch structures and 3-D finite element modelling of these structures. 3-D micro-model of segmental circular masonry arch was developed in ABAQUS in which masonry bricks and mortar were kept as separate entities. Various components of the masonry arch system like the arch barrel, spandrel walls, piers, backfill soil were modelled and assembled using suitable interactions. Load carrying capacity of masonry arch was determined with the change in various factors like the thickness of ring, boundary conditions of spandrel walls, etc. The failure mode of the finite element model of masonry arch structures was found to be similar to previous experimental studies.

***KEYWORDS:*** *masonry arch, thrust line, lower and upper bound approach, mechanism method, micro-modelling, four hinge mechanism*

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## INTRODUCTION

Arch structures are one of the oldest forms of construction. The construction of the arch structure made it possible to cover long spans, as the load resisting mechanism in the arch structure mostly utilizes the compressive strength of the material if the thrust line lies inside the kern.; due to this unique property, these structures became very popular before the advent of reinforced concrete. Arch structures were extensively used by Romans, and the construction at that time was purely based on empirical rules. The empirical rules developed by Romans were lost in medieval times because of very little construction in that period, as mentioned in the 'History of strength of materials' [1]. Later in the seventeenth and eighteenth-century many methods were developed to understand the behaviour of the arch structures. Work by Lahire [2] and Coulomb [3] was focused on determining the behaviour of arch structure and determining the appropriate thickness of the arch barrel. The notion of pressure line inside an arch was first introduced by Gerstner [4], the author compared the action of suspension bridge and arch structure. With the introduction of pressure line in arches, many graphical methods were developed in the nineteenth century. Mosely [5] studied the pressure line concept and concluded that infinite pressure lines could be obtained in a symmetric arch structure for a given configuration.

Elastic analysis of the arch was focused on determining the location of the thrust line in the arch; however, the elastic analysis relies on the known boundary conditions and very small movement in boundary conditions can change the equilibrium state of the arch structure. Plasticity theory was later applied to the arch structures. Heyman [6] explained the applicability of the limit analysis approach to masonry and stone arch structure. Limit analysis proved to be an effective tool to analyze masonry and stone arch structures. It provided the stability limits of the arch in terms of the location of the thrust line. Various modifications were made in this method to account for material strength, sliding failure in different studies to suitably use it to analyze the masonry and stone arch structures. Lower bound and upper bound approaches were developed to calculate the load-carrying capacity of the arch structures. Dynamic analysis of the masonry arch structures was first attempted by Oppenheim [7] in 1992. Masonry arch structures were modeled as a single degree of freedom system in which degrees of freedom was chosen as the rotation of the links which forms when the arch goes into four hinge mechanism. A similar type of study on masonry arch structure was performed by Clemente [8] in 1998, and the behaviour of masonry arch structure was explained as four hinge mechanism. De Lorenzis et al. [9] studied the behaviour of masonry arch structure under horizontal impulse base motion and provided an analytical solution to the problem.

In the present study, state of the art on masonry arch structures is explained briefly. Various static and dynamic studies performed on masonry arch systems are presented in the study. Based on the input from all the studies, a micro model of 3D masonry arch structure was developed in finite element package ABAQUS [10]. Static and dynamic testing was performed in ABAQUS [10] to observe the behaviour of the structure by changing various parameters.

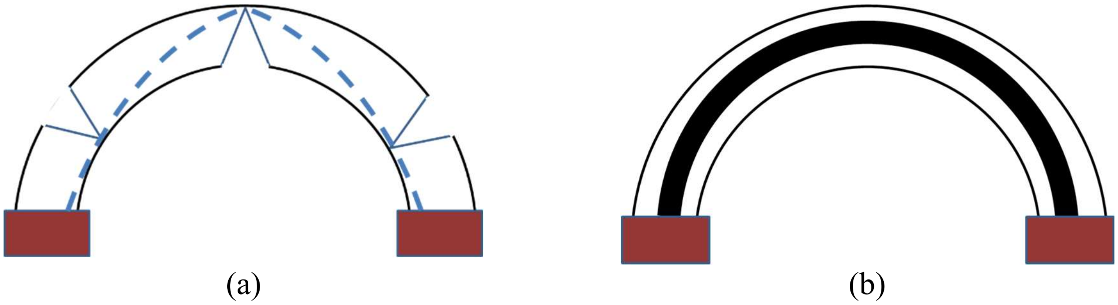
## STATIC ASSESSMENT OF MASONRY ARCH

### *Lower bound approach (Thrust line analysis)*

The freely hanging uniform chain takes the shape of a catenary under the action of gravity. Hooke [11] in 1675 compared the behaviour of the arch structure and a hanging chain. If we generalise this idea, it can be stated that the shape of a uniform chain carrying a certain set of loads if inverted will represent the path of compressive force in an arch structure carrying the same set of loads. Thrust line analysis originates from the same concept. The applicability of limit state analysis to the masonry arch system using the formation of plastic hinges was established by Heyman [6]. There were three main assumptions to this theory which are:

- Masonry in the arch has no tensile strength.
- The masonry in the arch has infinite compressive strength.
- Sliding between masonry units does not occur.

Based on the above three assumptions, the concept of thrust line was proposed in the masonry arch system. The thrust line is the line connecting all the sections in the masonry arch at the location of their total force. Two approaches were developed to determine the collapse load of a masonry arch. The lower bound approach was based on the location of the thrust line in the arch. Calculation of the thrust line can be calculated using the equations of equilibrium or by solving a linear programming problem. Several rules were developed in terms of the thrust line to define the stability of the masonry arch structures. If the thrust line touches intrados or extrados of the arch at a particular section, then a plastic hinge would form at that location, as shown in Figure 1(a). A geometric factor of safety was defined for the arch structure, which signifies that how much larger is the arch under consideration is compared to the just stable arch in which the thrust line is just touching the surface of the arch, as shown in Figure 1(a). If the value of the geometric factor of safety is three, then it will mean that the thrust line is lying in the middle one-third of the arch thickness, as shown in Figure 1(b).



(a) (b)

**Figure 1: (a) Limiting stable arch (b) Range of location of the thrust line with F.O.S. of 3**

Theory given by Heyman [6] defined the stability of the arch structure but does not take into account the material strength and sliding failures which will define the safety of the arch structure. Aita et al. [12] explores some of the limitations of Heyman theory and investigate the effect of limited material crushing strength and degrading system stiffness on behaviour of the arches under ultimate state. Harvey [13] in 1998 extended the safe theorem given by Heyman [6] by incorporating the possibilities of crushing failure and sliding failure. The term thrust line was replaced by the thrust zone which should be of sufficient depth at any section of the arch depending on the materials crushing strength. Sliding failure can be avoided if the thrust line crosses any joint with an angle more than $tan^{-1}(\mu)$, where $\mu$ is the friction coefficient. Thrust line has to be located for a given arch configuration and load pattern to analyze the arch structure. Based on the location of the thrust line in an arch structure, one can comment on the stability of the arch structure. In order to calculate the location of the thrust line for a given arch configuration, the catenary method was developed by a Spanish architect Gaudi as mentioned by Huerta [14]. The catenary method is a graphical method that is based on the correlation between tensile stresses in a freely suspended chain and compressive stresses in a masonry arch. In the nineteenth century, French engineer Mery also developed a funicular method to calculate the precise location of thrust line; this method was also known as Mery's method [15]. These two methods were used for a long time in the field to construct stable masonry arches. Nowadays, various computer programs based on linear programming and equilibrium equations can locate the thrust line for a given arch configuration. Recent development includes a solution provided by Galassi and Tempesta [16]; the authors provided a MATLAB code solution to calculate the thrust line for a user-defined arch configuration.

### *Upper bound approach (Mechanism method)*

The mechanism method was first developed for steel structures, but later it was applied to masonry arch structure by Heyman [6]. In this method, the masonry arch is assumed to be made up of a series of rigid block. The mechanism method assumes that the masonry arch becomes a mechanism when at least four hinges form in the structure, as shown in Figure 2. The position of these masonry hinges are not known prior to the analysis and has to be assumed. Three out of four hinges can be assumed at two supports and at the location of applied force, and the fourth hinge has to be assumed such that the collapse load is minimum. The problem can be solved with the moment equilibrium equations at the hinges or with the equations of virtual work. This method is an upper bound approach; hence it overestimates the collapse load for the masonry arch.

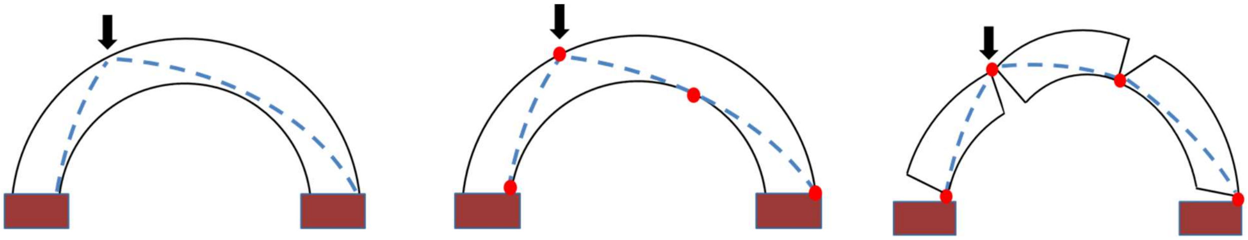
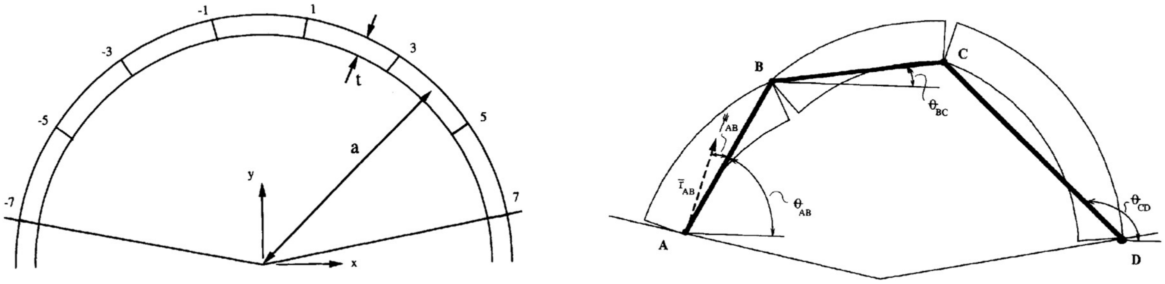


**Figure 2: Four hinge mechanism in a masonry arch**

Several models were developed based on the mechanism method to analyze the masonry arch structure. These methods are based on linear programming and vary depending on the material properties and geometry of the structure. One of the popular methods is the discrete limit analysis model, which was pioneered by Livesley [17] in 1978. In this method, a masonry arch structure is idealized using a series of rigid blocks. The size of these blocks is kept slightly larger than the actual blocks as the mortar is not modelled explicitly in these methods. Based on above approaches, a simple joint equilibrium formulation was developed by Gilbert et al. [18] in 2006. This method is used to analyze a masonry arch structure using modern linear programming algorithms. Based on the lower and upper bound approach, various analysis programs were developed over the years. These programs are software packages that analyze a masonry arch structure treating it as a two-dimensional structure. Two of the most popular programs are ArchieM [19] and Ring 2.0 [20].

## DYNAMIC ASSESSMENT OF MASONRY ARCH

True dynamic analysis of masonry arch structure was pioneered by Oppenheim [7] in 1992. Arch structure was assumed to behave as a single degree of freedom system under base excitation. Arch structure was analyzed as four hinged mechanism under horizontal base motion. Links will form in the masonry arch structure due to the formation of four hinges, as shown in Figure 3(b). The angle formed by these links with respect to their original position is considered as the degrees of freedom for the arch structure. These angles depend on each other hence effectively whole structure can be represented as a single degree of freedom system. An equation of motion was developed for a particular arch structure with seven equal blocks with an embrace angle of 157.5 degrees and thickness by radius ratio of 0.15, as shown in Figure 3(a).



(a) (b)

**Figure 3: (a) Geometry of arch with seven blocks (b) Mechanism of motion under base excitation (Oppenheim 1992)**

A single degree of freedom equation was derived for the arch using the Hamilton principle and Lagrange equation. Few important terminologies for masonry arch structure were established in this study which is crucial for the dynamic analysis of the arch structure. Results show that the masonry arch structure shows excellent resistance to earthquake motion if acceleration values are below a threshold value. The threshold acceleration was referred to as onset level and was found to be independent of arch radius '*a*' and mass per unit volume '*m*'. The onset value was found to be increasing with the increase in thickness to radius ratio; also, with the increase of angle of embrace, the onset value was found to be decreasing. Similar type of study was performed by Clemente [8] in 1998. The study was focussed on the response of arch structure under impulse base motion. Onset value of acceleration was determined using equation of equilibrium and effect of various parameters on onset acceleration were explored. The role of frequency of input signal was studied and masonry arch structure was found to be more sensitive towards low-frequency input signal.

De Lorenzis et al. [9] studied the effect of impulse base motion on masonry arches. The authors used the same arch dimensions, which were used in the study by Oppenheim [7]. The study provides an analytical solution to the response of the arch structure under impulse base motion. The study is focussed on the impact problem of the masonry arch during the closing of the hinges when the structure returns to its original configuration. It was assumed that the arch forms the symmetrical hinges under the reversal of load, as shown in Figure 4.

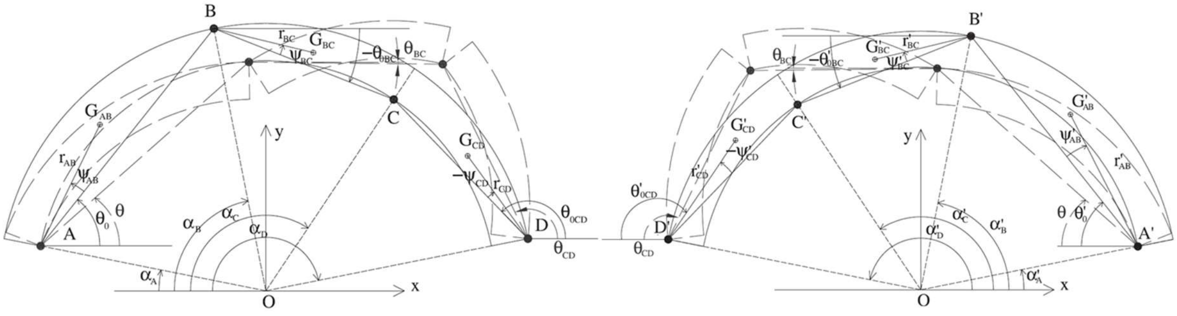
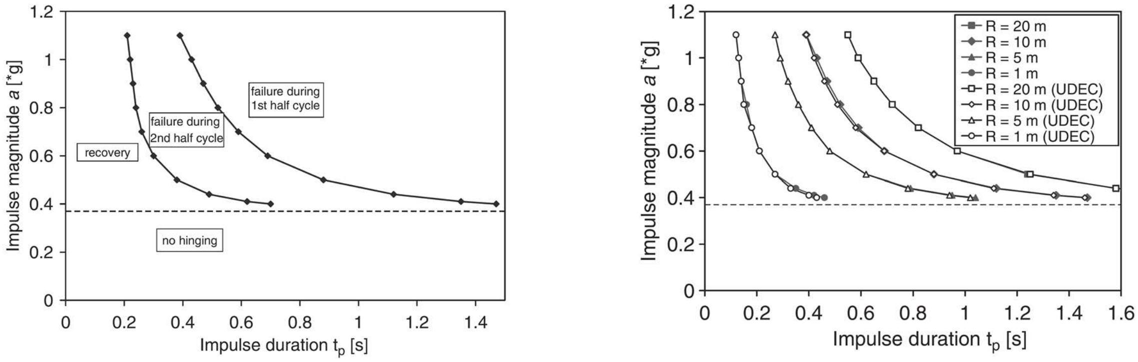


**Figure 4: Symmetricity of hinges during the opposite direction of horizontal motion ( De Lorenzis et al. 2007)**

The effect of impulsive forces which generates at internal hinges was considered in the study and the equation of motion for the arch structure was solved to obtain the response of the structure during each subsequent half cycles of the motion. The response of the considered arch was compared with the numerical results by analyzing the structure in a commercial program UDEC which utilizes the discreet element method. The analytical model was analyzed with the assumption of three voussoirs in the arch, and the numerical model was analyzed with seven voussoirs. The modelling parameters included the normal and shear stiffness, density, damping parameters etc. Failure domain was developed for rectangular impulse load of variable amplitude and frequency, as shown in Figure 5(a).



(a) (b)

**Figure 5: (a) Failure domain of Oppenheim's arch in response to the step impulse motion (b) Effect of arch size on failure domain (De Lorenzis et al. 2007)**

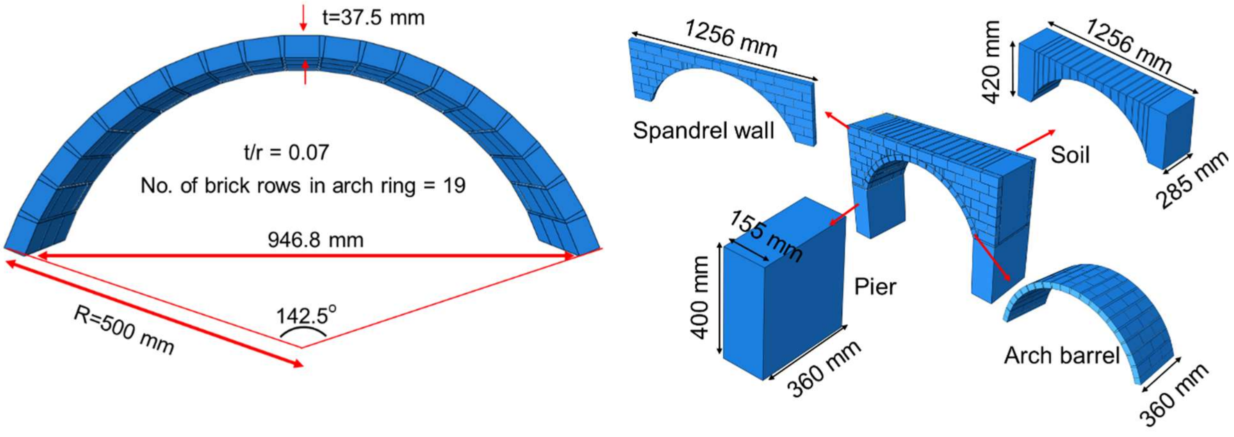
Authors compared the effect of various parameters like damping, coefficient of friction, the shape of impulse etc. on the failure domain of arch structure. The effect of scaling of arches on the failure domain was also investigated. It was derived that the larger arches are more resistant than the smaller ones by a factor equal to the square root of the ratio of the arch radii, as shown in Figure 5(b). The effect of damping on the failure mode of the arch structure was studied with the numerical model. It was observed that with very low damping in the structure, the failure mechanism of arch structures is not purely rocking based and sliding failures were observed along with hinging even with a very high value of friction coefficient.

## FINITE ELEMENT MODELLING OF MASONRY ARCH

In order to assess the existing masonry arch bridges against the existing loads and understand the failure mechanism, there is a need for detailed modelling techniques and experimental verification of results. Previous studies show that under static loading, the behaviour can be studied using the existing theories and computer programs based on those theories. However, the behaviour of masonry arch structure under dynamic loading is still relatively unexplored and cannot be studied by just using the existing theories. In the present study, a small-scale finite element model of a masonry arch structure was developed in ABAQUS. It has been proposed in the previous studies that while dynamically analyzing a small-scale model of masonry arch system, one can obtain results for any arbitrary scale and then derive the results for any other scale (De Lorenzis et al. 2007). However, care should be taken while scaling down such that the behaviour should not deviate from the actual arch behaviour.

### *Details of the finite element model*

The arch model with all the component and their dimensions is shown in Figure 6 (a) and (b). A small-scale model of a circular segmental masonry arch was developed in ABAQUS with a span to width ratio of 2.6.



(a) (b)

**Figure 6: (a) Dimensions of the arch barrel (b) Various components of masonry arch**

The model comprises half-scale brick units with a length of 120 mm, the width of 60 mm and a height of 37.5 mm. The dimensions of the small-scale model were chosen in such a way that future experiment work can be easily performed on the shake table in lab. The mortar in the arch barrel section was modelled separately, and interaction properties were provided using cohesion interaction in general contacts.

***Behaviour under static load***

Static pressure was applied on the top surface of the soil model to observe the behaviour under the effect of static pressure. The pressure was applied at a distance from 333 mm from the end of the span and the dimension of the rectangular patch was 51 mm along the span and 285 mm in the transverse direction. Four hinge failure mechanism was observed in the structure at collapse. Deflected shape of the arch at the state of collapse is shown in Figure 7. The failure mode observed in the arch structure was found to be comparable with previous experimental studies (Royles and Hendry 1991 [21]). Four hinge mechanism can be easily observed in the deflected shape of the structure. The failure mode of various components like spandrel walls and arch ring are similar to previous experimental studies, as shown in Figure 7.

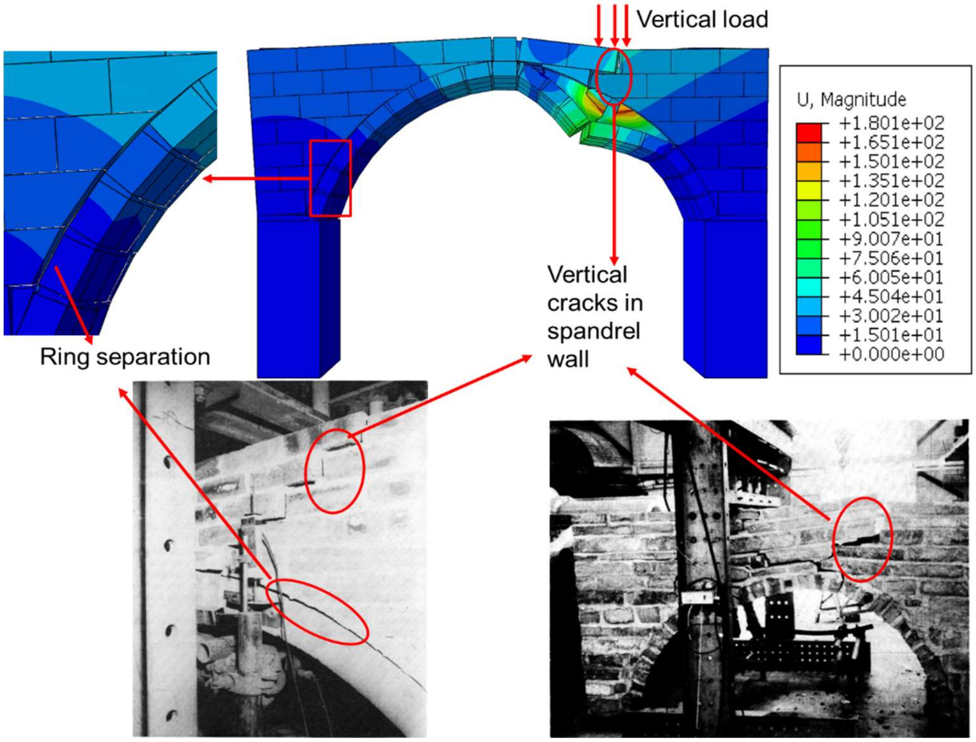


**Figure 7: Failure mode under of masonry arch under vertical static loading and comparison with previous experimental studies (Royles and Hendry 1991)**

The load capacity of a masonry arch is dependent on various arch characteristics like depth of fill material, thickness to radius ratio. In the present study, the depth of fill material was increased to compare the load capacity. Load carrying capacity was increased from 2964 N to 6669 N due to an increase in the depth of soil. It can be noted that while calculating the load-carrying capacity due to change in fill depth, the out of plane movement of spandrel walls were restricted.

***Behaviour under dynamic load***

The behaviour of masonry arch structure under sinusoidal impulse load was also observed. The arch barrel was subjected to the same set of sinusoidal impulse loading at the two supports in the direction of span. Four hinge mechanism was observed in the arch structure, two hinges formed at the base of the structure and the remaining two formed in the arch barrel, as shown in Figure 8. The motion of the base of the arch was restricted to move in vertical direction, all other displacements and rotation were allowed.

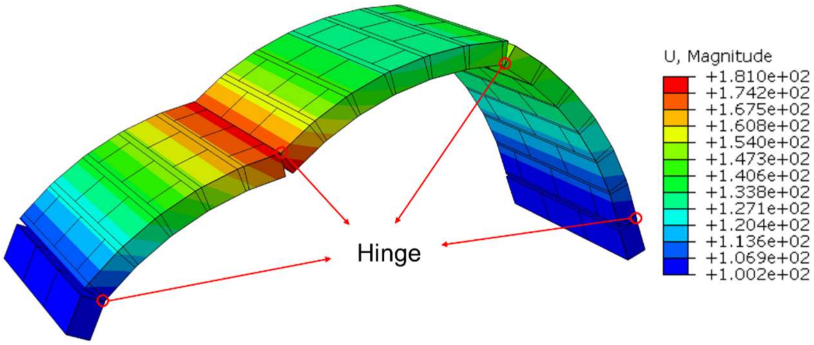


**Figure 8: Failure mode of the arch barrel under dynamic load**

Sinusoidal impulse load with variable frequencies was applied at the base of the structure, and the response was measured at the crown of the structure. The displacement amplitude of input signal was kept at 0.5 mm for all the signals so with increase in frequency in each subsequent cycle the PGA of base motion was also increased. The amplification of motion can be easily observed in Figure 9 when input motion frequency is reached at 12 Hz with PGA of 0.28g. It can be observed that the arch is going into hinging mechanism at frequency of 12 Hz at PGA of 0.28g and significant amplification of the motion is happening as shown in Figure 9.

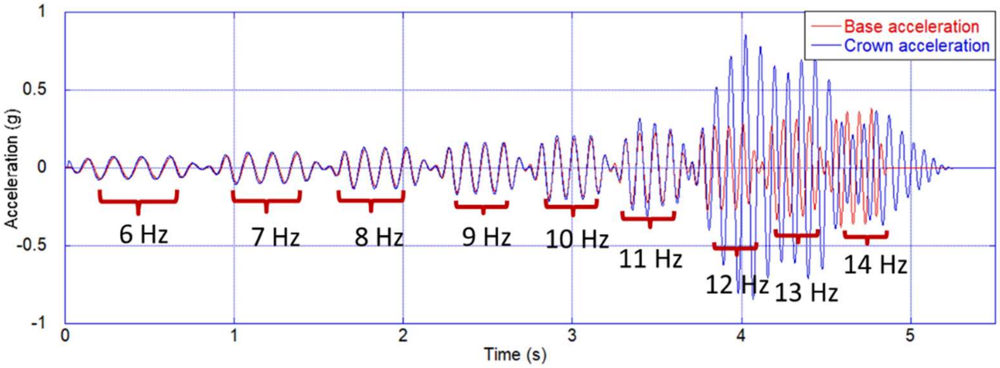


**Figure 9: Base vs crown displacement for 6 Hz to 12 Hz sinusoidal input with peak displacement amplitude of 5 mm**

In Figure 9, it can be observed that hinging mechanism is occurring at 0.28g. This value is suspected to be onset value. However, hinging mechanism can also occur at lower PGA than 0.28g if a lower frequency pulse is applied at the base because the masonry arch structure is more sensitive towards low frequency motion as reported in previous studies (Clemente 1997 [8]).

To determine the failure acceleration of the modelled arch structure, it was excited at base at 12 Hz frequency with increasing PGA in subsequent cycles. Displacements were obtained at base and crown to compare the motion. Initially at low PGA values the arch behaves as rigid body and hinge formation does not occur. When the acceleration values reach 0.3g which is close to the suspected onset value, hinge formation occur in the arch and at 0.5g failure occurs, that is arch does not return to its natural position as shown in Figure 10.

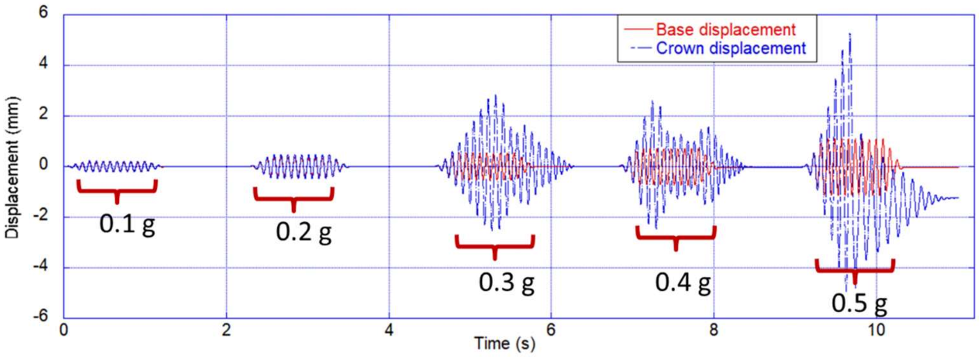


**Figure 10: Base vs crown displacement at 12 Hz sinusoidal frequency with increasing amplitude**

It can be observed that at 12 Hz frequency the failure occurs at 0.5g hence a point on the failure domain curve of the arch has been obtained. However, to obtain the failure domain and know the sensitivity of arch against various parameters, it should be excited at variable frequencies and amplitude. The failure observed at 0.5g acceleration included sliding followed by hinging, this type of failure mode is also reported in previous shake table experimental studies (Castellano et al. 2019[22]) as shown in Figure 11.

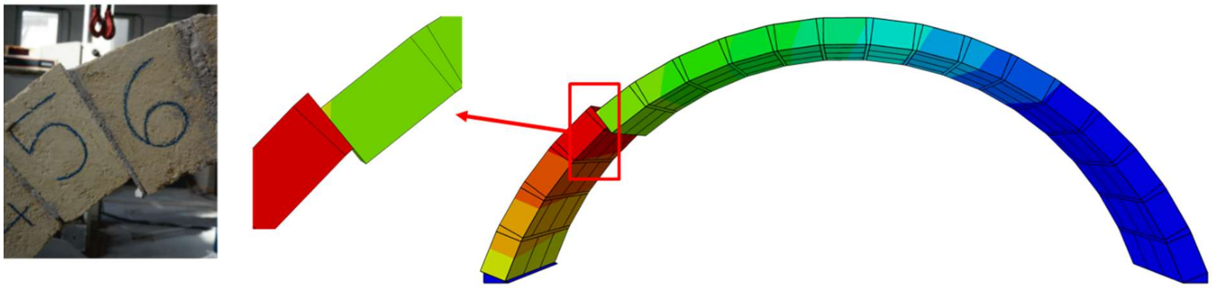


**Figure 11: Sliding failure after hinge formation in masonry arch (Castellano et al. 2019)**

## CONCLUSIONS

Masonry arch structures proved to be an effective way to resist large loads by just utilizing the compression strength of masonry. Over the years, analysis methods have evolved in both static and dynamic analysis of the masonry arch system. The thrust line method and mechanism method were developed to analyze the arch structure under static loading conditions. Various computer programs were developed to simulate the behaviour of the arch structure under static loading. All the static analysis methods approximate the arch structure as a two-dimensional structure; however, the load-carrying capacity of a masonry arch system can be significantly affected by the failure of other components in the transverse direction. Existing methods of static analysis can be used as a preliminary analysis tool to investigate the load-carrying capacity of a given structure, and detailed modelling is required to completely investigate the behaviour. The behaviour of masonry arch structures under dynamic loads was studied at a later stage. Masonry arch was represented as a single degree of motion system under dynamic loads. In the previous studies, the dynamic behaviour of the masonry arch system was studied only for the arch barrel and treating it as a two-dimensional structure. The dynamic behaviour of masonry arch system is complex and dependent on various factors like location of hinges, impact due to hinge formation etc. Previous dynamic studies show that the behaviour of masonry arch structure under dynamic loading is complex and depends on many input loading parameters like frequency and duration of pulse etc.

In the present study, a finite element micro model of a small-scale masonry arch system was developed, and its behaviour was explored under static and dynamic loading. The failure modes of arch structure and other components were simulated and compared well with previous experimental studies which validates the modelling technique. The general dynamic behaviour of arch structure in which it does not transform into mechanism till a particular PGA is achieved was observed in the model. This is a preliminary study of finite element modelling of masonry arch structure and the model needs to be calibrated based on the experimental testing of components and then the further studies can be performed on the arch structure.

## ACKNOWLEDGEMENTS

The financial support provided by the Ministry of Human Resource Development, Government of India, is gratefully acknowledged.

## REFERENCES

[1] Timoshenko, S. P. (1983), *History of Strength of Materials*, Dover, New York.

[2] La Hire, Philippe de. (1695), *Traité de mécanique*, où I'on explique tout ce qui est nécessaire dans la pratique des Arts, et les proprietés des corps pesants lesquelles ont eu plus grand usage dans la Physique.

[3] Coulomb, C. A. (1776), *Essai sur une application des règles de maximis & minimis à quelques problèmes de statique, relatifs à l'architecture,* Paris: De l'Imprimerie Royale.

[4] Gerstner, F. J., and Gerstner, F. A. (1834). *Handbuch der Mechanik*. Prag: Spurny.

[5] Moseley, H. (1835), *On the equilibrium of the arch*. Cambridge Philosophical Transactions, vol. 5, 293-313

[6] Heyman, J. (1966), “The stone skeleton.” *International Journal of Solids and Structures*, 2(2), 249-256

[7] Oppenheim, I. J. (1992). “The masonry arch as a four-link mechanism under base motion.” *Earthquake Engineering & Structural Dynamics*., 21(11), 1005-1017

[8] Clemente, P. (1998). “Introduction to dynamics of stone arches.” *Earthquake Engineering and Structural Dynamics*., 27(5), 513-522

[9] De Lorenzis, L., DeJong, M., and Ochsendorf, J. (2007). “Failure of masonry arches under impulse base motion.” *Earthquake Engineering and Structural Dynamics*., 36(14), 2119-2136

[10] Simulia. (2011). ABAQUS Analysis User's Manual (Ver. 6.13-3), *Dassault Systemes, Providence, RI, USA*

[11] Hooke, R. (1676). *A description of Helioscopes, and some other instruments*, John & Martin Printer to the Royal Society, London

[12] Barsotti, Riccardo & Aita, Danila & Bennati, Stefano. (2012). “Equilibrium of Pointed, Circular, and Elliptical Masonry Arches Bearing Vertical Walls.” *Journal of Structural Engineering*. 138(7), 880-888

[13] Harvey, W.J. (1988), “The application of the mechanism method to masonry arch bridges.” *The Institution of Structural Engineers.*, 66(5)

[14] Huerta, S. (2006), “Structural Design in the Work of Gaudí.” *Architectural Science Review*. 49. 324-339.

[15] Méry, E. (1840). “Mémoire sur l'équilibre des voûtes en berceau.*” Annales des ponts et chaussées, s. 1, le trim., t.* XIX: 50-70.

[16] Galassi, S. and Tempesta, G. (2019) “The Matlab code of the method based on the Full Range Factor for assessing the safety of masonry arches.” *MethodsX.*, 6, 1521-1542

[17] Livesley, R. K. (1978). “Limit analysis of structures formed from rigid blocks.” *International Journal for Numerical Methods in Engineering.*, 12(12), 1853-1871

[18] Gilbert, M., Casapulla, C., and Ahmed, H. M. (2006). “Limit analysis of masonry block structures with non-associative frictional joints using linear programming.” *Computers and Structures*, 84(14), 873-887

[19] Obvis Ltd. Home page, http://www.obvis.com

[20] RING home page, https://www.limitstate.com/ring

[21] Royles, R., and Hendry, A. W. (1991). “Model tests on masonry arches.” *Proceedings - Institution of Civil Engineers*. 91(2), 299-321

[22] Castellano, Anna & Fraddosio, Aguinaldo & Scacco, Jacopo & Milani, Gabriele & Piccioni, Mario. (2019). “Dynamic Response of FRCM Reinforced Masonry Arches.” *Key Engineering Materials*. 817, 285-292