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RANDOM-FRACTAL-METHOD-BASED GENERATION OF MESO-MODEL FOR CONCRETE HOLLOW BLOCK

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ABSTRACT

Concrete hollow block is one of the most widely used building materials. It is of great significance to simulate the meso-structure of block and describe the initial condition of aggregates. Based on the geometrical characteristics of concrete, it is considered that the fractal effect exists in the set of aggregates. Within this theoretical framework, the fractal dimension of the set of aggregates and cement are respectively defined. Furthermore, a mathematical model which describes the initial condition of concrete with the fractal dimension of aggregates is formulated. Moreover, an equivalent transferring method of concrete's gradation is introduced. Based on the results, a method of meso-simulation for concrete hollow block is formed. The sieving curve, the percentage of aggregate's volume in concrete, the weighted average grain-size and the fractal dimension are used as indicators. By comparing with specimens, the effect of simulation is verified, which shows that this method could meet the practical demand. On the basis of this generation algorithm, with the gradation of the aggregates affirmed, a systematic meso-simulation method for concrete block could be developed. Several implementations in the area of finite element analysis for future work are discussed.

KEYWORDS: *concrete hollow block, fractal geometry, aggregate, mesoscopic*

INTRODUCTION

Understanding the mechanical behaviour of concrete hollow block directly influences construction cost and building safety, which are of great significance.

Concrete meso-mechanics can explain the macro-scale mechanical behaviour of concrete. Therefore, it has become the primary focus area in the field of concrete production research, yielding important results [1]. There are two main research approaches within the framework of

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this theory: experimentation and numerical simulation [2]. Compared with experimentation, numerical simulation offers the advantages of low cost, convenient, and accurate data collection, and freedom from experimental restrictions [2]; hence, it has attracted considerable attention.

In general, numerical simulation involves three major steps in its analytical procedure: (i) creating the aggregate and cement models, (ii) meshing, and (iii) finite element analysis/discrete element analysis [2]. The quality of the concrete mechanical solution depends on the modelling accuracy, i.e., on the first step of the simulation. However, depending on the user's experience, creation of the numerical model is often a time-consuming task [3]. Therefore, it is important to study and discuss the modelling method.

Many researchers have introduced a number of ways to generate the concrete geometrical model. The modelling method typically includes two key procedures. The first procedure is generation of the aggregates, which includes polyhedron generation using either standard Voronoï/Delaunay tessellation [4]–[6]or a random spatial polygon growth [6],[9]. This procedure should follow the principle of keeping each aggregate convex, which is rather difficult. The second procedure is allocation of the aggregate models in the concrete model, which includes the 3D random particle allocation method introduced by J. P. B Leite [10], the layering disposition method for random aggregate arrangement introduced by Tang Xinwei [11], and the grid search method based on the background grid, which is combined with meshing[3],[9]. This procedure should follow the principles of making the generated concrete numerical model fit actual situations and avoiding the overlapping of aggregates.

The two key procedures described above are combined to obtain a complete concrete modelling algorithm. The main aspects of the modelling algorithm are computational efficiency and model quality. This paper classifies modelling algorithms into three categories. In the first category, random aggregate models are first generated and then allocated in the concrete model sequentially, e.g., the method introduced in [10], which is a traditional method. Based on this method, it is necessary to judge whether the current particle is completely inside the specimen and overlaps previously placed ones before each particle allocation. The complexity of this method is relatively high; the computation time increases with the number of allocated aggregates. Algorithms derived from this method include those introduced in [11]. In these methods, the judgment of whether the current particle overlaps previous ones is omitted; this improves the computational efficiency considerably but increases the space between aggregates. In the second category, the aggregate models are generated and allocated simultaneously, e.g., the method introduced in [9]. Furthermore, some methods combined with meshing do not require the overlapping of aggregates to be judged, thereby saving a significant amount of time and facilitating follow-up work. Qin C [3] showed that it is easy to generate irregularly shaped aggregate models using this type of algorithm, and there is usually a difference between the gradations of the numerical model and actual conditions. In the third category, the location and boundaries of the aggregates are first determined and the aggregates are then generated, e.g., the method introduced in [6] and an algorithm based on random fractal theory introduced by us in

[12]. According to the random fractal method (RFM), the boundaries of aggregates around an aggregate can be defined simultaneously with the generation of this aggregate. As with the second category of generating algorithms described above, these algorithms save time but provide a lower coarse aggregate content.

Each of the three types of algorithms described above has its advantages and disadvantages. A major advantage of the RFM is that it involves the application of an index (i.e., the fractal dimension of concrete) that can describe the geometrical character of concrete quantitatively. Such quantitative characterization is useful for extending this algorithm, as it can provide a theoretical basis for follow-up meso-mechanical studies. Therefore, in this paper, we present the theoretical basis, generation algorithm and implementation of RFM.

METHOD

Theoretical Basis

It could be considered that the set of aggregates and cement is a fractal. Based on this condition, the theories within the theoretical framework of fractal geometry can be applied. The possible availability of the fractal effect of aggregates and cement enables us to use a mathematical expression with the fractal dimension to describe the spatial geometry property of the aggregate condition. Furthermore, this assumption makes it possible to generate the aggregate simulation model using the fractal method.

In the theory of fractal geometry, if two sets have the same fractal dimension, they are considered to be the same [13]. Therefore the RFM could be expressed essentially as follows: a geometrical pattern to be generated with the same fractal dimension as that of actual concrete, using the iteration-based modelling algorithm, can be considered as the aggregate numerical model.

Besides, according to these two purposes of modeling: (i) limit the change in aggregate shape in simulation process and simplify the simulation process; (ii) facilitate element meshing in the finite element analysis stage, thereby preventing abnormal element generation as well as stress or strain concentration. Two other assumptions were introduced: all the simulated aggregates are convex; there always a space between any two aggregates.

Generation Algorithm

The main operation processes for simulating a concrete sample are shown in Figure 1. The three most important steps of this flowchart are set cell, aggregate generation, and edge modification, which are detailed in the following subsections.

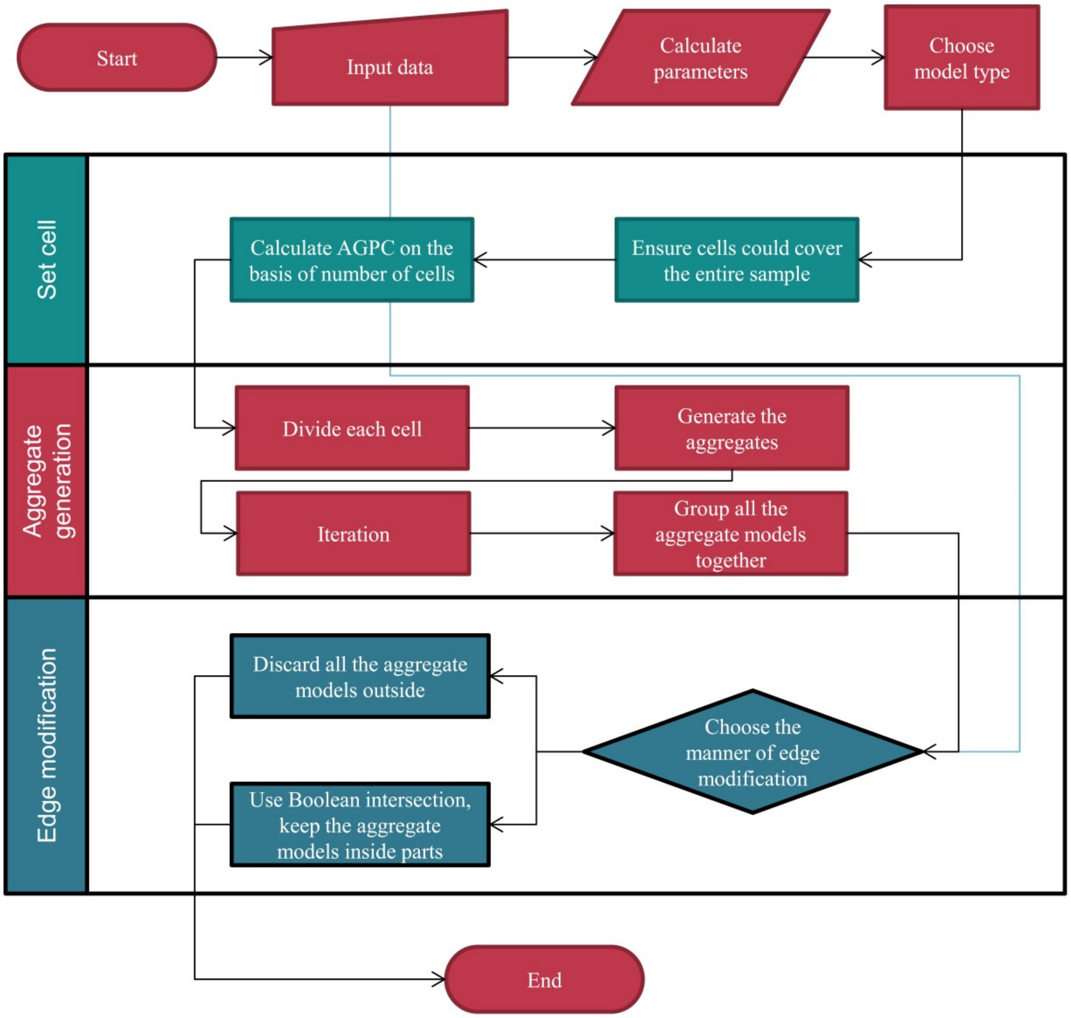


Figure 1: Flowchart of Simulation Method

Set Cell

Consider an initial model as a cell. According to the specific purpose, the cell can be a two-dimensional (2D) or three-dimensional (3D) pattern. A cell is considered as an auxiliary pattern. An auxiliary pattern is used for convenience and is virtually non-existent. It is drawn temporarily to facilitate the construction of the model; it neither is shown after the process of modelling nor participates in the calculation.

If $F_1, F_{2,\cdots,} F_n$ is a countable sequence of sets and each F_i has the same dimension, then $\dim\bigcup_{i=1}^{n} F_i = \dim F_i$. Thus, the fractal dimension of the model combined with several cells would be equal to D as long as the fractal dimension of each cell is equal to D. This is the theoretical basis for simulating concrete using arrayed cells.

Based on this principle, the number of cells N^{cell} can be calculated as follows. Let $C_i, i = 1, 2, 3$, denote the lengths of the concrete sample in three directions. If the sample is irregularly shaped, C_i should be the edge length of the smallest cuboid that can hold the sample. Therefore, the number of cells that can ensure that the entire sample can be covered is given by Equation 1.

$$N^{cell} = \prod_{i=1}^{3} \frac{C_i}{ceil\left(\frac{\min(C_i)}{S}\right)} \tag{1}$$

Aggregate Generation

Then a fractal pattern can be generated in each cell by the iterate algorithm represented as follows:

(i) divide the cell: Divide the cell into several sub-auxiliary patterns in accordance with a certain rule, ensuring that these sub-auxiliary patterns are closed and convex; (ii) generate the aggregate(s): change the sub-auxiliary graph(s) at the specified location to a real graph; the real graph is treated as the model of an aggregate; (iii) iteration: Treat each remaining sub-auxiliary pattern as a new cell, and repeat steps 0 and 0 until the program ends.

Edge Modification

A corollary can be obtained [13]: Any part selected from concrete is fractal, where the set of aggregates and the set of cement have the same fractal dimension as the original concrete. Based on this corollary, the simulated model can be cut in a random shape, while the geometric properties and fractal dimension remain constant.

SIMULATION AND STATISTICAL ANALYSIS

Two simulation examples are presented in this section. One is the simulation of a concrete hollow block for the purpose of explaining the simulation method for a complex concrete production; the other is the generation of a group of concrete cube samples to be measured and analysed.

Simulation of Concrete Block

The simulation process for a hollow concrete block is shown in Figure 2.

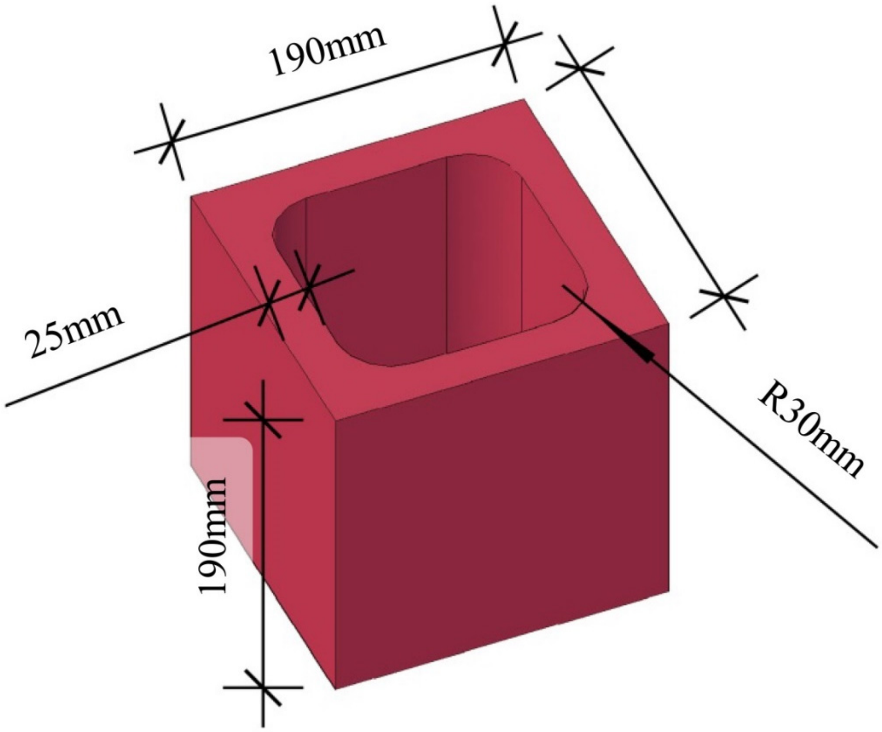


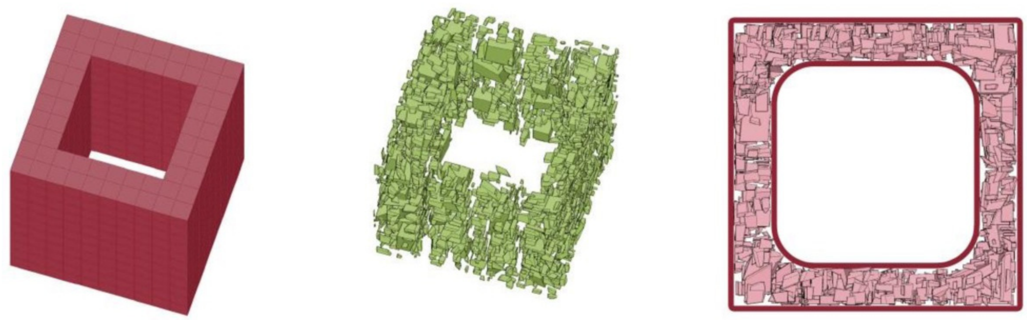
Figure 2: Dimensions of Concrete Hollow Block

This type of block is produced by gravel aggregates; the coarse aggregate content is 40%. The sieving result of the aggregates is summarized in Table 1, and the fractal dimension of gradation can be estimated in the following manner. Considering the ratio of each sieving size x_i to the maximum aggregate size $x_{\max}$ as the x-coordinate and the value of the mass retained $P\left(x_i\right)$ corresponding to the size of each x_i as the y-coordinate, several points can be plotted in a log-log plot. Then, the slope of the straight line fitted to these points can be determined as 3 minus the fractal dimension [12], which is obtained as 2.6325.

The cell was set as a cube with $S = 20mm$ on each side, and the arrayed cells were set. It would be feasible to set an array of $10 \times 10 \times 10$ to completely cover the entire sample, but all the models generated in the hole would be would be discarded eventually. That is more than 30% of the aggregate models would be useless. Therefore, to reduce the computation burden, the arrayed cells were set as shown in Figure 3(a), i.e., 640 cells were set. Then, the aggregates were generated in each cell, as shown in Figure 3(b). Finally, edge modification was performed. The overhead view of the final model is shown in Figure 3 (c).

Table 1: Result of Sieving Test

Total mass (kg)	IS sieve size (mm)	Separated sieve residue (kg)	Separated mass retained (%)	Cumulated sieve residue (%)	Mass retained (kg)	Mass retained P(x) (%)
1	10	0.065	0.065	0.065	0.935	0.935
	5	0.210	0.210	0.275	0.725	0.725
	2.5	0.163	0.163	0.438	0.562	0.562
	1.25	0.126	0.126	0.565	0.436	0.436
	0.63	0.097	0.097	0.661	0.339	0.339
	0.315	0.076	0.076	0.738	0.262	0.262
	0.16	0.058	0.058	0.795	0.205	0.205
	PAN	0.204	0.204	0.999	0.001	0.001



a) Step 1
set cell

b) Step 2
aggregate generation

c) Step 3
edge modification
(overhead view)

Figure 3: Schematic Diagram of Modelling Process

Simulation of Concrete Cube Sample

The models generated here are analysed as the overall sample in the following section. The dimension of the models was determined by referring to the sampling method presented by WANG et al. [14],[15]: the size of the overall sample should be at least 15 times larger than that of the maximum aggregate size, the size of the sub-sample should be at least seven times that of the maximum aggregate size. Thus, 100 cube models with edges equalling 500 mm were generated according to the gradation shown in Table 1, a randomly located cube with an edge length of 150 mm was taken from each overall sample as a sub-sample, and each overall sample was collected four times. The model generated by RFM as well as a schematic diagram of the sampling method was shown in Figure 4.

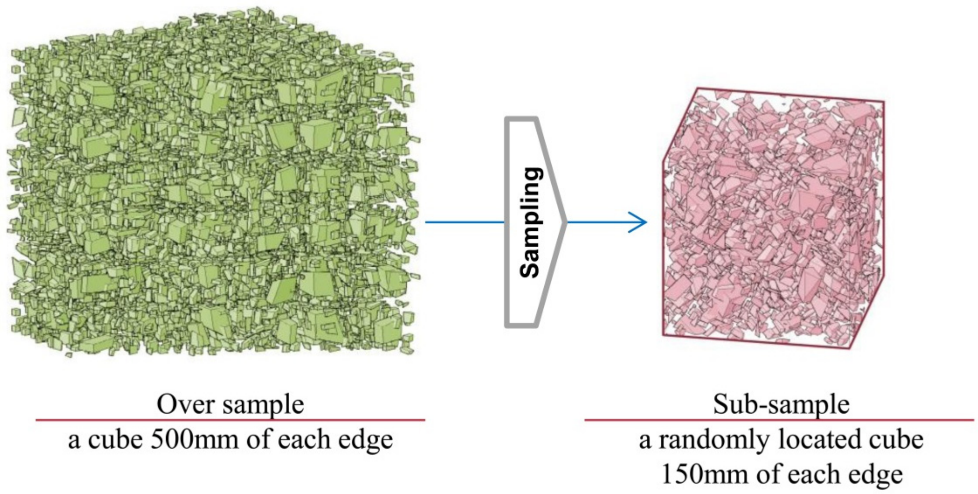


Figure 4: Concrete Simulation Model Generated by RFM and Sampling Method

Sample Statistical Analysis

To verify the modelling effect, the group of concrete cubic meso-models generated in above section was subjected to sample statistical analysis.

After sampling, the gradation curves of the overall sample and the sub-sample could be plotted. Further, three indices were selected as the meso-structure indicators: coarse aggregate content, weighted mean diameter of aggregates, and fractal dimension of sample.

The coarse aggregate content is 40% in this model. The weighted mean diameter of aggregates, $\overline{d}$, is calculated from the weight of each aggregate volume in the space:

$$\overline{d} = \frac{\sum d_i \times pv_i}{\sum pv_i} \tag{2}$$

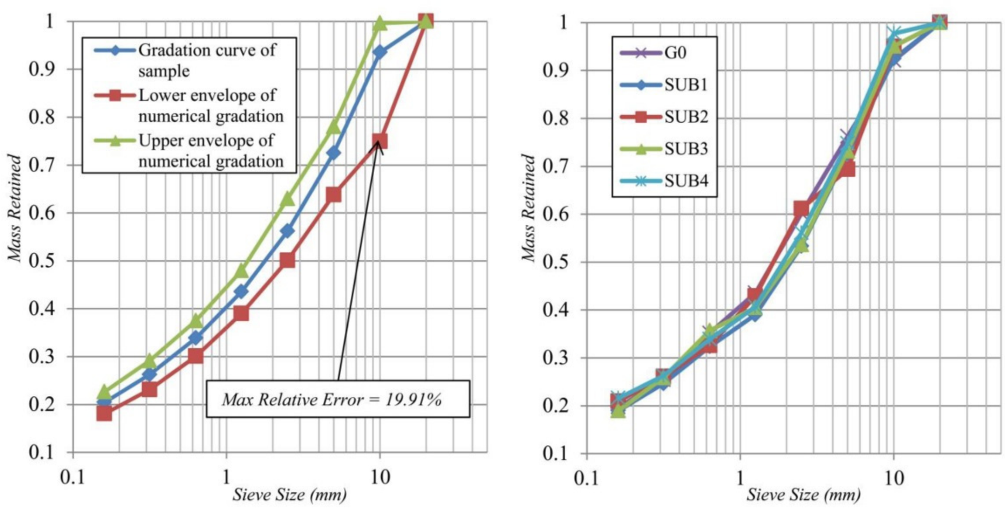
where d_i is the equivalent spherical diameter, which is the diameter of a sphere whose volume is equal to that of an irregularly shaped object, and pv_i is the coarse aggregate in level i . The equivalent spherical diameter $\overline{d}$ of the aggregate under consideration is equal to 2.36 mm. Each aggregate (with a different size) contributes differently to the capacity of the concrete. The distribution of the aggregates and the way in which the aggregates contribute to the capacity of concrete can be represented using the weighted mean diameter of the aggregates.

The fractal dimension could be obtained using the box-counting method [13]. In this paper, the fractal dimension could be considered as the Hausdorff dimension or the box-counting dimension.

RESULTS AND DISCUSSION

Gradation Curve

The density of each aggregate can be considered to be the same. Based on the number of aggregates obtained using a computer, the gradation curves of the simulation models generated by RFM are plotted in Figure 5(a), and the gradation curve of the referenced object is also plotted in the same figure. Further, one of the simulation models is randomly selected and treated as the overall sample. The gradation curves of the overall sample and four sub-samples are respectively plotted in Figure 5 (b). These curves show a similar form overall and there are some small differences influenced by random parameters. This indicates that the numerical model of concrete generated by the two methods fit the aggregate gradation.



(a) Gradation curves of sample and envelopes of gradation of meso-models

(b) Gradation curves of overall and sub-samples of meso-models with pebble aggregates

Figure 5: Gradation Curves

Statistical Result of Other Measurement Values

The statistical results of measurement of all of the models are summarized in Table 2. Based on the statistical method introduced in above section, the coarse aggregate content, weighted mean size, and box-counting dimension are analysed.

Table 2: Statistical Analysis of the Results of Measurements of Concrete Meso-simulation

	pv (%)	$\overline{d}$ (mm)	D_{box}
Mean	39.898	2.441	2.503
Sd	1.632	0.393	0.010
Max	43.715	3.498	2.527
Min	36.516	1.214	2.480
Med	39.993	2.400	2.503
CV	4.09%	16.092%	0.398%

Note: D_{box} represents the dimension calculated by box-counting; Sd represents the standard deviation; Max and Min represent the maximum and minimum values, respectively; Med represents the median; CV represents the coefficient of variation.

Table 3: Sampling Measurement of Models Generated by Convex-Polyhedron-3

	pv		$\overline{d}$		D_{box}	
	val (%)	ε (%)	val (mm)	ε (%)	val	ε (%)
G 0	40.394		2.261		2.521	
Sub 1	42.657	5.60	2.287	1.11	2.499	0.88
Sub 2	40.003	0.97	2.307	2.00	2.500	0.84
Sub 3	39.953	1.09	2.109	6.73	2.504	0.69
Sub 4	38.830	3.87	2.188	3.24	2.518	0.11

Note: val represents the measurement value; ε is the relative error between each sub-sample and overall sample; G0 represents the overall sample; Sub1~Sub4 represent the four sub-samples.

One of the models generated was selected; the measurements of the overall sample and four sub-samples are listed in Table 3. The results show that each index remains stable when the sub-sample changes, and the difference between the measurements of the sub-sample and the overall sample is small. However the CV and relative error of weighted mean size are much higher than the others. This phenomenon might be caused by the number of aggregate with large size. The reason needs to be further analyzed.

CONCLUSION AND FUTURE WORK

This paper introduced a versatile concrete meso-modelling method. The following conclusions could be drawn.

a) In the framework of fractal geometry, a modelling method could be developed.
b) Taking a concrete hollow block as an example, the implementation of RFM for simulating a complex was detailed. Further, a group of concrete cube samples were generated using RFM.
c) The numerical cube models were statistically analysed. The gradation curves of the models had forms similar to the actual sieving results. Further, the coarse aggregate content and fractal dimension of the numerical models were identical to those of the actual situation. However the weighted mean diameter of aggregates represented unstable, the reason needs to be further analyzed and discussed.

It was reported that the applied effect and processing efficiency of RFM meet the needs of concrete meso-modelling. A precise numerical model is the basis for future concrete block meso-scale studies on the mechanical behaviour, chlorine diffusion, etc., which are related to many other physical properties apart from the geometrical characteristics. Thus, the following discussions should be included in future work: based on the known gradation status of concrete, the material properties of the aggregate and cement and the interface between them should be taken into consideration. Furthermore, numerical simulation of the failure process and prediction of the capacity of concrete should be studied.

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