



13TH CANADIAN MASONRY SYMPOSIUM
HALIFAX, CANADA
JUNE 4TH – JUNE 7TH 2017



**INFLUENCE OF MATERIAL SPATIAL VARIABILITY ON THE RELIABILITY OF
MASONRY WALLS IN COMPRESSION**

Müller, Dominik¹; Förster, Valentin² and Graubner, Carl-Alexander³

ABSTRACT

Masonry is one of the oldest and most traditional construction types. Thus, corresponding safety concepts are often still based upon experience instead of being calibrated by structural reliability methods. Because of that, reliability analyses of masonry structures are needed to see if safety factors should be adjusted. Since masonry is a non-homogenous material, considering the spatial variability of material properties is very important when assessing the reliability of masonry walls. Therefore, it is useful to know if and to what extent spatial variability increases or decreases the reliability of masonry walls. Amongst others, this depends on the length of a wall due to the capability of load redistribution. Also, it depends on the slenderness of a wall or rather the governing failure mode which could be local compression or stability failure. This paper shows the effect of spatial variability on the load-bearing capacity of masonry walls in terms of mean value, scatter and design value. For this purpose, walls of varying length and slenderness were analysed with and without the consideration of spatial variability by performing Monte Carlo simulations.

KEYWORDS: *masonry, Monte Carlo simulation, reliability, spatial variability, stability*

MOTIVATION

In the last decades, a lot of effort has been put into the development of precise mechanical models for the load-bearing capacity of masonry walls so that a safe and economical design can be guaranteed. Nevertheless, there are large uncertainties regarding the input parameters for these models which cannot be eliminated due to the natural scatter of material properties, for example. Therefore, all design codes establish safety concepts that should provide a certain reliability level. A safety concept or more precisely the safety factors defined by a safety concept

¹ Research Assistant, Institute of Concrete and Masonry Structures, Technische Universität Darmstadt, Franziska-Braun-Straße 3, 64287 Darmstadt, Germany, mueller@massivbau.tu-darmstadt.de

² Research Assistant, Institute of Concrete and Masonry Structures, Technische Universität Darmstadt, Franziska-Braun-Straße 3, 64287 Darmstadt, Germany, foerster@massivbau.tu-darmstadt.de

³ Professor, Institute of Concrete and Masonry Structures, Technische Universität Darmstadt, Franziska-Braun-Straße 3, 64287 Darmstadt, Germany, graubner@massivbau.tu-darmstadt.de

have to be well calibrated. A resulting reliability which is too low leads to structures that are not safe enough. If the reliability level is too high, structures designed according to this safety concept are not economical. It is obvious that the accuracy of mechanical models would be wasted if the corresponding safety factors are not well calibrated.

Since unreinforced masonry has a very limited tensile strength, it is mainly designed for compression. In Figure 1, safety factors for the design of masonry in compression according to different national annexes of Eurocode 6 are shown. Note that the partial safety factor format according to Eurocode is defined as in eq. (1), which leads to safety factors larger than 1.

$$\gamma_F \cdot E_k \leq \frac{R_k}{\gamma_M} \quad (1)$$

Where E_k and R_k are the characteristic values of load effect and resistance. The value γ_F is the partial safety factor for the load effect and γ_M is the partial safety factor for the resistance.

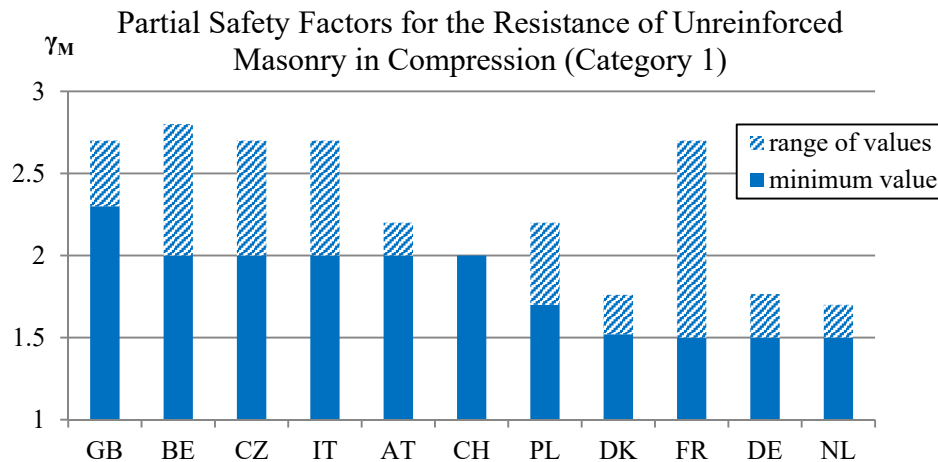


Figure 1: Comparison of Safety Factors in National Annexes of Eurocode 6 [1]

It can be noticed that the safety factors according to different national annexes of Eurocode vary a lot. To a certain extent, minor differences in the mechanical models defined by the national annexes as well as national construction characteristics are a reason for this discrepancy. Nevertheless, the extent of the discrepancy leads to the assumption that the majority of current safety factors are still based on experience and historical developments. Hence, it is very important to conduct reliability analyses which lead to probabilistically calibrated safety factors.

One aspect that is often neglected in reliability analyses is the spatial variability of masonry properties. No material is perfectly homogenous. This is particularly true for masonry due to being composed of separate units and the mortar in between. Therefore, material properties like the compressive strength or the elastic modulus do not deviate homogeneously from a certain mean value but the material properties also differ from unit to unit (and even within a unit). There are a lot of reasons why it is very important to consider spatial variability in reliability

analyses. First of all, a lot of effects can only be explained by spatial variation. This includes the location of failure, which is not always at the same location as the maximum stress, for example. Spatial variation also causes a size effect which means that the mean value of the relative bearing strength of larger structural members is lower than that of smaller members. In addition to that, experiments show that the scatter of the load-bearing capacity also depends on the member size, see e.g. [2]. According to EN 1996-1-1, this increased scatter for small cross-sections has to be considered in design by reducing the specified compressive strength with a factor of

$$0.7 + 3A \leq 1.0 \quad (2)$$

This reduction factor, which has to be applied to walls with cross-sections A smaller than 0.1 m^2 , can only be explained and calibrated by considering spatial variability.

A research project [3] regarding the reliability of slender ultra-high strength fibre reinforced concrete columns, which was conducted at the Technical University of Darmstadt, indicated that the consideration of spatial variability can lead to higher reliability indices. This means that theoretical safety deficits resulting from a reliability analysis may disappear if a more realistic approach of modelling the spatial variation is chosen. There have already been a few research projects that dealt with the reliability of masonry walls, see e.g. [2], [4], [5] and [6]. Most of them did not take the spatial variation of material parameters into account, an exception can be found in [7], for example. Here, the importance of considering the spatial variability for the reliability of masonry walls subjected to out-of-plane bending is emphasised. To the best of the authors' knowledge, there are no existing research projects that investigated the reliability of masonry walls subjected to compression systematically and took spatial variation into account. This paper should be understood as a preliminary study regarding the reliability of masonry walls in compression with consideration of spatial variability.

RELIABILITY OF STRUCTURES

Performing a reliability analysis is equivalent to determining the probability of failure P_f of a certain structure. Since probabilities P_f are usually quite small in the field of structural engineering, the use of the probability of failure is not very handy as a dimension for reliability. Therefore, the reliability of a structure is usually given by the reliability index β . The relationship between failure probability P_f and reliability index β is defined via the cumulative distribution function Φ of the standardised normal distribution as

$$P_f = \Phi(-\beta) \quad (3)$$

A high reliability index β corresponds to a high reliability and vice versa. Safety factors or design values can only be calibrated if a target reliability is given, which must be chosen in a way that the calibrated safety factors neither cause unsafe nor uneconomical structures. According to EN 1990, safety factors for common structures shall result in a reliability index of $\beta = 4.7$ ($P_f = 1.3 \cdot 10^{-6}$) for a reference period of 1 year or $\beta = 3.8$ ($P_f = 7 \cdot 10^{-5}$) for 50 years,

respectively. In general, safety factors for the resistance cannot be calibrated without knowledge about the distribution of the load effect. However, as a simplification, EN 1990 defines sensitivity factors $\alpha_S = -0.7$ and $\alpha_R = 0.8$ which make it possible to calibrate safety factors separately for load effect and resistance, respectively. The design resistance R_d (resistance including safety factor) for a reference period of 1 year can therefore be calculated by

$$P(R \leq R_d) = \Phi(-\alpha_R \cdot \beta) = \Phi(-0.8 \cdot 4.7) \approx 1 \cdot 10^{-4} \quad (4)$$

Thus, the required design value only depends on the distribution of the resistance. If the spread of the resistance is high, the ratio of design value to mean value is low and vice versa. In this paper, for the quantification of the influence of spatial variability on the reliability of masonry walls, the distribution of the resistance of masonry walls in compression is determined and the corresponding required design resistances are calculated.

FAILURE MODES OF MASONRY WALLS IN COMPRESSION

General Remarks

Depending on the eccentricity of the applied compression force, the slenderness of the wall and the stress-strain relationship of the material, a masonry wall in compression can fail due to local compression failure (which may include second order effects) or due to stability failure. A compression failure occurs if the compressive resistance of the critical cross-section is reached. The term stability failure means that the load cannot be increased anymore although the cross-sectional bearing capacity is not reached at any part of the wall. This happens if an increase in deflection causes a higher increase in the moment due to second order effects than in the counteracting resistance due to stiffness of the member. If a wall fails due to compression failure, the compressive strength is the decisive material parameter. If it fails due to stability and the eccentricity is low, the elastic modulus is decisive.

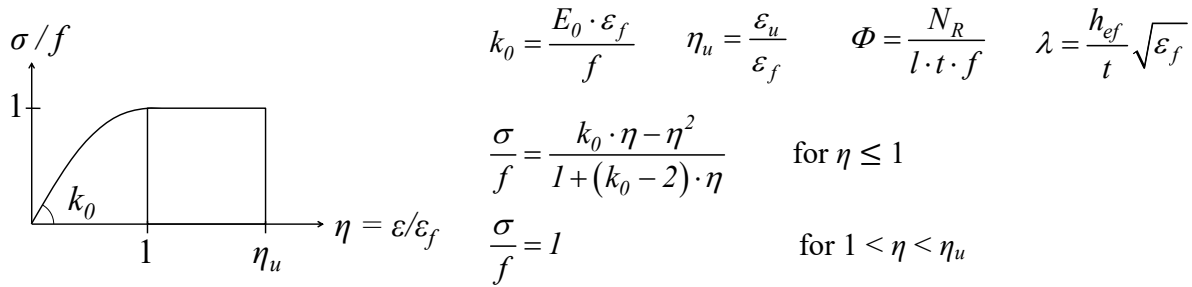


Figure 2: Stress-Strain Relationship and Parameter Definition

In [8], an easy to use and still very accurate approximation formula is developed to calculate the load-bearing resistance of slender unreinforced masonry members in compression. The initial elastic modulus E_0 , the compressive strength f , the strain at reaching the compressive strength ϵ_f and the ultimate strain ϵ_u of masonry can be freely chosen according to Figure 2. The equations use standardised dimensions: Standardised initial elastic modulus k_0 , standardised ultimate strain

η_u , standardised load-bearing capacity Φ and standardised slenderness λ . In Fig. 2, N_R is the compressive load-bearing capacity for a given eccentricity and considering second-order effects, l and t are the length and the thickness of the wall and h_{ef} is the buckling length of the wall. A value of $k_0 = 1$ results in a line and a value of $k_0 = 2$ in a quadratic parabola for the stress-strain relationship. The post-peak behaviour is modelled as a horizontal line with limited length. Resulting standardised capacities according to [8] are shown in Figure 3. In this diagram, the transition between stability and compression failure can be identified since it is at the inflexion point of the capacity curve. It can be seen that the definition of a tensile strength f_t only has a stronger influence on the capacity if the eccentricity and the slenderness are high. For this paper, walls of different slenderness λ are analysed, the first slenderness leads to a compression failure and the second to a stability failure.

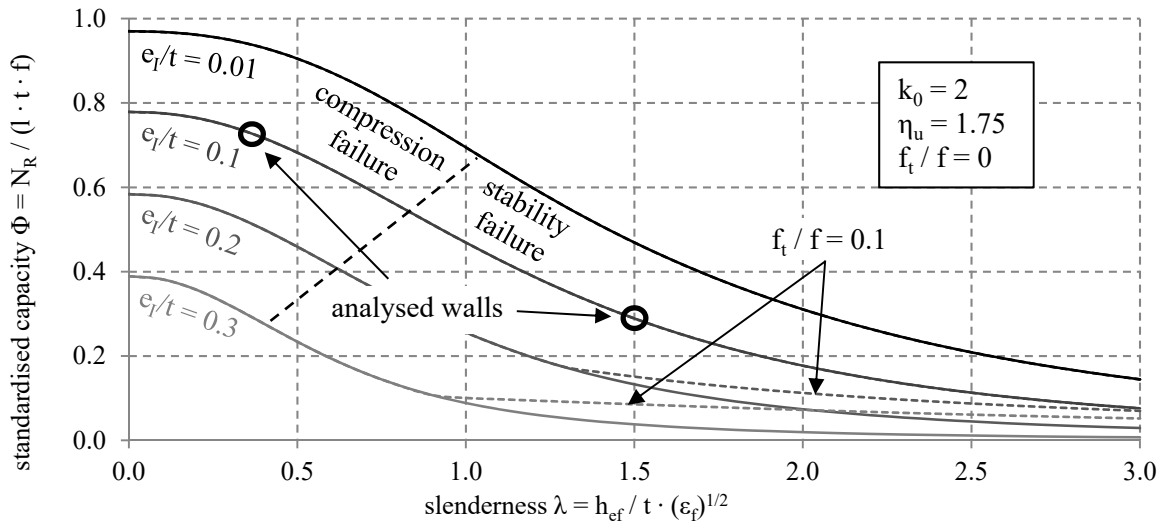


Figure 3: Load-Bearing Capacity of Masonry Walls in Compression according to [8]

Failure Mode and Spatial Variability

If the strength of a system, which consists of different members, is stochastically modelled including spatial variability, two ideal systems can be defined as boundaries for the behaviour, see e.g. [9]. First, there is the series system. Its strength is determined by the weakest member of the system, i.e. the system fails if the weakest link has reached its strength. The second boundary is the parallel system with perfectly ductile elements. This means that the strength of the system is the sum of the member strengths or the average of the member strengths if strength is measured in the dimension of stress, respectively. The behaviour of a masonry wall lies in between these two boundaries. If a wall fails due to compressive strength, it behaves more like a series system, since the weakest course of the masonry wall determines the strength (if bending moment and compression force are constant). The strength of each course, nevertheless, behaves more like a parallel system since the load can be redistributed between the units in one course to a certain extent. The extent of this redistribution depends a lot on the post-peak behaviour i.e. the

ductility of the masonry material. If a wall fails due to stability, the local strength of the wall is not decisive anymore but the global stiffness of the wall which depends on the elastic moduli of all the units in the wall. The behaviour is close to that of a parallel system because a (weighted) average of the elastic moduli determines the load-bearing capacity.

MONTE CARLO SIMULATION

General Remarks

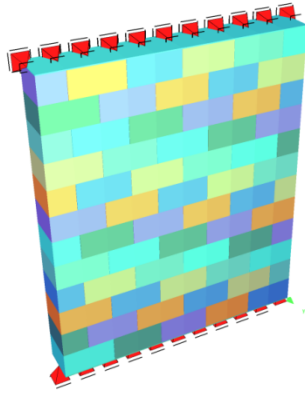
When spatial variability is considered, a high number of random variables is required which makes it difficult to calculate a failure probability or a distribution function of the resistance analytically. Therefore, Monte Carlo simulations were performed. The Monte Carlo method is an alternative to analytical methods of calculating probabilities or probability distributions. The idea behind this method is to perform a very high number of simulations of the problem. These are conducted by using random numbers for all the basic variables which have been defined. The higher the number of the simulations is, the better the quality of the estimation of the failure probability or the distribution function gets.

Finite Element Model

The masonry walls were modelled with the finite element software Sofistik 2016 [10], see Fig. 4. For simplification, the material was considered to be isotropic and no differentiation between stone and mortar was made which means that the mechanical parameters for composite masonry were used and no bed joint was modelled. These simplifications may be discussed but seem to be sufficient for this preliminary study. To avoid a load transfer along the head joints which could be unfilled, the head joints were modelled as 2 mm wide gaps between the units. As parameters for the stress-strain relationship $k_0 = 2$ and $\eta_u = 1.75$ were chosen which is given for calcium silicate full block masonry in [11]. For simplification, the post-peak behaviour was modelled with a horizontal line. The stress-strain relationship is very important for these investigations since it has a large influence on the possible load redistribution. Therefore, the results of this paper can be viewed as a first example and calculations have to be performed for different material behaviour, e.g. for more brittle materials, as well. The tensile strength was chosen as $f_t/f = 0.1$ but does not have a big influence on the results since the eccentricity to thickness ratio e/t of the compression force is 0.1 which means that the cross-sections at the ends of the wall are completely in compression. All walls have 12 courses which lead to a wall height of 3.0 m, whereas the wall length was varied. The supports at top and bottom were modelled as pinned. Two types of walls were modelled, one with low slenderness which shows compression failure and one with high slenderness failing due to stability, see Fig. 3. To change the slenderness λ , thickness and elastic modulus were varied. All other parameters, e.g. the number of units, were chosen equally so that the results can be compared.

$$\text{Low slenderness wall: } t = 0.365 \text{ m} \quad E_0/f = 1000 \quad \rightarrow h_{ef}/t = 8.2 \quad \lambda = h_{ef}/t \cdot \varepsilon_f^{1/2} = 0.37$$

$$\text{High slenderness wall: } t = 0.115 \text{ m} \quad E_0/f = 600 \quad \rightarrow h_{ef}/t = 26.1 \quad \lambda = h_{ef}/t \cdot \varepsilon_f^{1/2} = 1.51$$



Height: $h = h_{ef} = 3.0 \text{ m}$
 Unit height: 250 mm
 Unit length: 498 mm
 Courses: 12
 Eccentricity: $e / t = 0.1$
 $k_0 = E_0 \cdot \varepsilon_f / f = 2$
 $\eta_u = \varepsilon_u / \varepsilon_f = 1.75$
 $f_t / f = 0.1$

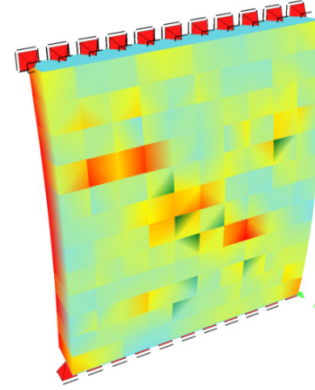


Figure 4: Finite Element Model

Stochastic Model

Since the compressive strength f and the initial elastic modulus E are the decisive material parameters for the walls that are analysed, these two parameters were chosen to be random variables in the investigation. A correlation $\rho_{f,E}$ between elastic modulus and compressive strength was taken into account. It is important to consider this correlation between compressive strength and elastic modulus since this correlation usually decreases the reliability of masonry walls. The scatter of tensile strength was neglected since the tensile strength has almost no influence on the load-bearing capacity of the analysed walls. Geometrical parameters like the thickness of the wall or an eccentricity due to imperfections could also be modelled as random variables. However, the scatter of geometrical parameters was not within the scope of this paper.

Table 1: Stochastic Parameters

Basic Variable	Distribution Type	CoV	Correlation
compressive strength f	log-normal	20 %	$\rho_{f,E} = 0.7$
elastic modulus E	log-normal	25 %	

One challenge when stochastically modelling material parameters is the choice of the right distribution types and coefficients of variation (CoV). Both can be obtained by the statistic assessment of experiments. As mentioned before, the scatter depends on the sample size. Therefore, it has to be differentiated between CoVs that have been obtained for material properties of single units and CoVs obtained for samples consisting of more units. This differentiation is usually not found in literature, although this effect could strongly influence the results of reliability analyses. In [11], CoVs for the compressive strength of different masonry materials are given, ranging from 14 % for masonry made of large sized autoclave aerated concrete units to 20 % for calcium silicate masonry. Here, a log-normal distribution with a CoV of 20 % is chosen for one unit, see Table 1. The CoV for the elastic modulus is selected as 25 %, which is given as an estimate in [11]. For the correlation coefficient between elastic modulus and compressive strength $\rho_{f,E} = 0.7$ was assumed, which is also used in [2].

The spatial variation of material properties is usually modelled as a random field with a certain correlation function. For masonry it seems appropriate to model spatial variability as a unit-to-unit variability. A correlation coefficient ρ_{spat} between the elastic moduli and compressive strengths of different units i and j was defined, see Fig 5. This correlation coefficient is independent of the location of the units i and j . Reasons for this correlation are, for example, that workmanship affects the strength of the whole wall and that all units for one wall are usually within one production batch.

...	...	E_j, f_j
E_i, f_i
...
E_1, f_1	E_2, f_2	...

ρ	E_i	f_i	E_j	f_j
E_i	1	$\rho_{f,E}$	ρ_{spat}	$\rho_{f,E} \cdot \rho_{spat}$
f_i		1	$\rho_{f,E} \cdot \rho_{spat}$	ρ_{spat}
E_j			1	$\rho_{f,E}$
f_j	<i>symmetric</i>			1

Figure 5: Correlations between Material Properties

Generation of Random Properties

Random numbers are usually given as a random value between 0 and 1. Random properties can then be generated by applying the corresponding inverse cumulative distribution function to this random number. When random variables are correlated with each other, this gets more complex. To achieve the desired correlation for this application, the following approach was developed.

The product of log-normal random variables is also log-normally distributed which is why random properties for the compressive strength f_i and the elastic modulus E_i of a certain unit i can be described by a product of four independent log-normally distributed random variables:

$$E_i = W \cdot E_w \cdot U_i \cdot E_{u,i} \quad (5)$$

$$f_i = W \cdot f_w \cdot U_i \cdot f_{u,i} \quad (6)$$

The random variables W and U_i determine the correlation $\rho_{f,E}$ between E_i and f_i since they are the same for elastic modulus and compressive strength. W describes the “shared” deviation of compressive strength and elastic modulus of the whole wall from a mean value and U_i is the “shared” deviation of compressive strength and elastic modulus of a single unit. The variables W , E_w and f_w determine the spatial correlation ρ_{spat} between the individual units since they are the same for all the units and describe the deviation of compressive strength and elastic modulus of the whole wall. The mean values of W , U_i , E_w , and f_w are 1. The final multiplication with $E_{u,i}$ and $f_{u,i}$, which contain the additional deviation of the compressive strength and the elastic modulus of a single unit, leads to the material parameters of the individual units. The mean values of $E_{u,i}$ and $f_{u,i}$ are the mean values of elastic modulus and compressive strength. The coefficients of variation v of the single random variables can be determined according to eq. (7) to (10), which were derived by using the common formulas for the correlation coefficient and the variance of products of random variables.

$$v_W = \sqrt{\rho_{spat} \cdot \rho_{f,E} \cdot v_E \cdot v_f} \quad (7)$$

$$v_{E_w} = \sqrt{\frac{\rho_{spat} \cdot (v_E^2 - \rho_{f,E} \cdot v_E \cdot v_f)}{1 + \rho_{spat} \cdot \rho_{f,E} \cdot v_E \cdot v_f}} \quad (8)$$

$$v_{U_i} = \sqrt{\frac{\rho_{f,E} \cdot v_E \cdot v_f \cdot (1 - \rho_{spat})}{1 + \rho_{spat} \cdot \rho_{f,E} \cdot v_E \cdot v_f}} \quad (9)$$

$$v_{E_{u,i}} = \sqrt{\frac{(v_E^2 - \rho_{f,E} \cdot v_E \cdot v_f) \cdot (1 - \rho_{spat})}{(1 + \rho_{spat} \cdot v_E^2) \cdot (1 + \rho_{f,E} \cdot v_E \cdot v_f)}} \quad (10)$$

v_{f_w} and $v_{f_{u,i}}$ can be determined equivalently to eq. (8) and (10).

Results and Discussion

Monte Carlo simulations were performed for the two wall types of different slenderness with a varying length of 1, 3 and 5 units and for a spatial correlation ρ_{spat} of 0, 0.25, 0.5, 0.75 and 1. This means that 30 walls were analysed. For each of these walls, 400 random simulations were conducted and statistically evaluated. For the distribution function of the resistance, a log-normal distribution with mean value and CoV according to the simulation results was assumed. Based on that, the required design value was calculated according to eq. (4). For the calibration of safety factors, model uncertainties would have to be considered, too, which have not been included in the calculated design values yet. Fig. 6 and 7 show the results for both wall types in terms of mean value and required design value. Note that a correlation coefficient of $\rho_{spat} = 1$ means that there is no spatial variability i.e. the wall is homogenous, whereas $\rho_{spat} = 0$ means that there is no correlation between the units and therefore full spatial variability.

It can be seen that the two wall types behave differently when spatial variability is considered. The mean capacity of the walls failing due to local compression decreases if spatial variability is considered. For this failure mode, the wall is closer to a series system, which means that the wall fails if the weakest course fails. The walls failing due to stability behave more like a parallel system, which means that the capacity of the system is determined by the average unit stiffness and therefore, the mean value of the load-bearing capacity is not affected by spatial variability. For both wall types, the mean values without spatial variation ($\rho_{spat} = 1$) match well with the deterministic results shown in Fig. 3. The space between mean value and required design value represents the scatter of the corresponding wall capacity. It can be seen that this scatter decreases for all walls if spatial variability is considered. In almost all cases, this leads to a higher design capacity, which means that spatial variability increases the reliability. This is even true for most of the walls failing due to compression. With more spatial variability, the scatter decreases that strongly that it overcompensates the decrease of the mean value. Still, the positive effect due to spatial variability is bigger for walls failing due to stability since the mean capacity does not decrease. One exception for the positive effect of spatial variability is the wall with low slenderness and just one unit in each course since no load redistribution is possible for this wall. Here, the design value does not increase with more spatial variability. This leads to a gap between the design value for the wall of 1 unit and those of 3 and 5 units, which is illustrated by the red area. This gap indicates that, in principle, a reduction factor for small cross-sections,

like defined in eq. (2) according to EN 1996-1-1, is justified. Because of the missing ability of load redistribution, the spread of the capacity for these walls is bigger and therefore the design capacity should be reduced. There is almost no difference between the walls with courses of 3 and 5 units. This indicates that load redistribution is limited to a few neighboring units and is not possible along the whole course.

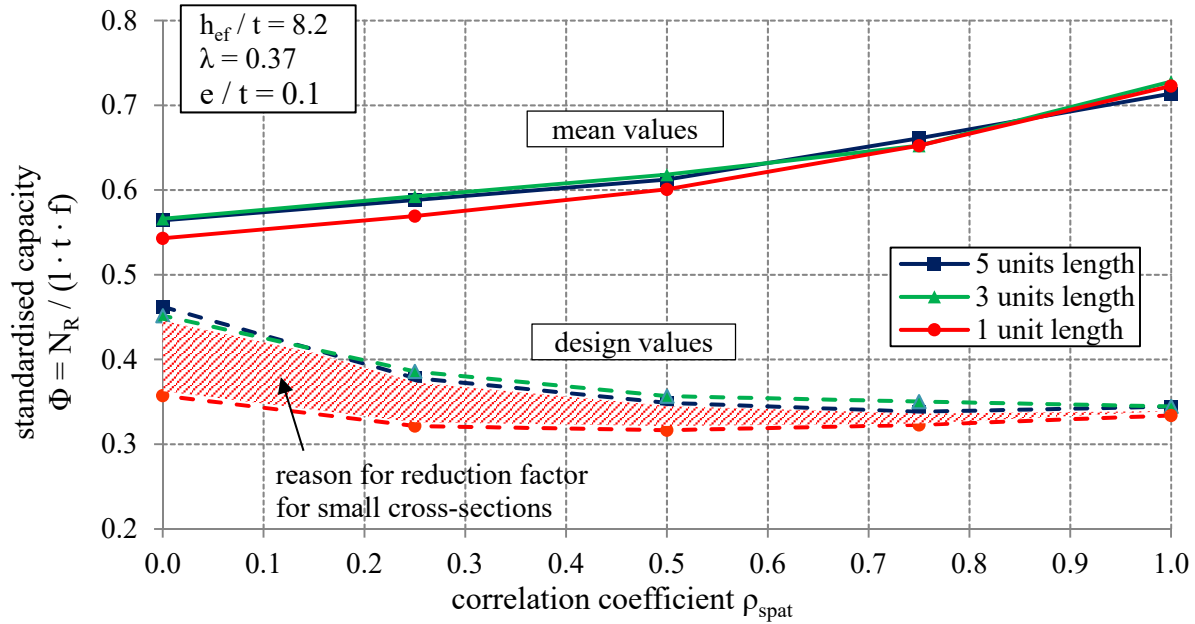


Figure 6: Mean and Design Capacity of Walls Failing due to Compression

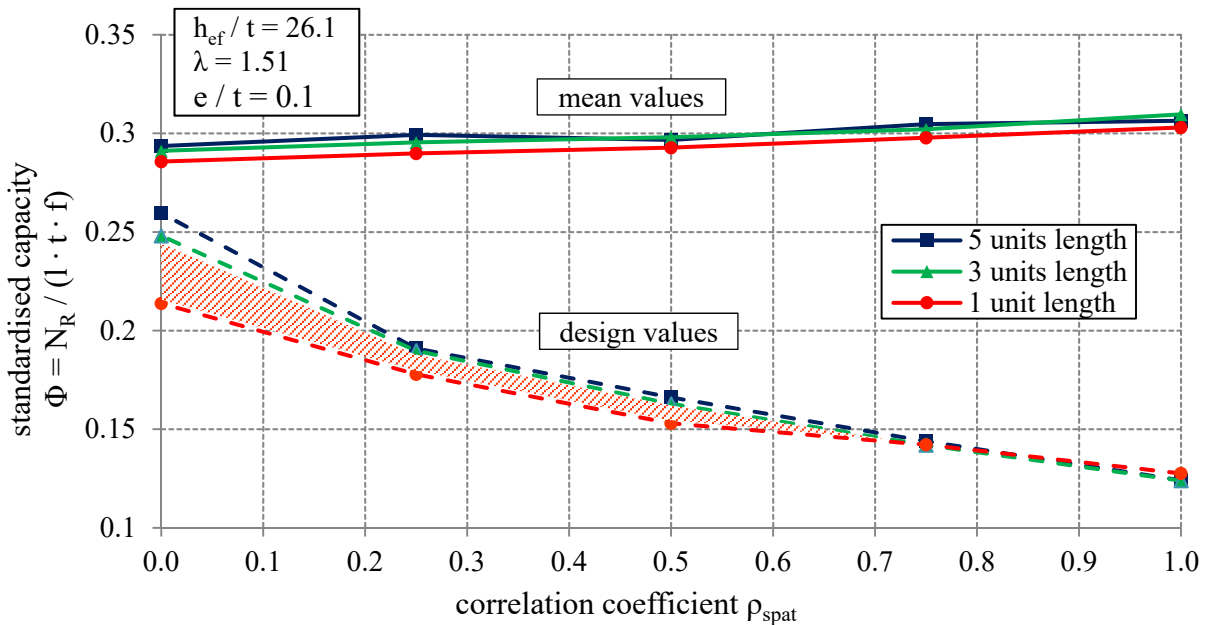


Figure 7: Mean and Design Capacity of Walls Failing due to Stability

CONCLUSIONS

Big differences between the safety factors of different design codes indicate that probabilistic analyses are needed to calibrate these factors and thereby improve the corresponding safety concepts. It was emphasised that spatial variability of material parameters should be considered in such reliability analyses. The results of Monte Carlo simulations showed that spatial variability has an influence on mean value, scatter and required design value of the load-bearing capacity and therefore on the reliability of the wall. It was pointed out that this effect differs depending on the failure mode and that, in most cases, the consideration of spatial variability will lead to higher reliability indices determined for masonry walls in compression. Further research should focus on the improvement of mechanical and stochastic models which can describe masonry walls including their spatial variability. Therefore, tests are required to gain reliable stochastic material parameters like spatial correlation coefficients, for example. With these improved models, reliability analyses can be performed for the calibration of safety factors.

REFERENCES

- [1] Graubner, C.-A. and Koob, B. (2015). "Analysis and comparison of the NDPs of various national annexes of Eurocode 6". *Mauerwerk – European Journal of Masonry*, 19(6), 424-440.
- [2] Schueremans, L. (2001). *Probabilistic evaluation of the structural reliability of unreinforced masonry*, doctoral thesis, Katholieke Universiteit Leuven, Belgium.
- [3] Tran, N. L.; Grziwa, U. and Graubner, C.-A. (2015). "Spatial variability of material properties and its influence on structural reliability of UHPFC columns." *Proc., Fib Symposium, Concrete – Innovation and Design, Copenhagen*, Denmark.
- [4] Brehm, E. (2011). *Reliability of unreinforced masonry bracing walls*, doctoral thesis, Technische Universität Darmstadt, Germany.
- [5] Glowienka, S. (2007). *Reliability of unreinforced structural masonry members made of big sized units*, doctoral thesis, Technische Universität Darmstadt, Germany, in German.
- [6] Ellingwood, B. (1985). "Limit states criteria for masonry construction." *J. Struct. Eng.*, 111(1), 108-122.
- [7] Li, J.; Masia, M.; Stewart, M. and Lawrence, S. (2014). "Spatial variability and stochastic strength prediction of unreinforced masonry walls in vertical bending." *Engineering Structures*, 59, 787-797.
- [8] Glock, C. (2004). *Load-bearing capacity of unreinforced concrete and masonry walls – Model for non-linear analysis and consistent design concept for slender walls under compression*, doctoral thesis, Technische Universität Darmstadt, Germany, in German.
- [9] Nowak, A. S. and Collins, K. R. (2013). *Reliability of structures*, second edition, CRC Press, Boca Raton, FL, USA.
- [10] Sofistik (2016). Finite element analysis program package. SOFiSTiK AG, Oberschleißheim, Germany.
- [11] Joint Committee on Structural Safety (2011). "Masonry Properties." In: *JCSS Probabilistic Model Code*, www.jcss.byg.dtu.dk.