BUCKLING OF REINFORCED AND UNREINFORCED MASONRY WALLS - A UNIFIED SOLUTION FOR EUROCODE 6

Bakeer, Tammam[1] and Jäger, Wolfram[2]

ABSTRACT

Several empirical formulae were proposed for the practical estimation of the load bearing capacity of reinforced and unreinforced masonry walls under buckling. Critics were reported on the methods of Eurocode 6. The solutions provided in the current version of Eurocode 6 show inconsistency at a specific range of masonry materials. The authors have recently proposed a solution for unreinforced masonry walls to replace the current formula in EC6. Some other works proposed solutions for reinforced masonry based on the nominal curvature. However, the current work shows that one unified solution can be proposed for both unreinforced and reinforced masonry walls and no need to handle each case separately. The proposed solution is based on representing the relationship between the capacity reduction factor Φ and the slenderness ratio λ by using two curves. One is a parabola for the material failure region and the other one is a hyperbola for the stability failure region. The proposed solution for unreinforced masonry have been checked against experimental data and the calibration parameter has been determined. The same concept has been used to generalize the proposed solution for reinforced masonry walls as well.

KEYWORDS: *Eurocode 6, empirical method, buckling, regression-based models, capacity reduction factor, stability failure, material failure*

INTRODUCTION

In the original draft of Eurocode 6, the capacity reduction factor has been approximated by a linear formula but this has been criticized because the formula gives rise to negative capacity reduction factors at high values of slenderness and replaced later on in ENV with the exponential formula of Kirtschig [9]. However, critical remarks also have been made on the empirical formulae of

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Eurocode 6 as well, because it doesn't consider softer types of masonry in some European countries e.g. Denmark [6].

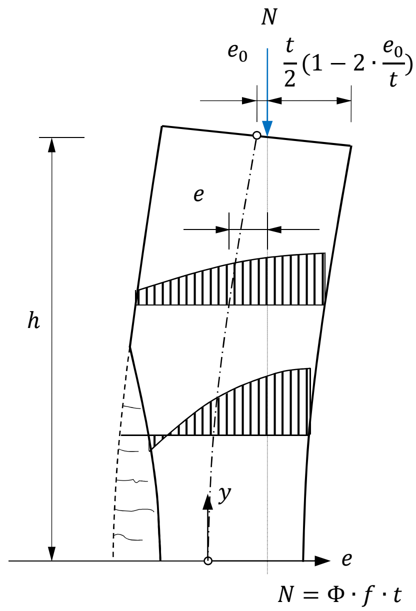

Figure 1: The stress distribution at transverse cross sections of a masonry wall of thickness *t* subjected to a buckling under eccentric loading *N*

Figure 1 shows the deformation and stress state of masonry wall of a height h and a thickness t under an eccentric loading and considering the second order effect. It is more appropriate for the presentation of the empirical formulae to use a relative form description for the buckling problem of masonry walls. For practical use and standards, the load bearing capacity is represented by the capacity reduction factor Φ for the compressive strength allowing the actual conditions:

$$\Phi = \frac{N}{f \cdot t} \tag{1}$$

where f is the compressive strength of masonry, N the vertical load bearing capacity per unit length, and t the thickness of the wall.

The capacity reduction factor Φ is influenced by the relative eccentricity of the load applied at the ends of the wall $r_{e_0} = e_0/t$ and the slenderness ratio of the wall λ which in turn depends on the geometry, the stiffness of the cross-section, the boundary conditions, and the existence of any lateral loads. The slenderness ratio λ is defined as following:

$$\lambda = \frac{h}{t}\sqrt{\frac{f}{E}} \tag{2}$$

INCONSISTENCY IN THE CURRENT EC6 FORMULA

The formula in the annex G of EC6, is based on the Gaussian bell-shaped function, which is introduced by Kirtschig [12]. The goal was to have an approximation which gives the load bearing capacity for slender walls in a better way than any linear or other approximation eg. the former DIN 1053-1-solution [11]. Kirtschig used the value $E/f_k = 1000$ to determine the variables which currently can be found in the Eurocode 6:

$$\Phi = \left(1 - 2r_{e_0}\right) \exp\left(-\frac{1}{2}u^2\right) \quad with \quad u = \frac{\lambda - 0{,}063}{0{,}73 - 1{,}17 r_{e_0}} \tag{3}$$

This approximation was not calibrated over different values of $E/f_k = const.$ [13].

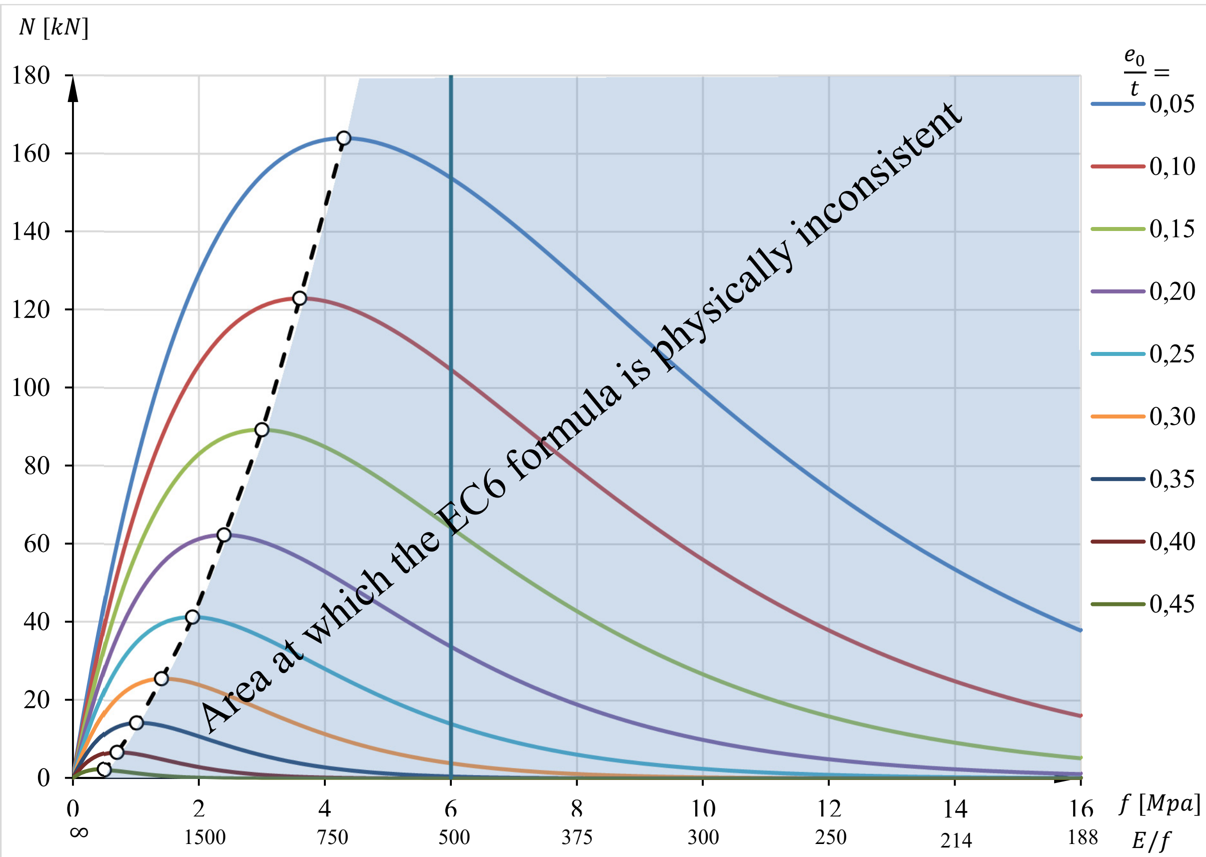

Figure 2: Plots for the load bearing capacity as a function of the compressive strength f for the EC6 formula. The following parameters are assumed for the calculation, $E = 3000\ [Mpa]$, $\frac{e_{mk}}{t} = 0{,}05 - 0{,}45$; $h_{ef}/t_{ef} = 25{,}9$

The inconsistency in the formula (3) can be detected by drawing the relationship between the load bearing capacity N and the characteristic compressive strength of masonry f_k under consideration of constant elastic modulus E and $(h, t, e_0, \dots etc.)$. The load bearing capacity with respect to the compression strength been plotted in 0, using the following parameters: $E = 3000$ [Mpa], $\frac{e_{mk}}{t} = 0{,}05 - 0{,}45$, $h_{ef}/t_{ef} = 25{,}9$. It can be found that the function $N(f_k)$ of the load bearing capacity with respect to the characteristic compressive strength is a decreasing function for $K_E = \frac{E}{f_k} < 500$. The 0 shows the area at which the EC6 formula is invalid or at which is too conservative. The diagram clears the reason for this inconstancy: the EC6 formula has a peak. After this peak N decreases as f_k increases which is illogical. The reason for this is, that two different types of failure were represented by one formula levelling these both types. The bell shaped curves had been calibrated in the area $700 < K_E \leq 1000$.

PROPOSAL FOR UNREINFORCED MASONRY

Based on numerical investigations made in [1, 4, 5], the authors proposed an empirical formula for unreinforced masonry in [2]. The formula were derived for the perfectly plastic material and it makes distinguish between material failure and stability failure:

$$\Phi_m = \begin{cases} A_1 - \dfrac{1}{3{,}15\,\zeta A_1}\cdot\lambda^2, & \lambda < A_1\sqrt{1{,}575\,\zeta} \\ \dfrac{0{,}79\,\zeta A_1^3}{\lambda^2}, & \lambda \geq A_1\sqrt{1{,}575\,\zeta} \end{cases} \tag{4}$$

where ζ is factor takes into account the degree of nonlinearity.

$$A_1 = 1 - 2\frac{e_m}{t} \tag{5}$$

The slenderness λ is given by:

$$\lambda = \frac{h_{ef}}{t_{ef}}\sqrt{\frac{f_k}{E_{0k}}} \tag{6}$$

The factor ζ should be determined by having enough number of experiments. The above formula has been calibrated with test data and found that $\zeta \approx 1$. Therefore, eq. (4) becomes:

$$\Phi_m = \begin{cases} A_1 - \dfrac{\lambda^2}{3{,}15A_1}, & \lambda < 1{,}26A_1 \\ 0{,}79\dfrac{A_1^3}{\lambda^2}, & \lambda \geq 1{,}26A_1 \end{cases} \tag{7}$$

where E_{0k} is the initial characteristic elastic modulus, and h_{ef}, t_{ef}, t are defined in clause 6.1.2.2 of EC6, e_m is the eccentricity due to loads; The values of Φ_m are represented in graphical form in Figure 3 and Figure 4.

In the stability failure, the load bearing capacity is only dependent from the initial elastic modulus E_0. It is necessary here to use a design value for the elastic modulus E_{0d}. Assuming equal uncertainties in the compressive strength of masonry and the elastic modulus, it gives

$$\frac{f_d}{E_d} = \frac{f_k/\gamma_M}{E_k/\gamma_M} = \frac{f_k}{E_{0k}} \tag{8}$$

The initial elastic modulus E_0 shall be used to calculate the stability failure. This is different from the secant elastic modulus E defined in section 3.7.2. and given by EN 1052-1 [7]. From there follow the mean value as secant modulus in the height of 1/3 of f. If there are no test data provided, the initial elastic modulus should be taken as $E_0 = (1{,}1 - 1{,}2)\,E$. The relationship of the characteristic value of the initial modulus depending from the test according to EN 1052-1 is:

$$E_{0k} = \frac{E_{0mean}}{1{,}2} = \frac{(1{,}1..1{,}2)E_{mean}^{EN\,1052-1}}{1{,}2} = E_{mean}^{EN\,1052-1} \tag{9}$$

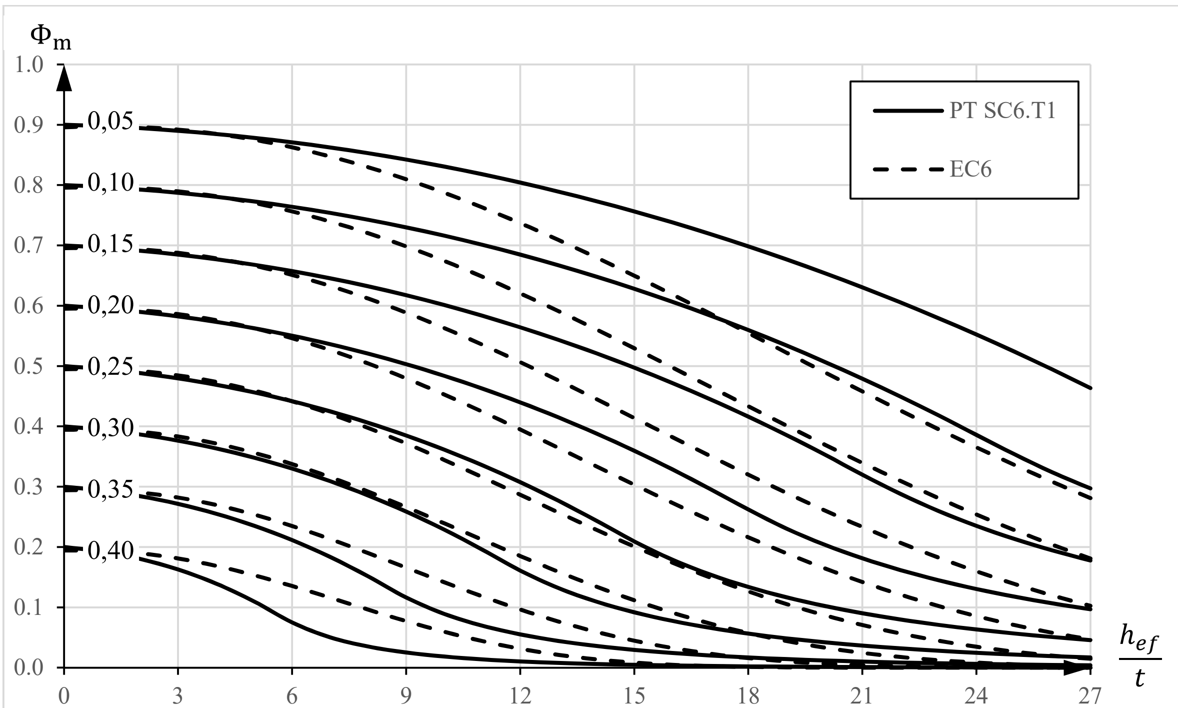

Figure 3: Comparison of the different methods for $E_{mean} = 1000\,f_k$. The curves with the dotted lines in the Figures 1 and 2 should not be plotted in the final version of EN 1996-1-1.

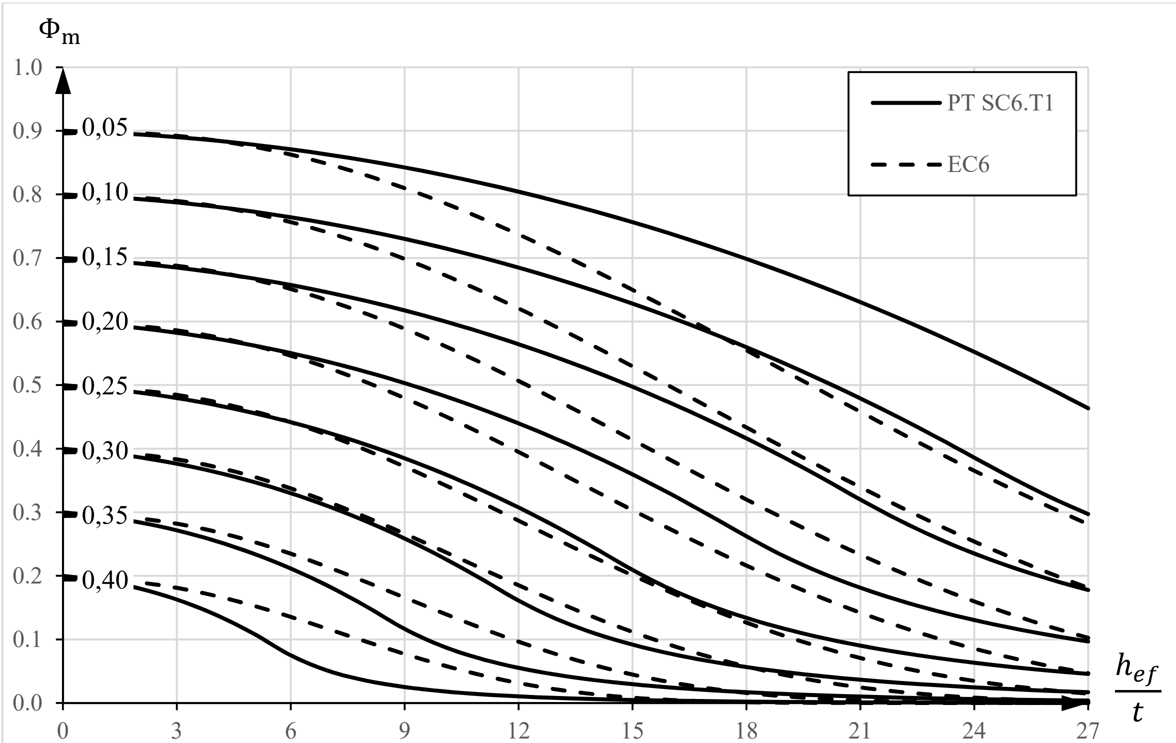

Figure 4: Comparison of the proposed method with EN 1996-1-1 for $E_{mean} = 700\ f_k$. The curves with the dotted lines in the Figures 1 and 2 should not be plotted in the final version of EN 1996-1-1.

CALIBRATION OF THE SOLUTION AGAINST EXPERIMENTS

The formula (7) has been compared with experimental data for different masonry materials obtained from [8, 10, 14-16]. The comparison between the values of the reduction factors obtained experimentally and the values estimated empirically are given in Figure 5, Figure 6 and Figure 7.

Considering the uncertainty in the material parameters like the compressive strength and the elastic modulus, besides the uncertainty in the boundary conditions of the test, the empirical formula shows good fitting with the experimental data. An explanation about the influence of the uncertainty in the material parameters on the load bearing capacity is given in [3].

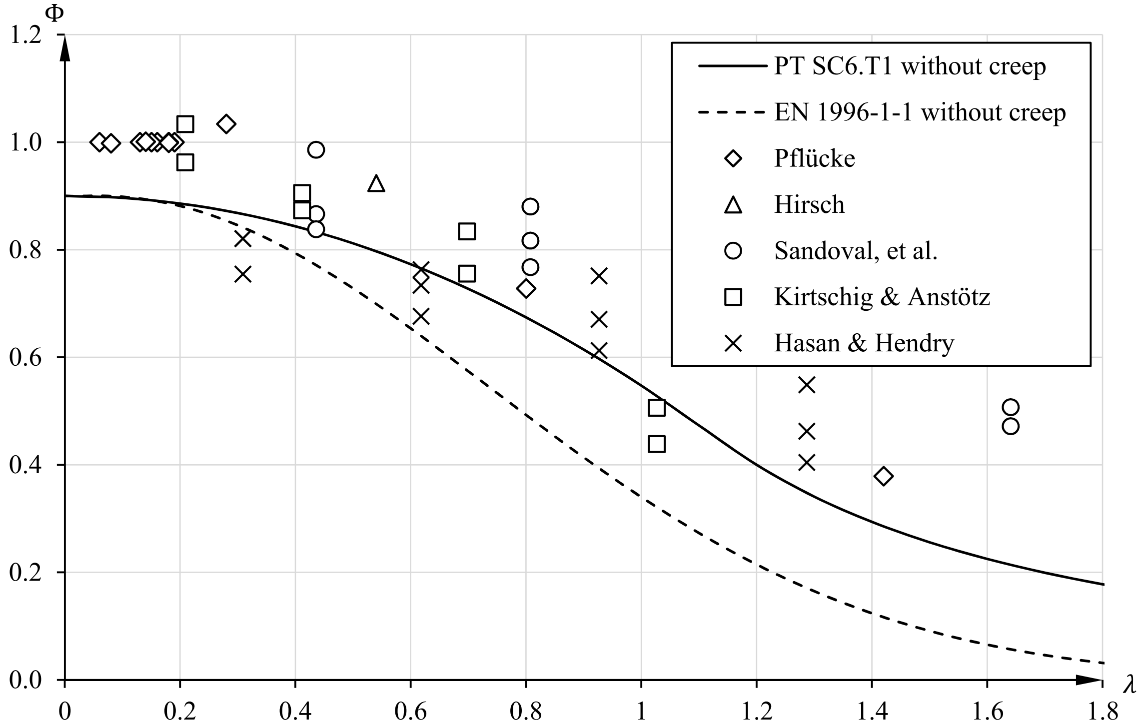

Figure 5: Comparison of the proposed formula with experimental data and the EC6 formula for e/t=0.

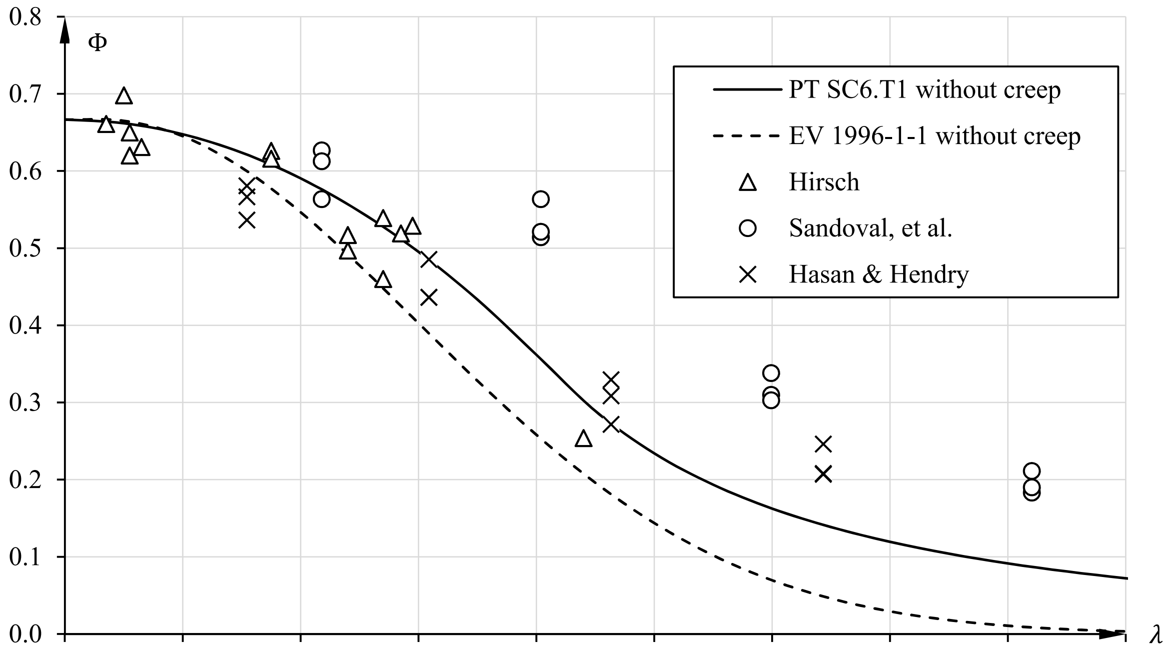

Figure 6: Comparison of the proposed formula with experimental data and the EC6 formula for e/t=1/6.

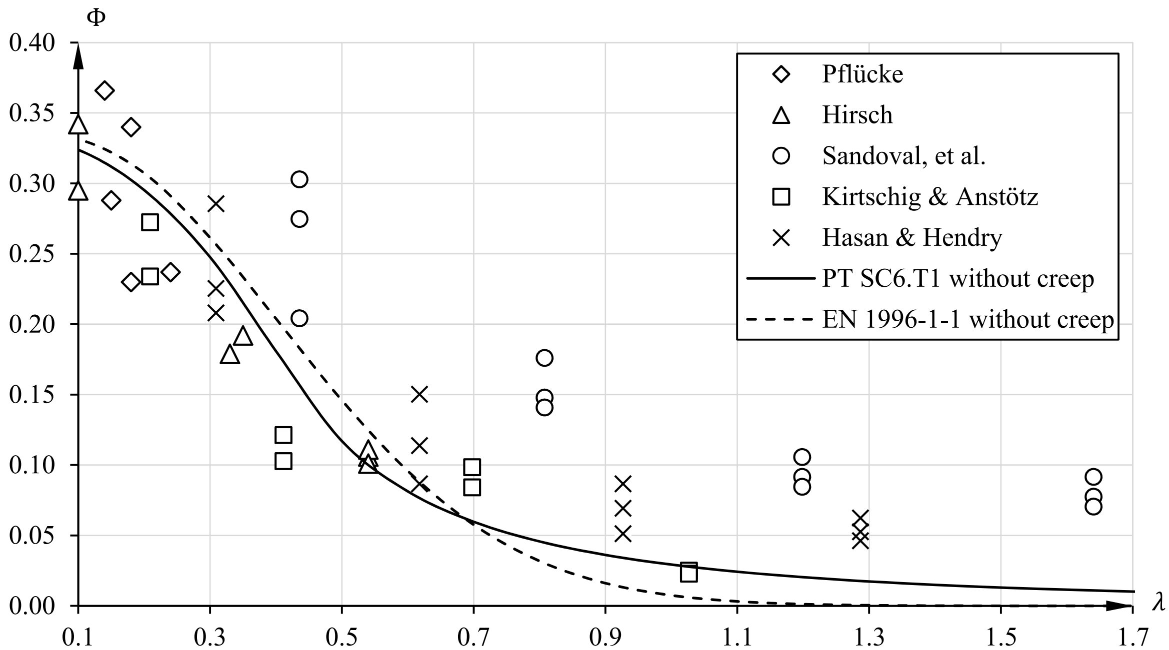

Figure 7: Comparison of the proposed formula with experimental data and the EC6 formula for e/t=1/3.

GENERALIZED METHOD FOR URM AND RM

The authors suggest to represent the relationship between the capacity reduction factor Φ and the slenderness ratio λ by using two curve with a tangent point at $\lambda = \lambda_t$. The load bearing capacity for short reinforced or unreinforced masonry walls are controlled by the material failure under compression, this range of failure can be represented by a parabola which characterized mainly by two parameters:

$$\Phi = \Phi_e - C \cdot \lambda^2 \tag{10}$$

The parameter Φ_e is the capacity reduction factor of the reinforced/unreinforced wall due eccentricity e. This value can be calculated for any masonry reinforced or unreinforced cross section from the equilibrium equations at the cross section. The load bearing capacity for slender reinforced or unreinforced masonry walls are controlled by the stability failure, this range of failure can be represented by a hyperbola:

$$\Phi = \frac{A}{\lambda^2} \tag{11}$$

The stability failure is mainly characterised by the elastic properties of the wall and not influenced with the material nonlinearity and can be used in the same way for reinforced or unreinforced masonry walls. To calculate the constant A, eq. (11) can be compared with the capacity reduction factor of Euler load:

$$\Phi_{Euler} = \frac{\pi^2}{12\lambda^2} \tag{12}$$

This gives $A \approx 0{,}8$ for concentrated loads.

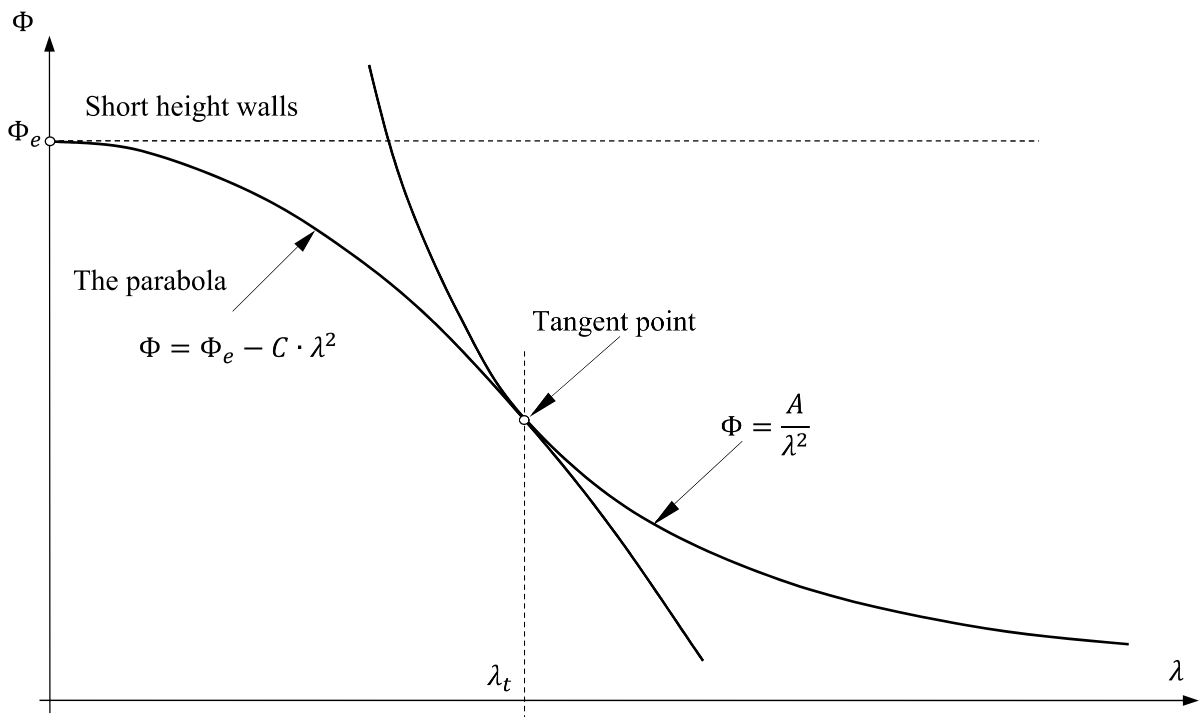

Figure 8: Graphical interpretation of parabola and hyperbola approximation.

For eccentric loading, based on the empirical solution in [2, 4], the above equation can be generalized to:

$$A = 0{,}8\frac{r_B}{r_N}A_1^3 \tag{13}$$

where

$$r_B = \frac{B_r}{B_u};\ \ r_N = \frac{N_{0,r}}{N_{0,u}};\ \ A_1 = 1 - 2\frac{e}{t} \tag{14}$$

B_r the elastic flexural stiffness of the cross section of the reinforced wall. B_u the elastic flexural stiffness of the cross section of the unreinforced wall. $N_{0,r}$ the load bearing capacity of the reinforced wall under concentric loading with no second order effect. $N_{0,u}$ the load bearing capacity of the unreinforced wall under concentric loading with no second order effect. The tangent point between the parabola and the hyperbola can be obtained by solving eq. (10) and eq. (11). This gives

$$\lambda_t = \sqrt{\frac{2A}{\Phi_e}} \tag{15}$$

A generalized empirical method to calculated the vertical load bearing capacity of reinforced or unreinforced masonry subjected to mainly eccentric or concentric vertical loading can be summarized as following:

$$N_R = \Phi \cdot N_0 \tag{16}$$

where: N_0 the load bearing capacity of the reinforced wall under concentric loading with no second order effect. Φ is a capacity reduction factor takes into account the eccentricity and slenderness and can be estimated as follows:

$$\Phi = \begin{cases} \Phi_e - \dfrac{\Phi_e^2}{4} \cdot \dfrac{\lambda^2}{A}, & \lambda < \lambda_t \\ \dfrac{A}{\lambda^2}, & \lambda \geq \lambda_t \end{cases} \tag{17}$$

Φ_e the capacity reduction factor of the reinforced/unreinforced wall due eccentricity e:

$$\Phi_e = \frac{N_e}{N_0}; \tag{18}$$

Where N_e the load bearing capacity of the wall with eccentricity equal to e and no second order effect. A is a parameter takes into account the elastic properties of the reinforced / unreinforced section. For unreinforced masonry the above equations can be reduced to $r_B = 1;\ r_N = 1;\ \Phi_e = A_1; N_0 = f \cdot t;\ N_e = 1 - \frac{2e}{t};\ B_r = B_u = Et^3/12$ the above equation turn into equation (7).

CONCLUDING REMARKS

The buckling formula provided in the current version of Eurocode 6 for unreinforced masonry walls shows inconsistency at a specific range of masonry materials. This inconsistency has been cleared up by proposing a solution that distinguish between the material failure and stability failure of masonry walls.

In the current version of EC6, the buckling in reinforced masonry walls has been considered for slenderness ratios greater than 12 by adding an additional design moment for the second order effect (6.6.2.(7) EC6), other solutions suggested for reinforced masonry walls were developed originally for reinforced concrete members, like the column model methods or the nominal curvature. However, the paper shows that no need to handle buckling problem, in reinforced masonry different than from unreinforced masonry walls, and unified solution can be used for both cases.

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