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# NON-FUNICULAR MASONRY DOMES

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## ABSTRACT
The author and his students have built several counter-intuitive and in some cases unprecedented masonry domes. Forms include the anticlastic or bell-shaped *pseudomes* and *antidomes* that descend from their foundation ring to form a basin. An *ambidome,* which rises in the normal manner but descends to a pendant oculus, is shown in Figure 1. The domes are all unreinforced and un-mortared masonry, and un-bound except by a tensile foundation ring. The completely flat *floordomes* achieve span-to-depth ratios up to 27:1. Some are made of voussoirs that taper upward rather than down. Yet all the structures are built of loose blocks held in place by gravity alone. Our experiments to date are small in scale, but no matter how unbelievable they appear in cross section, physical demonstration of their inherent stability is incontrovertible. None of the domes are understandable as arches rotated about a vertical axis, but seen as vertical stacks of fully circular horizontal arches they begin to make sense. This conceptualization emphasizes the hoop compression that pre-stresses and stabilizes extant shallow domes, and reminds us of the horizontal and vertical shear forces that act throughout all "compression-only" domes and arches. Some of the forms look more useful than others, but all of them are instructive. They show that our funicular conception of domes is incomplete, and suggest ways of broadening our perspective. Whether unreinforced masonry is valued for social or environmental sustainability or to avoid rust-driven failures, any improvements to masonry theory should be of value.

**KEYWORDS:** *funicular, non-funicular, masonry, compressive shells, arches, domes*

## INTRODUCTION
Modern construction is characterized by the tensile capacity and the cheapness of steel, whether in the form of structural sections, fastenings and connectors, tie rods and cables, or brick ties and reinforcing bars. Un-reinforced masonry can feel like a pre-industrial relic, and the study of compressive spans and shells either a historical pursuit or a computer-generated novelty. But there are two compelling reasons to understand and exploit pure masonry construction. In the developed world, rust is a huge economic burden. Our infrastructure is not crumbling, as the

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cliché has it, but corroding. Where masonry and concrete constructions fail, this is generally due to oxidizing shelf angles, cavity ties, and re-bar. With some Roman aqueducts 2000 years old and still carrying water, it should be possible to interest bond-rating agencies in compression-intensive structures. At the same time, in the less developed world, where tensile materials cost scarce cash, where employment is a benefit, and where trees in their growing state are a precious resource, locally sourced stone and brickwork are the ecologically and socially sustainable material options. Both industrialized and developing economies, therefore, can benefit from a renewed interest in un-reinforced masonry spans and surface structures.

The Western understanding of masonry arches and domes leans heavily on the funicular paradigm initiated by Hooke in the $17^{th}$ century and elaborated by Poleni, Gaudí, Otto, and others in the years since. The spectacular shells devised in recent years by Block, Ochsendorf, and others are elaborated by modern calculators, but are conceived entirely within the established paradigm. Perhaps the best single account of these developments, from the seminal ideas to their contemporary cutting edge is Adriaenssens, et. al. *Shell Structures for Architecture: Form Finding and Optimization* [1]. The present paper, on the other hand, explores forms outside the funicular realm, continuing the investigations reported in Jannasch [2]. It examines non-funicular structures that predate Hooke's original insight, or were built outside the Western tradition. It also describes novel domes developed by the author and his students not anticipated in received theory. The structures are un-mortared and the smallest are made of 3d printed plastic. The slipperiness of this material puts our geometries to a stricter test than adhered masonry would, subjecting them not only to hinging failures but also to the ejection of elements due to slippage along joints. To date, our experimental structures have been small. But as shown by Heyman [3], the critical consideration in masonry, even in the most attenuated gothic cathedral, is neither strength nor stiffness of material but stability. And stability of configuration can be tested at scale. In this respect, the success of our models points the way directly to larger buildings, and more importantly, to a sturdier theory. It is worth noting here that all arches and domes generate horizontal thrust that must be contained at the base whether by tensile members or geological resistance. Prolonging a dome or arch downward or thickening it toward the base doesn't reduce this horizontal component but merely obscures it. Our structures work within this imperative.

**Figure 1: Ambidome exterior and interior: 3d printed ABS blocks, 240mm diameter**

**THIN ARCHES AND MARGINALLY STABLE DOMES**

Masonry domes are much more than rotations of freestanding masonry arches, and exhibit more complex behaviours than compression along a line of thrust. Our innovative structures illustrate and exploit these other behaviours. Nonetheless, a review of arch concepts is a sound basis for these investigations. Definitions vary, but we take *catenary* to mean the form described by any free-hanging continuum, whether of constant or variable section. *Funiculars* are a broader class of load trajectories that may additionally reflect superimposed loads. In both cases we apply the term both to upright and inverted (or tensile and compressive) forms, letting context provide the distinction. The *plain* catenary is the funicular of exclusive and constant self-weight; the *parabola* is the funicular of negligible self-weight plus dead loads distributed uniformly along the horizontal. Comparing a catenary and a parabola of the same height and span, we find that the loads on the parabola are more closely spaced along the arc length at the middle than at the ends: it thus bends more sharply at the midpoint than its catenary cousin, and runs straighter to the ends. The *more acute* parabolically loaded chain thus falls inside a catenary of the same aspect ratio. We can also imagine a funicular that is thickened in proportion to the load carried. A catenary refined in this way weighs less per arc length in the middle and increasingly more toward the ends. Such an *equally stressed* catenary will be more *obtuse* than the plain catenary, and hang to its outside. Catenaries may also be weighted for purely formal purposes. The voussoirs in the very thin arch shown in Figure 2 are cut from the same bevelled stock, ensuring that the structure follows a circular arc. The length of the voussoirs was cut in direct proportion to the weights required to pull a hanging chain into a circular arc.

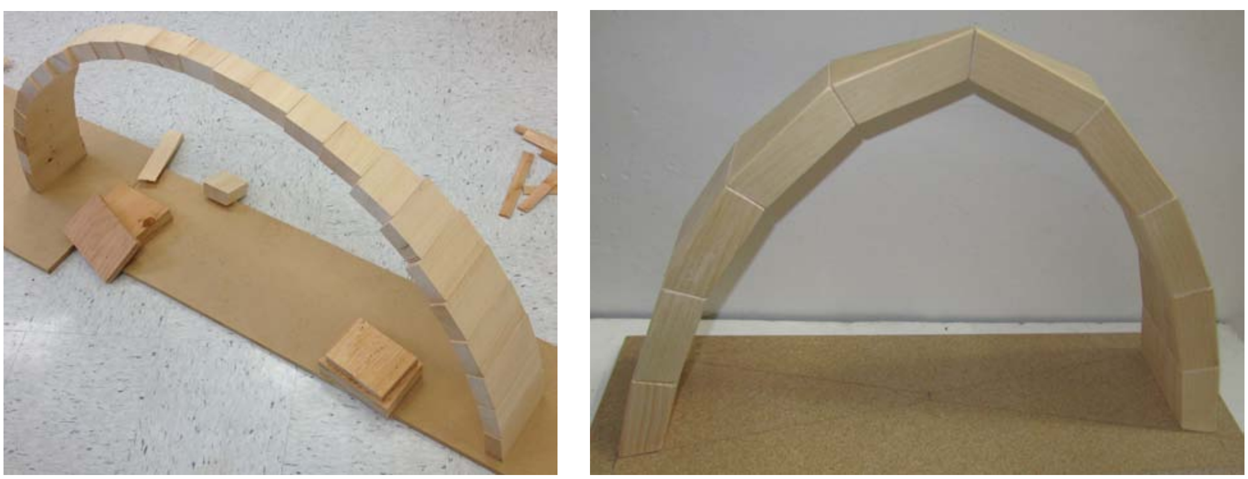

**Figure 2: Arc-Weighted and Bowtie-Weighted Catenaries (1m and .5m span)**

An important example of a weighted catenary is that of an arch selectively widened to fall on a bowtie plan. The example shown was worked out from a triangular voussoir sloping downward from the apex of the arch. The thrust of this topmost element was calculated, and the slopes of the remaining stones – of the same arc length – were adjusted, iteratively downward, to generate the same horizontal forces at each joint. These forces vary with the slope of the voussoir, but the relationship is not straightforward: as the slope of the voussoir is adjusted by rotating it about its upper edge, the lower edge of the trapezoid changes length, modifying both the mass of the voussoir and its centre of gravity. In any case, the *bowtie* catenary is evidently heavier toward the bases than at the midpoint, and is thus more obtuse than a plain catenary of the same height and span. This arch exhibits no behaviour perpendicular to its central plane. Rotating the arch about its vertical axis would therefore generate a dome with no tangential or hoop action, whether compressive or tensile. We have known since Poleni's explanation in the $18^{th}$ Century that the radial slice or *lune* of a dome is not weighted uniformly, and that it defines a different funicular than an arch of continuous section. The bowtie-weighted catenary arch is exactly congruent to the profile of a funicular dome: the profile that we call, in Poleni's honour, a *lunar catenary*.

Any distribution of loads along the arc length of a chain generates a characteristic funicular. Hooke's great contribution was to declare that the form of this chain inverted into a thin arch yields a compressive funicular that will be stable under the same loads. An arch more obtuse than its own funicular can be called *hypofunicular*. It will burst outward. An arch more acute than its required funicular can be called *hyperfunicular*. It will collapse inward. The critical difference between arches and domes is that while both hypo- and hyperfunicular arches will collapse, and the hypofunicular dome will collapse, the hyperfunicular dome will not. (See Figure 3.) The lunes of a hyperfunicular dome press inwards upon each other, generating compressive or negative hoop stresses. Compared to a thin funicular dome, which will collapse under the least live load, the hyperfunicular dome (restrained, of course, at the base) can be pre-stressed against substantial live loads. It is more stable than a dome notionally "optimized" to a funicular form. Related problems of optimization and form-finding of arches and domes are discussed in Jannasch [4].

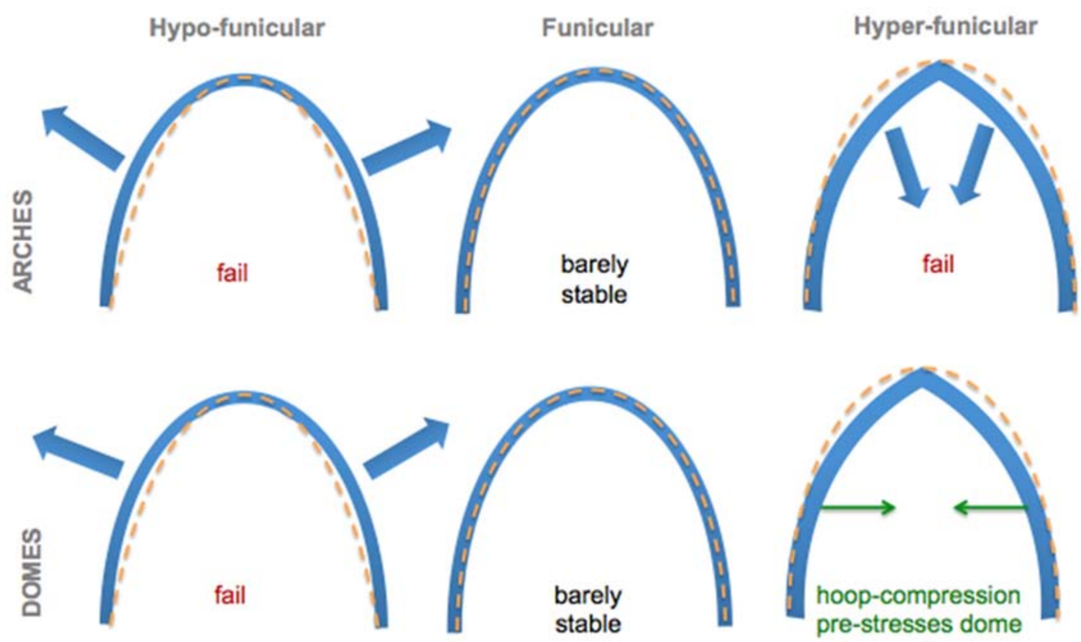


**Figure 3: Hyper-Funicular Domes are Robust**

## HYPERFUNICULAR DOME FORMS

Both parabolas and plain catenaries are more acute than the lunar catenary, so domes built to these curves are hyperfunicular. Heyman [5] shows that a uniformly thick spheric dome is stable as long as it doesn't extend downwards beyond 51.82º latitude. In our terms, spheric domes shallower than this are hyperfunicular. A straight-sloped cone also yields a robust hyperfunicular dome, leading us to reconsider Wren's cone at St. Paul's, London, seen in Figure 4. A drawing shown by Addis [6] portrays a section of St.Paul's overlaid with a graphically inverted hanging chain. The image suggests that the cone is merely a lazy attempt at a catenary. A better interpretation of the cone would be as the straight-sided funicular of the significant central load and the negligible weight of the cone. But neither story fully credits the insight and pragmatism of Wren, or of his likely collaborator, Hooke. In fact, Wren knew that each course of the dome, upon completion, formed a fully circular horizontal arch, pre-stressed as the bricks pulled downward and inward against each other. He saw that the cone was stable for every load case, from the first course through every stage of construction; before and after the surcharge of the lantern. Wren demonstrates his appreciation of the horizontal arches acting throughout the cone with two features. First, he lands loads from the exterior timber dome onto discrete rings of the conical structure, without introducing a funicular knuckle. Then, the windows in the body of the cone are circular. This is not a decorative touch: the openings are invisible to the interior and are purely functional. Their roundness shows that Wren knew the framing arches to be carrying horizontal loads tangential to the cone as well as vertical loads along its ruling lines. This logic plays out at the uppermost ring of windows that are more conventionally parallel-sided – because at this point they fall between radial ribs that transfer the load of the lantern vertically downwards, without generating tangential forces. Domes, especially those terminating in an oculus or lantern, can't really be explained as arches rotated about a vertical axis. A much sounder explanation of domes is a vertical stack of complete horizontal arches. With this understanding in mind, some previously counter-intuitive domes may be built.

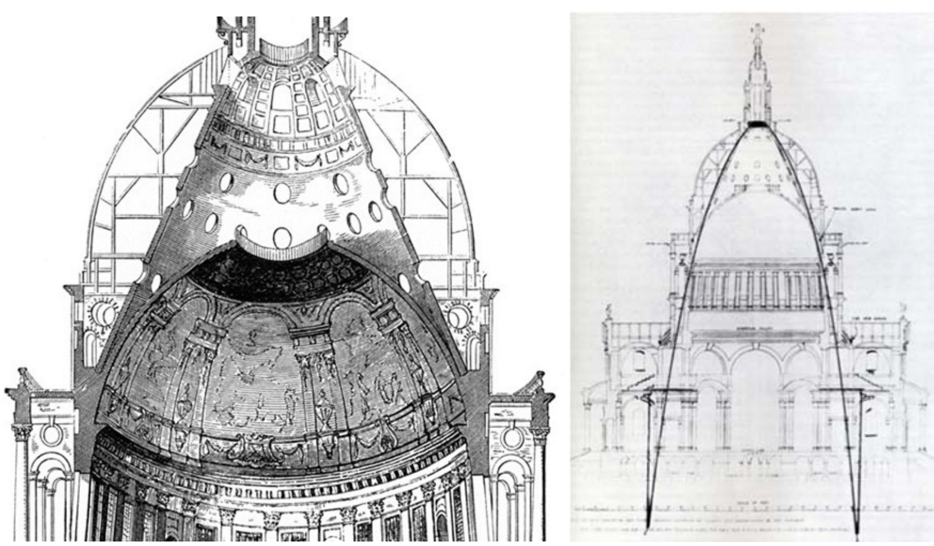

**Figure 4: Wren's Conical Dome at St. Paul's, London, and a Mistaken Interpretation**

Our team has built domes with concave or bell-shaped cross sections that we call *pseudomes* for their affinity to the geometer's pseudosphere, shown to the left in Figure 5. The hoop-wise compression that stabilizes these domes increases rapidly downward, both with the accumulation of self-weight and with decreasing slope. One special advantage of the anticlastic surface is that each voussoir tapers inward in plan and outward in section, which makes it hard to dislodge in either direction. For this reason we call the forms *radically* hyper-funicular.

Heyman [8] alerts us to an anticlastic dome appearing in 1840, in Mackenzie's [7] account of the fan vaulting at King's College, Cambridge. Mackenzie wasn't proposing such a structure be built, he was asking readers to mentally invert the real fans to better understand their shell action. In grasping the hoop-compression in fan vaults and the possibility of pseudomes he was ahead of other structural thinkers. We reversed his thought-experiment by inverting a small pseudome into a fan, in the middle of Figure 5. The tiny base leaves the structure vulnerable to tipping, but the fan itself is internally stable. A hyperboloid or catenoid cooling tower could also be built as a compression structure, with one tension ring at the top and another at the bottom. A small *enantiodome,* as we call such forms, is underway.

**Figure 5: Pseudome (2.3m), Fandome (0.8m) and Checkerdome (.5m)**

One curious opportunity is a chequered openwork dome in which successive courses are interrupted in their circular progression but overlap vertically just enough to provide continuous hoops. An initial sketch is shown in the right of figure 5. A masonry pergola suggests itself as more durable than one of timber or steel. Assembly and false-work clearly pose challenges, but hopefully the greater value is in helping to expand our sense of masonry's potential.

## SUBNORMAL AND SUPERNORMAL COURSING

In the structures considered so far, the bedding joints between courses are assumed to be normal (in the sense both of "usual" and of "perpendicular") to the meridians. Bedding joints steeper than normal would accentuate the wedging action of the conical courses being drawn downward. Although increasing the horizontal thrust would seldom have much value, models with sharply wedged courses nonetheless demonstrate that normalcy is far from necessary. A more useful technique is to slope the joints shallower than normal to the meridians, thus reducing radial thrust. Both *supernormal* and *subnormal* coursing are shown in Figure 6. The extreme case of sub-normalcy is the corbelled dome, where the slope and the thrust are zero. Although eliminating thrust would seem to be a useful achievement, we nonetheless discount corbelled structures as somehow more primitive than their arched counterparts. Consequently, the modern European tradition originating with Hooke has not paid corbels much attention. The printed plastic example shown in Figure 7 shows just how corbelled domes can defy sectional logic.

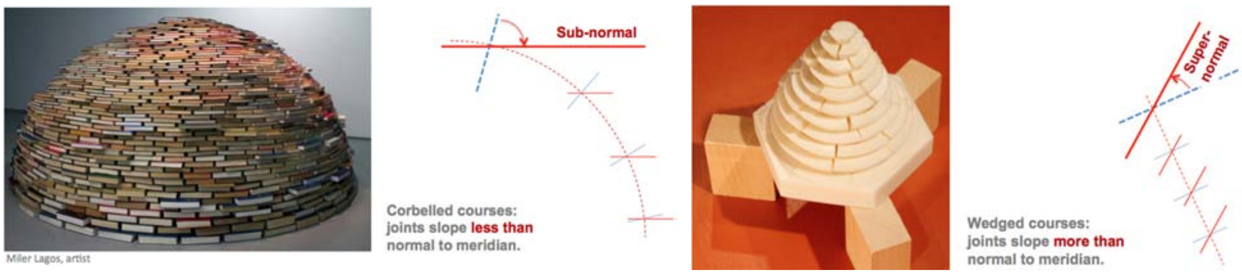


**Figure 6: Subnormal and Supernormal Bedding Joints**

Between the normal joint of the arch and the horizontal joint of the corbel, there is a whole realm of subnormal angles. While they don't eliminate horizontal thrust, non-zero sub-normal joints do reduce it. The structural forms they produce are intermediate between corbel and arch and appear to be unnamed in the European literature. They are also under-exploited in modern European practice. In his account of the domes of Cairo, Bernard O'Kane [9] quotes John Ochsendorf – an international expert on masonry shells – to say that the apparently hypo-funicular domes of Cairo stand "contrary to the known laws of engineering mechanics". Subnormal coursing indicates one direction in which the corpus of laws known to modern Westerners (and regulating their work) might be expanded.

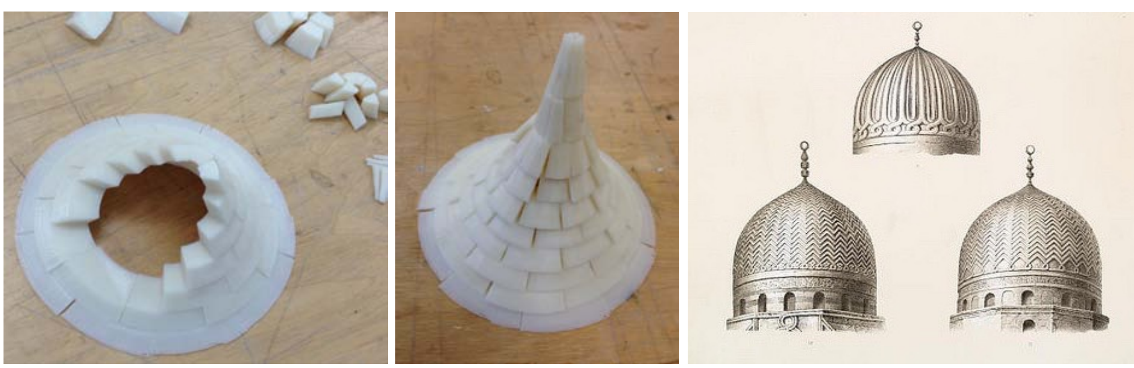

**Figure 7: Corbelled and (Presumably) Sub-normally Coursed Domes**

## FLAT VAULTS OR FLOORDOMES

Subnormal joints enable some of the most counter-intuitive domes to be built, such as the flat floordomes. Figure 8 shows two examples. Our own diminutive model demonstrates how the voussoirs sliding down and into each other form a cone that cannot fall through the supporting masonry. The span-to-depth ratio is conservative, especially compared to its companion: Juan de

Herrera's flat vault in the Basilica of San Lorenzo, at El Escorial, near Madrid. This remarkable structure stands outside the funicular paradigm established by Hooke and predates it by a century or more. Bill Addis [10] shows it as 7.81m in diameter and only 0.285m thick. However, where Addis describes it as a two-way spanning arch, this doesn't seem plausible. Our experimental arches based on Addis' sectional drawing all failed, apparently due to the shallow angle of the keystone. Herrera's floor depends on circular compression that circumvents the central masonry.

**Figure 8: 3d Printed Plastic Floordome (150mm dia.) and Herrera's Vault (7.81m dia)**

In the 3d printed plastic floordome in Figure 8, all the bedding faces fall on parallel cones. In the nine-petaled concrete block structure in Figure 9, all the diagonal edges radiate from a central point below the floor, so that the faces, even if skewed, radiate in conventional arch-like fashion. Both examples are thick enough that one can imagine tracing a conventional line of thrust through them. The thinner plastic structure to the right, however, simply can't be read as an arch. The cones get shallower toward the middle, so that as voussoirs in a straight arch, the blocks would simply be ejected downward.


**Figure 9: Conventional and Inverted Tapering of Voussoirs**

Floordomes of this type may successfully be inverted onto a central support, in the manner of a parasol. It occurred to us that rather than 3d printing the blocks, we could have cut them from a slice of onion taken across the polar axis, above the equator. This would also have provided the geometric necessity: of having the outer bedding joints more steeply inclined than the outer ones. The surrealism of this novel construction is captured in the designation *onionbrella.* (Figure 10.)

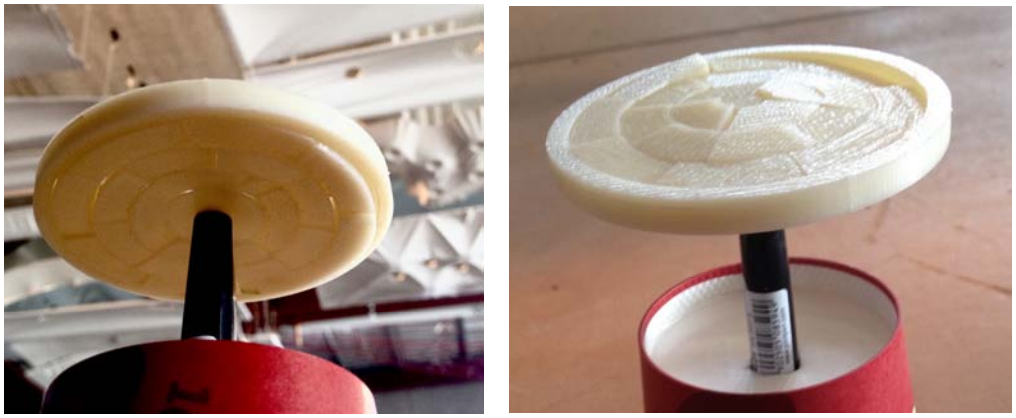

**Figure 10: Floordome Inverted Into Onionbrella (75mm)**

In some floordomes, including Herrera's, the stones are longer in the radial direction than they are thick, and express a propensity not only to slide diagonally but to rotate in- and downward. What prevents this rotation is the wedging action of each stone into the space between its neighbours. At the wider part of the block, above its axis of rotation, angular motion forces it directly into the wedge. As the block narrows, the radius of action lengthens - but the block moves into the wedge only obliquely. As it turns out, the obliquity of action cancels the length of action so virtual displacement into the wedge is identical throughout the radius. (See Figure 11.) The entire upper surface of the block is thus stressed equally in a hoop-wise direction. This distribution of compression goes some way toward explaining such an implausible structure.

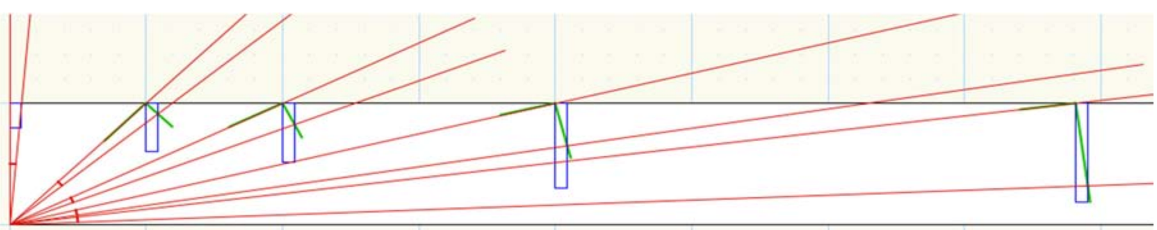

**Figure 11: Wedging Action of a Floordome Voussoir Is Equal Along its Radial Length**

**ANTI- AND AMBIDOMES**

The floordome in Figure 12 has a similar span-to-depth ratio as Herrera's, though fewer courses. The blocks turned out somewhat smaller than designed, hence the wooden shims. These shims are still slightly undersized, so that the floor sags a bit below level. Domes that bulge downward we call anti-domes. Those built from the kit of cone parts are the most explanatory. Depending on the diameters of the courses selected, these antidomes can step down at different slopes. In the steepest one, shown to the right in Figure 11, the courses to the inside and the outside of any given course fall entirely above and below each other, so that the course in question is acting in explicit horizontal shear and explicit vertical tension. Antidomes have the least applicability of all these novel domes but they remind us that there's more to any dome than meridional thrust. And if we return to the ambidome in Figure 1, we can see that the annular shelter, the pendant oculus, and the strong vertical axis are all generated by the central antidomical courses.

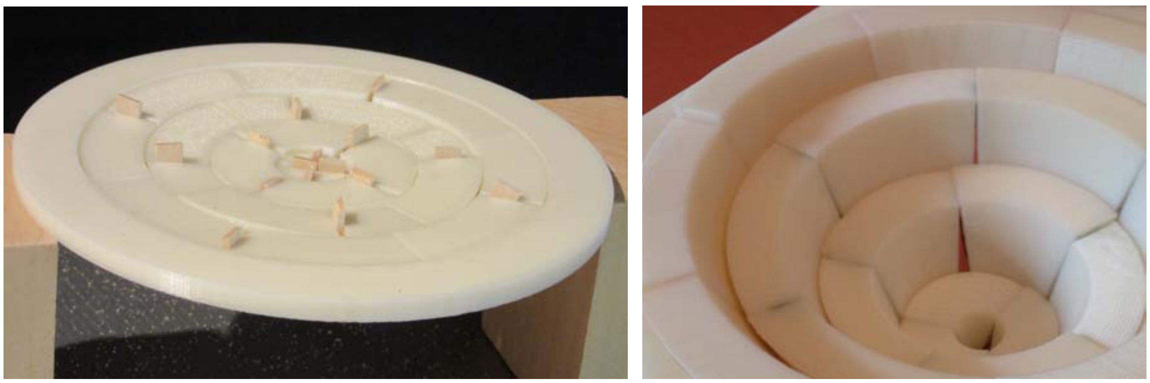

**Figure 12: Shallow and Deep Antidomes**

Some observers wonder that tensile action in the antidomes seems to "come out of nowhere". Perhaps the orthogonal analysis is to blame. At a granular level, orthogonal shear forces are better understood as coincident diagonal compression and tension, as is borne out in the diagonal "shear" failure of concrete beams. Of the two accounts in Figure 12, it may be the more elegant.

**Figure 13: (Blue) Shear Action in Floor Dome and Arch; Resolved into Diagonal Forces**

**CONCLUSIONS**

At this point, how much application these novel structures will find is anyone's guess. The ambidome and the pseudomes have a particular spatial character that we believe will be exploited architecturally. Pseudomes and conedomes and even the synclastic hyper-funiculars offer considerable structural safety. Flat vaults are being developed and used in India, independently of our work, for example by Meera Prajabati [11]. But regardless of their direct applicability, all these investigations provide structural designers with important lessons. It's common to speak of masonry domes (and arches) as "compression-only" but we should not neglect the shear actions so clearly demonstrated in the anti-domes. Often these actions can be ignored safely, but sometimes, for example where material is weak or the shell thin, or pierced by substantial openings, shear forces must be explicitly accounted for. An elemental structural action available to dome-builders is hoop compression: this inherent pre-stress is virtually free of cost and can provide a significant margin of stability. Whether we use unreinforced masonry for durability or for sustainability, these actions should be in every masonry designer's conceptual tool kit. This is not to say that tension members are an inherent problem always to be eliminated: there are situations where they are desirable and tensile materials that are durable. The point is only that many more things are possible in unreinforced masonry than we have imagined. And beyond the scope of masonry design, a critical awareness of our "axiomatic" assumptions must always be a good idea, even where these assumptions are enshrined in professional legalisms, in the inertia of everyday practice, or in the black boxes of commercial software.

**ACKNOWLEDGEMENTS**

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**IMAGES**

All images are by the author and the Antidome team except Figure 4 left, Dimmock (Project Gutenberg), Figure 4 right, RIBA Drawings Collection, Figure 7 left Miler Lagos, and Figure 8 right, Prissse d'Avennes (islamic-arts.org.)