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FINITE ELEMENT-BASED PROBABILISTIC BEHAVIOR ANALYSIS OF SLENDER REINFORCED MASONRY WALLS UNDER OUT-OF-PLANE LOADING

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ABSTRACT

Due to uncertainties inherent in masonry structures, large scatter is typically observed in the mechanical behaviour predicted experimentally and analytically. In order to lay down the basis for reliable structural design, rigorous evaluation of the uncertainty in the structural behaviour of masonry structures is of paramount importance. The present study investigates the probabilistic behaviour of reinforced masonry walls subjected to out-of-plane loading. To this end, a well-validated finite element (FE) model is used for probabilistic structural analysis. Material and geometric uncertainties are incorporated in the FE model, and the contribution of each uncertain variable to the overall probabilistic behaviour is evaluated. The relative importance of parameters concerning the lateral load capacity is assessed using variance-based global sensitivity analysis. It is concluded that the variance in the predicted probabilistic lateral load capacity is most influenced by the variance in the reinforcement properties such as yield strength and depth (bar location) followed by masonry compressive strength, while the other parameters have a minimal contribution.

KEYWORDS: *masonry, probabilistic analysis, finite element modelling, sensitivity analysis*

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INTRODUCTION

The behaviour of masonry walls can be predicted using different finite element techniques such as the micro finite element (FE) modelling approach [1], macro-FE modelling approach [2], and simplified analytical procedures [3]. In the micro approach, the constitutes of masonry structures (i.e., block, mortar, and grout) are modelled explicitly, aiming to capture complex failure modes. In contrast, the macro approach incorporates a homogenized equivalent for the masonry composites based on the macroscopic characteristics. Nevertheless, the behaviour is highly affected by different uncertainties that remain outside of the scope of deterministic prediction models [4]. Therefore, the safety margin is typically increased by adopting a conservative approach. Accordingly, to lay down the basis for a reliable structural design, rigorous evaluation of the uncertainty in the behaviour of masonry structures is essential.

Previous studies [5,6] investigated the effect of uncertainties on the behaviour of plain masonry structures by incorporating the randomness in material and geometric properties into a micro-model using Monte Carlo technique (MC) [7]. The stochastic analysis outcomes were used to assess the effect of spatial variability in a masonry wall. Predicting the behaviour of masonry walls based on a micro modelling approach is computationally challenging. Therefore, a macro modelling approach is considered a viable alternative, particularly when the global behaviour is of interest. Accordingly, the macro approach is adopted in this paper to study the probabilistic behaviour of slender reinforced masonry walls under out-of-plane loading.

To gain a deeper insight into the effect of variations of different input parameters on the response of interest, the developed model can be further used parametric studies or sensitivity analysis [8]. Sensitivity analysis is categorized into two main categories: local and global. In the local sensitivity analysis, the effect of input parameters on the considered response is assessed on a one-factor-at-a-time basis employing gradient-based techniques. In contrast, global sensitivity analysis quantifies the output variance by simultaneously accounting for the uncertainty of all input parameters, which allows a global assessment of their relative contribution, including interaction effects [8,9].

Although global sensitivity analysis provides more reliable measures for the relative influence of input parameters, it is often associated with a high computational cost [10]. A common way to overcome this problem is to adopt a surrogate model, which resembles an approximation of the original computational model with an enhanced computational performance [11]. To this end, different surrogate models have been developed, such as polynomial chaos expansion [9], Gaussian process regression (kriging) [11], and polynomial-chaos-based kriging [12]. Surrogate model-based global sensitivity analysis has been employed in several past studies [5,6,8,13]. In this paper, the polynomial chaos expansion-based surrogate model is employed to conduct global sensitivity analysis.

To summarize, a developed FE model based on the macro-modelling approach is adopted in this paper to study the probabilistic behaviour of slender reinforced masonry walls under out-of-plane loading considering the combined effect of uncertainty associated with different parameters. Afterwards, a variance-based sensitivity analysis was carried out to quantify the contribution of the parameters to the load-carrying capacity variance.

CASE STUDY

The probabilistic behaviour of reinforced concrete masonry walls is investigated based on masonry wall 1 tested in the experimental program carried out by American Concrete Institute and Structural Engineers Association of Southern California [14]. The wall considered was fully grouted with a nominal thickness (t) of 10 inches (254 mm) and a slenderness ratio (h/t) of 30 where (h) is the wall height. Material wise, the masonry prism strength (f_m) was 17 MPa and the wall was reinforced with five #4 (#13) bars of grade 60 (420 MPa). The wall was tested under pinned-roller conditions. Initially, the wall was loaded by an axial load (P) of 5.67 kN with an eccentricity e, equal to 7.62 cm plus half of the wall thickness. Afterwards, an air-bag loading device was used to apply uniform lateral pressure (q) until excessive deformation and extensive stiffness degradation were exhibited in the wall. More details about different experimental aspects can be found in the experimental report [14].

Numerical Model

The wall considered is modelled in an open-source finite element software framework, *Opensees* [15]. The wall is represented by 14 displacement-based fiber beam elements each with 5 integration points. At each integration point, the fiber section is defined, consisting of 20 masonry and 5 steel fibres, for which realistic uniaxial nonlinear material models are assigned to represent the stress-strain relationships for the corresponding materials. Specifically, masonry is assumed to behave elastically in tension until it reaches the peak tensile strength (f_t). After peak tensile strength is reached, the post-peak behaviour can be approximated by a linear decay until the maximum tensile strain (ε_{tu}) [2]. In compression, masonry is assumed to exhibit a parabolic pre-peak behaviour. After reaching the peak compressive strength (f_m) and the corresponding strain (ε_o), linear softening is exhibited to the ultimate crushing strain (ε_u) and the corresponding residual stress (f_{mu}) [3]. Thus, the masonry fiber is represented by a uniaxial concrete material model (i.e., *concrete02*), which mimics the aforementioned behavioural assumptions. On the other hand, the steel reinforcement bars are modelled using the bilinear steel material model (i.e., *steel01*) with kinematic hardening, which is in coherence with the provided stress-strain characteristics for the reinforcement bars used in the considered experimental program. It should be noted that the parameters of the adopted material models (i.e., *concrete02* and *steel01)* are deduced from the properties of the tested masonry prisms and steel bars. In this way, plastic behaviour can be well captured over the cross-section and along the wall height. The schematic view of the FE model for the wall considered is illustrated in Figure 1.

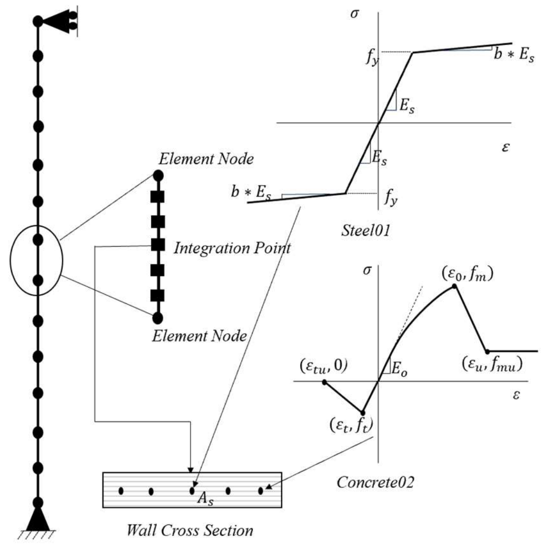


Figure 1: **Schematic view for the FE model of wall 1**

Numerical Simulation Results

Figure 2 shows the comparison of the FE-predicted and experimental load-displacement curves for the masonry wall. The numerical model is able to closely and accurately trace the behaviour until failure. The elastic behaviour, which is mainly influenced by masonry compressive and tensile characteristics, was well-captured. Furthermore, good agreement is observed regarding different behavioural phases such as cracking capacity, post-cracking stiffness, yielding onset and post-yielding stiffness. Due to the nature of the test specimen and apparatus, loading was stopped when a noticeable stiffness degradation and excessive deformation are observed. This prevents the experimental investigation of the near-peak behavioural characteristics. Nevertheless, the numerical model is able to predict the behaviour at significantly larger deformations with a slight increase in the predicted peak capacity compared to the experimental testing.

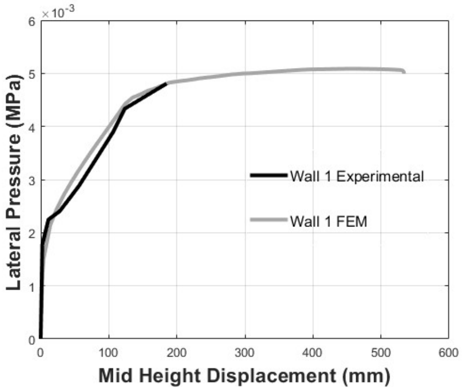


Figure 2: Comparison of the FE-predicted and experimental load-displacement curves for wall 1

PROBABILISTIC ANALYSIS

In order to understand the wall behaviour after considering material and geometric uncertainties, the FE model developed is used to perform probabilistic behaviour analysis of masonry walls. To accurately capture the statistical behaviour of nonlinear structural response [16], the Monte Carlo sampling technique is integrated with the FE model. This allows propagating the uncertainty in the basic random variables of the wall to the variation of the overall wall behaviour, characterized by the load-deflection curve. The basic random variables considered are masonry compressive strength (f_m), the strain corresponding to the peak strength (ε_o), masonry tensile strength (f_t), steel yield strength (f_y), elasticity modulus of steel *(E),* steel reinforcement location *(d),* wall height *(H)*, wall alignment *(l)*, and wall self-weight *(S.W)*. It should be noted that H and l account for construction tolerance allowed by CSA A371 [17], and the other basic random variables account for the material and geometric uncertainties with their statistical descriptors determined based on literature findings. A summary of the statistical properties is shown in Table 1, assuming these random variables are statistically independent. It is worth noting that $f_{m,nominal}$ is taken as 10 MPa, $f_{y,nominal}$ corresponds to the characteristic yield strength of grade 60 bars, $d_{nominal}$ is taken as 123 mm which corresponds to half of the actual block thickness and $S.W_{nominal}$ is taken as 43.2 kN representing the self-weight of the walls.

Table 1: Statistical characterization of random variables considered

Random Variable	Mean	Coefficient of variation	Distribution	Reference
f_m	$1.6\,f_{m,nominal}$	0.236	Gumbel	[18,19]
ε_o	0.002	0.2	Normal	[20,21]
f_t	0.5 MPa	0.27	Normal	[20,22]
f_y	$1.14\,f_{y,nominal}$	0.07	Normal	[19]
E	200000 MPa	0.033	Normal	[23]
d	$d_{nominal}$	$4/d_{nominal}$	Normal	[19]
l	±14.8		Uniform	[17]
H	±13		Uniform	[17]
$S.W$	$1.05\,S.W_{nominal}$	0.1	Normal	[24]

Probabilistic Analysis Results

To ensure an accurate representation of the probabilistic behaviour of the masonry wall, a sufficient number (i.e., 2000) of Monte Carlo Simulations is performed to investigate the probabilistic behaviour considering the combined effect of the aforementioned uncertainties, as shown in Figure 3. the 2000 samples are shown to be sufficient as they provide good coverage of the input multidimensional space and provide converged statistical characteristics of the quantity of interest (i.e., lateral capacity). The results show that a relatively large variation is observed, which indicates a large scatter associated with the behaviour of masonry walls, as showed by the 5th and 95th percentile of the load-displacement curves. It is also observed that the experimental curve of the wall lies in between the mean and the 95th percentile. To investigate the uncertainty in the lateral load capacity of the wall, the capacities obtained from the stochastic simulation results are fitted to a normal distribution with a mean (μ) of 0.0045 MPa and a coefficient of variation (COV) of 0.094, which best represents the statistical nature arising from the uncertainty in the basic random variables.

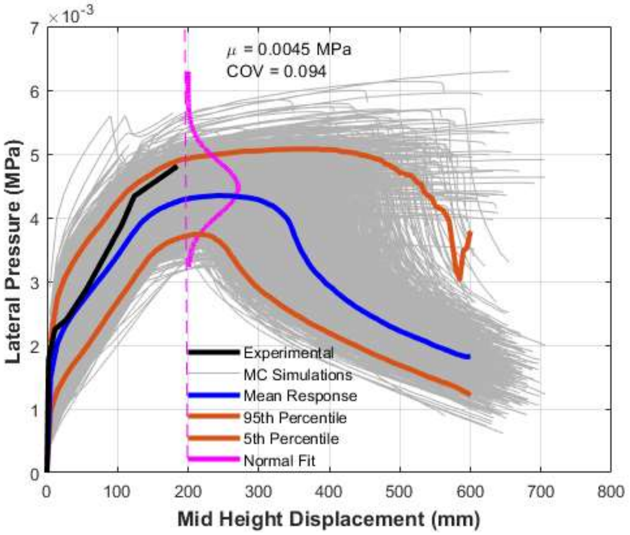


Figure 3: Probabilistic analysis results

SENSITIVITY ANALYSIS

The scatter plots between the lateral load capacities and all nine basic random variables are shown in Figure 4, using the samples generated from the MC simulation to visualize the effect of input parameters on the lateral load capacity. The lateral load capacity was found to be mostly correlated to steel reinforcement parameters, such as yield strength (f_y) and rebar location (d), and masonry compressive strength (f_m), because a strong positive trend is observed between them and the lateral load capacity, as also indicated by the high R-squared values here.

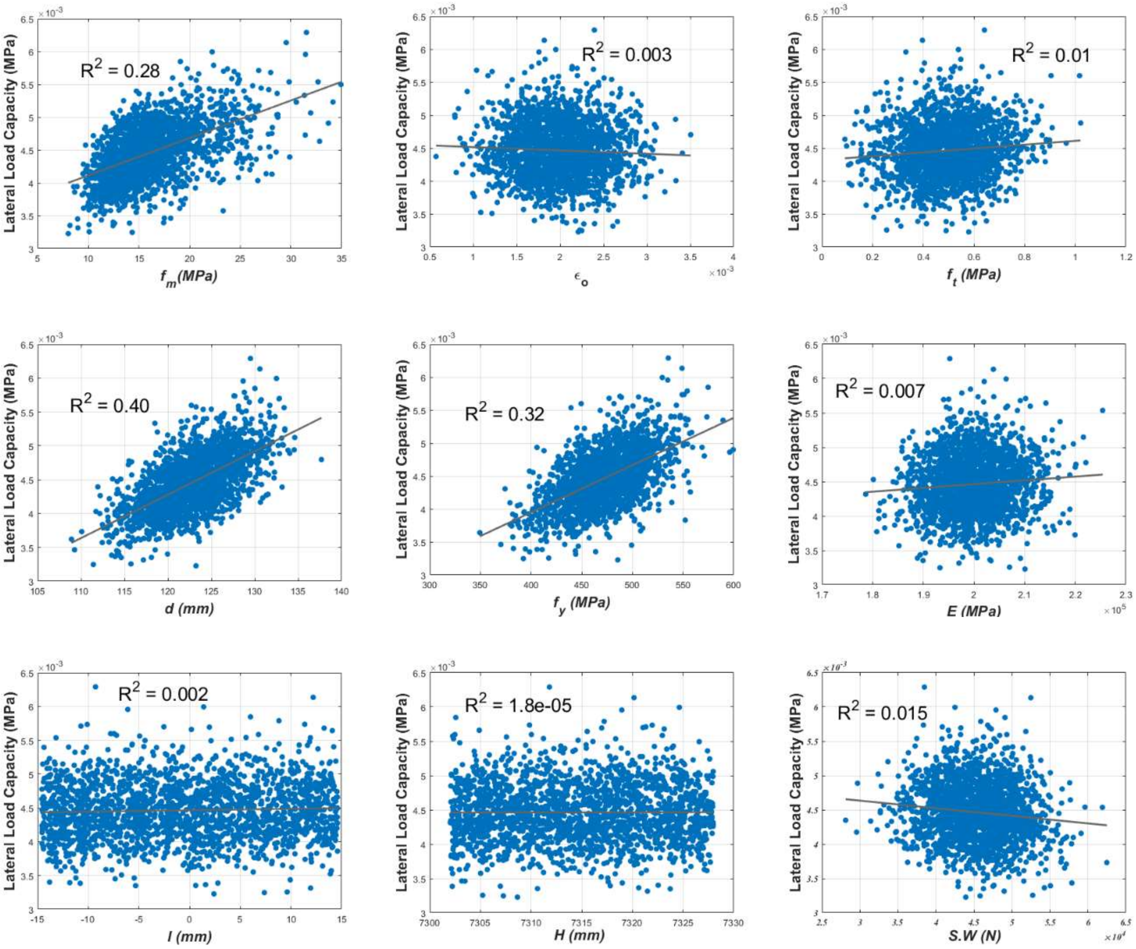


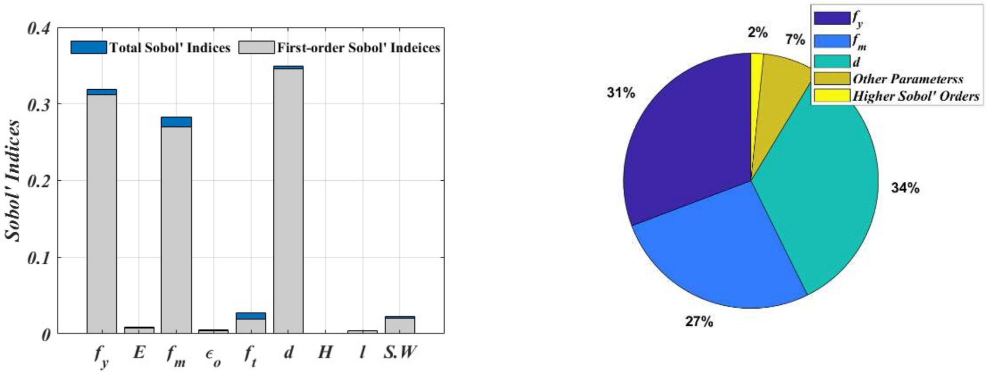
Figure 4: Scatter plots to show the relations between lateral capacity and basic random variables

To quantify the contribution to the variance in the lateral load capacity of the wall from the uncertainty in each basic input random variable, a global sensitivity analysis is conducted employing Sobol indices [25] as presented in the following section.

PCE-Based Sobol Indices

As concluded by previous studies [26], Sobol' indices are reliable measures for decomposing the output variance with respect to the different input parameters. However, a large number of realizations is required to obtain the Sobol' indices, which can result in a high computational cost [10]. To tackle this problem, the original computational model can be replaced by a fast-to-evaluate surrogate model, which resembles the original model with enhanced computational performance. Among different surrogate models, polynomial chaos expansion (PCE) [27] is selected to resemble the finite element model of the masonry wall. Afterwards, the polynomial chaos expansion-based surrogate model is employed to conduct global sensitivity analysis as it allows to analytically derive the Sobol' indices within its framework with no additional computational cost [28]. Thus, the 2000 samples simulated from MC simulation were used to construct the PCE surrogate model and thereafter perform the PCE-based Sobol' sensitivity analysis, as shown in Figure 5. It is interesting to note the same important variables, e.g., yield strength of steel (f_y), steel rebar location (d), and masonry compressive strength (f_m), as indicated by R-squared values, are the ones which contribute the most to the variance in the lateral load capacity.

In addition to the total Sobol' indices for the masonry wall, the first-order indices, which evaluate the amount of partial variance including one variable only, are also provided in Figure 5. It is shown that the first-order indices from the three important variables contribute to a significant amount (i.e., 92%) of the total capacity variance. In contrast, the other parameters and the interaction effects are insignificant, as indicated by their corresponding first-order and higher-order Sobol' indices.



(a) (b)

Figure 5: Sobol' indices:(a) Total and first-order indices, and (b) variance decomposition.

CONCLUSIONS

The probabilistic behaviour of a reinforced concrete masonry wall (Wall 1) was investigated based on a validated FE model by considering material and geometrical uncertainties. A considerable variation of the masonry wall behaviour was observed. Accordingly, the lateral load capacity of the wall was quantified statistically. Additionally, the contribution of each input basic variable to the capacity variance was quantified through variance-based global sensitivity analysis. It was found that the capacity variance was mostly attributed to yield strength of steel (f_y), steel rebar location (d), and masonry compressive strength (f_m). Note that the numerical model was assumed to be accurate without considering model error (uncertainty) which can also be significant compared to other influential parameters.

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