13TH CANADIAN MASONRY SYMPOSIUM
HALIFAX, CANADA
JUNE 4TH – JUNE 7TH 2017

LOAD-DISPLACEMENT BACKBONE MODEL FOR FLEXURE-DOMINATED REINFORCED MASONRY SHEAR WALLS

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ABSTRACT

The lateral load-displacement response of reinforced masonry shear walls (RMSWs) has been extensively studied experimentally over the last decades. However, few simple analytical load-displacement models, exist in the literature, capable of predicting the complete RMSW response including the RMSW post peak behaviour. In this study, a backbone model is proposed capable of predicting the load-displacement relationship for flexure dominated RMSWs up to 20% strength degradation. The proposed backbone model is a quad-linear connecting the origin point with four key points corresponding to crack initiation, yielding, ultimate strength and 20% strength degradation. This study builds on the model proposed by Ashour and El-Dakhakhni [1]. A stress-strain material model for masonry and steel is utilized in the current study. Moreover, the model predictions were calibrated and validated against twenty-five RMSW tested under quasi-static cyclic loading having various shear span to depth ratio, vertical and horizontal reinforcement ratio and levels of axial stress. The model results show an overall acceptable level of accuracy including the post peak response. The RMSW lateral force corresponding to the four aforementioned points were perfectly predicted utilizing the proposed material models. Furthermore, it can be inferred that a simple reduction factor (i.e. computed from simple regression) multiplied by the RMSW stiffness at different level of loading can be used in calculating the corresponding RMSW displacements. The model procedure is simple, and the predictions are promising. Consequently, this model can be adopted in different design and assessment frameworks.

KEYWORDS: *analytical model, backbone model, concrete block wall, quad-linear model, reinforced masonry, shear walls*

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INTRODUCTION

Modeling is, in most cases, more economical than experimental studies. However, experimental studies are essential to validate and/or calibrate these models. *Numerical* and *Analytical* modeling approaches were followed in the available literature to simulate the reinforced masonry shear walls (RMSWs) response as component or within a system under seismic loading. Numerical modeling involves modeling the component or system response using in most micro- or macro-modeling techniques. On the other hand, analytical modeling is based on basic mechanics (i.e. equilibrium and compatibility) and sometimes empirical formulas generated from experimental data fitting.

A complete predicted load-displacement backbone model including the post peak wall response is an essential tool that can be beneficial in various ways. For example, the initial stiffness, yield and ultimate strength, ductility and damage states are some important engineering information that can be extracted from such model. Adding to that, the RMSW load displacement hysteresis can be generated using the computed backbone curve and any available hysteresis model (e.g. Sengupta and Li [2]). Therefore, such models represent an essential tool for forced-, displacement- and performance-based seismic design and seismic risk assessment of RMSW component and systems.

This study builds on what was originally proposed by Ashour and El-Dakhakhni [1] by computing the RMSW lateral load and displacement at four distinctive points; crack initiation, yielding, peak and 20% strength degradation. However, in the current paper the model was calibrated against more data available in the literature (i.e. 25 RMSWs). The RMSW lateral force corresponding to these four loading stages were calculated based on strain compatibility and forces equilibrium. Force predictions were enhanced in the current study by implementing a material model for masonry and the steel. Adding to that, this study validates the equation proposed by Paulay and Priestly [3] used to calculate K_y utilizing the effective moment of inertia, I_e, and effective cross-sectional area, A_e, as suggested by Priestley and Hart [4].

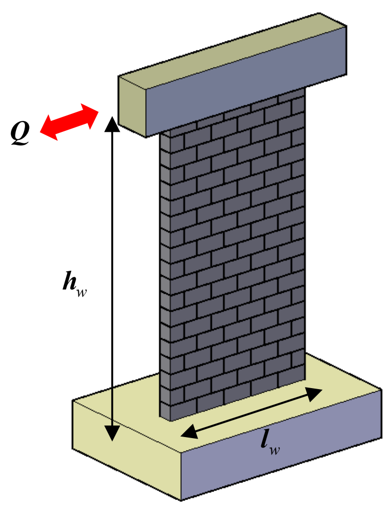


Figure 1: RMSW height and length

Twenty-five quad-linear backbone load-displacement curves were computed using the proposed model and compared to the observed experimental results. It is worth mentioning that the model calculations are simple and were carried out using Excel spread sheet and it takes only seconds to enter the data and compute the curves. However, this models is still open for more enhancements by considering more flexure-dominated RMSWs available data in the literature.

DATABASE

Twenty-five RMSWs presented in Table 1 were used to validate the RMSW lateral force predictions and to calibrate the lateral top displacement. The database consisted of full-scale flexure dominated RMSW tested by Sherman [5], Ahmadi [6], and Kapoi [7] under quasi-static cyclic loading. These walls had aspect ratio ranging between (0.7 ~ 4.5), different levels of axial stress, and various vertical and horizontal reinforcement ratios and arrangements (see Table 1). Moreover, two of the walls (i.e C7, C8) tested by Kapoi [7] had two layers of vertical rebars at the wall ends. As shown in Figure 1, the wall height h_w is the distance between the top of the footing and the loading beam's centerline.

Table 1: Database utilized in the current study

Reference	Wall #	Specimen ID in the reference	Boundary conditions	Wall dimensions		Aspect ratio	Shear span to depth	Vertical reinforcement		Horizontal reinforcement		P/(f'm*Ag)
				h_w	L_w			Number of bars	ρ_v	Number of bars	ρ_h	
				mm	mm		M/(Q.dv)		%		%	Mpa/Mpa
Sherman [5]	1	WSU-Wall 1A	Cantilever	1829	1006	1.8	2.50	5	0.73	9	0.33	0.063
	2	WSU-Wall 1B	Cantilever	1829	1006	1.8	2.50	5	0.73	9	0.33	0.063
	3	WSU-Wall 2A	Cantilever	1829	1006	1.8	2.50	5	0.33	9	0.33	0.125
	4	WSU-Wall 2B	Cantilever	1829	1006	1.8	2.50	5	0.33	9	0.33	0.125
	5	WSU-Wall 3	Cantilever	1626	1819	0.9	1.25	9	0.33	3	0.12	0.000
	6	WSU-Wall 4	Cantilever	1626	1819	0.9	1.25	9	0.33	8	0.33	0.063
	7	WSU-Wall 5	Cantilever	1219	1819	0.7	0.97	9	0.33	6	0.33	0.000
	8	WSU-Wall 6	Cantilever	1219	1819	0.7	0.97	9	0.33	6	0.33	0.063
Ahmadi [6]	9	UT-W-13	Cantilever	3658	1219	3.0	4.0	6	0.72	9	0.16	0.050
	10	UT-W-14	Cantilever	3658	1219	3.0	4.0	6	0.33	9	0.16	0.100
	11	UT-W-15	Cantilever	3658	1219	3.0	4.0	6	0.72	9	0.16	0.100
	12	UT-W-16	Cantilever	3658	1219	3.0	4.0	6	0.33	9	0.16	0.150
	13	UT-W-17	Cantilever	3658	813	4.5	6.0	4	0.72	18	0.33	0.050
	14	UT-W-18	Cantilever	3658	813	4.5	6.0	4	0.33	18	0.33	0.100
	15	UT-W-19	Cantilever	3658	813	4.5	6.0	4	0.72	9	0.16	0.100
	16	UT-W-20	Cantilever	3658	813	4.5	6.0	4	0.33	9	0.16	0.150
	17	UT-PBF-05	Cantilever	3658	813	4.5	6.0	4	1.29	18	0.33	0.000
Kapoi [7]	18	WSU-Wall C1	Cantilever	1829	1016	1.8	2.5	5	0.33	9	0.33	0.000
	19	WSU-Wall C2	Cantilever	1829	1016	1.8	2.5	5	0.33	9	0.33	0.063
	20	WSU-Wall C3	Cantilever	1829	1016	1.8	2.5	3	0.59	9	0.33	0.063
	21	WSU-Wall C4	Cantilever	1219	1829	0.7	1.0	5	0.55	12	0.66	0.063
	22	WSU-Wall C5	Cantilever	1626	1829	0.9	1.3	5	0.55	16	0.66	0.063
	23	WSU-Wall C6	Cantilever	2642	1422	1.9	2.5	7	0.72	7	0.18	0.000
	24	WSU-Wall C7	Cantilever	2642	1422	1.9	2.5	8	0.82	26	0.36	0.000
	25	WSU-Wall C8	Cantilever	2642	1422	1.9	2.5	8	0.82	26	0.36	0.063

MODEL OVERVIEW

A trilinear backbone analytical model was proposed by Ashour and El-Dakhakhni [1] capable of computing the load and displacement of flexurally-dominated RMSW. The load and corresponding displacement were computed at three particular points corresponding to, yield initiation, ultimate strength, and 20% strength degradation. Then a crack initiation fourth point was added by Ashour and El-Dakhakhni [8] to enhance the model predictions in the elastic zone (see Fig.2). The four points were calculated using first principles (enforcing equilibrium and compatibility conditions), given the wall cross-section dimensions, the arrangement of reinforcement, material characteristics, and boundary conditions. The model predictions were then validated against three RMSW components and two buildings.

The model proposed by Ashour and El-Dakhakhni [8] had some limitations. The model used, K_y, proposed by Paulay and Priestly [3] (i.e. indicated as K_P in the current study) without any validation to calculate the yield displacement (see Eqs 1-4). On the other hand, the RMSW lateral displacement corresponding to the ultimate strength and 20% strength degradation were calculated based on a reduction factors (i.e. 0.6 and 0.2) multiplyed by the RMSW stiffness as shown in Eq. 5 and 6. However, these reduction factors were calculated based on the best fitting of three RMSW. In this study twenty-five walls were used to first investigate if a linear relation exists between K_y, K_u, and $K_{0.8u}$. Consequently, calculating new reduction factors calibrated against larger database. Moreover, a stress-strain material model was used, in the current study, to calculate the masonry stress, strain and youngs modulus at different loading stages.

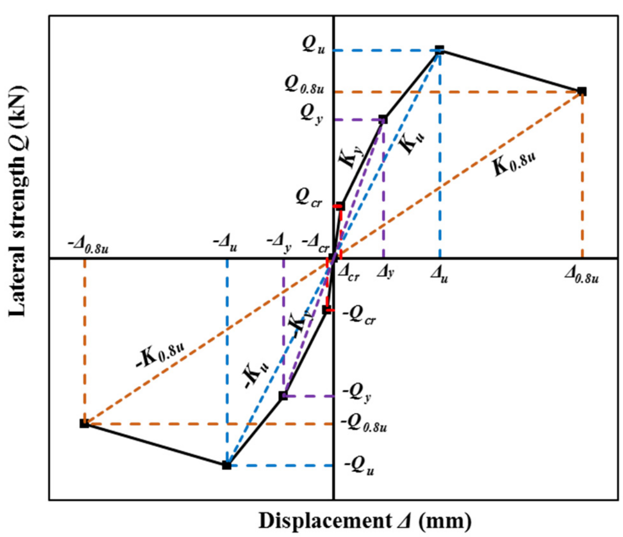


Figure 2: Backbone model. (Adopted from Ashour and El-Dakhakhni, [8])

$$K_p = 1/\left(\frac{h_w^{\,3}}{12E_m I_e} + \frac{1.2\,h_w}{G_m A_e}\right) \tag{1}$$

$$G_m = 0.4 \times E_m \tag{2}$$

$$I_e = \alpha I_g,\ A_e = \alpha A_g,\ \alpha = \left(\frac{100}{f_y} + \frac{P_u}{f'_m A_g} \right) \tag{3}$$

$$K_y = K_P \tag{4}$$

$$K_u = 0.6 \times K_y \tag{5}$$

$$K_{0.8u} = 0.2 \times K_y \tag{6}$$

MATERIAL MODEL

A stress strain masonry model originally developed for concrete by Hogenstad [9] was used in the current study to calculate the stress and corresponding strain at different loading stages. Figure 3. present the parabola defining the relation between the masonry stress, f_m, and the masonry strain, ε_m, as a function of masonry ultimate stress, f'_m, and the corresponding strain, ε_0 (see Eq.7).

$$\boldsymbol{f}_m = f'_m \left\{ \frac{2\varepsilon_m}{\varepsilon_0} - \left(\frac{\varepsilon_m}{\varepsilon_0} \right)^2 \right\} for \varepsilon_m < \varepsilon_0 \tag{7}$$

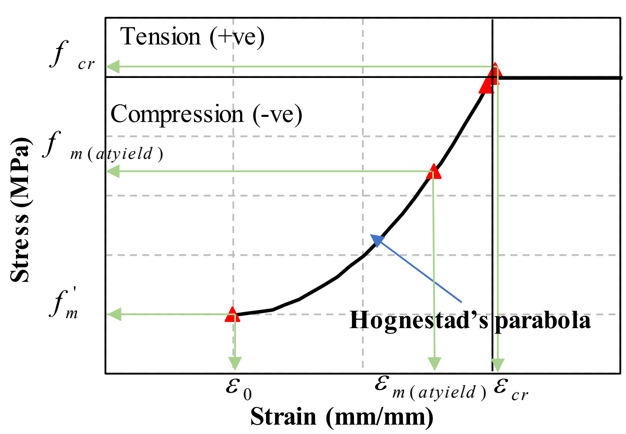


Figure 3: Masonry stress-strain model.

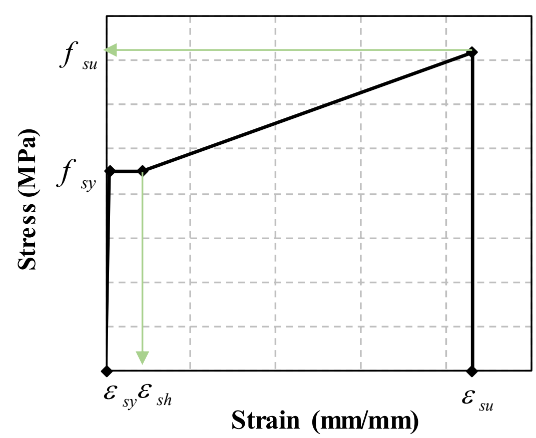


Figure 4: Rebar stress-strain model.

A trilinear stress strain relationship was implemented to calculated the stress in rebars corresponding to different strain levels considering strain hardening effects. In the current study the steel yield stress, ultimate strength, yield strain, strain corresponding to hardening initiation and ultimate strain were extracted from the material testing curves documented in the experimental studies [5] [6] [7].

LATERAL RMSW FORCE RESISTANCE CALCULATIONS

Cracking initiation

The first point in the model corresponds to the onset of the cracking in masonry, Q_{cr}, was calculated when the tensile stress in masonry reaches f_t. The flexure tensile strength, f_t, was assumed 0.65 MPa as recommended by CSA S304 [10]. Therefore, the cross-sectional moment capacity at this loading stage can be calculated and the lateral wall strength corresponding to crack initiation can be estimated using Eq.8.The corresponding masonry strain and young's modulus were computed using the aforementioned material model. Thus, the corresponding wall stiffness and lateral displacement can be computed using Eqs. 9, 2, 10. (See Figure 5a)

$$Q_{cr} = \frac{M_{cr}}{h_w}, \quad (8) \qquad K_g = 1 \Big/ \left(\frac{h_w^{\ 3}}{3E_m I_g} + \frac{1.2\, h_w}{G_m A_g} \right) \quad (9) \qquad \Delta_{cr} = \frac{Q_{cr}}{K_g} \quad (10)$$

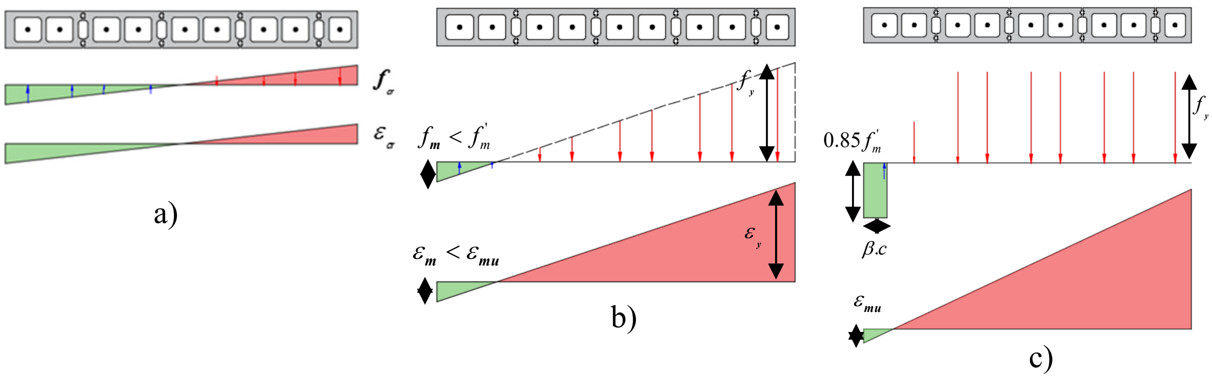


Figure 5: Stress and strain distribution over the RMSW cross-section at: a) crack initiation, b) yielding, c) ultimate strength

Yielding

The yield strength, Q_y, was computed at the onset of the outermost reinforcement bar yielding. Similarly, the RMSW moment capacity at yielding and corresponding wall strength can be computed using equilibrium and compatibility. (See Eq.11 and Figure 5b)

$$Q_y = \frac{M_y}{h_w} \qquad (11)$$

Ultimate

Q_u (the third point) was calculated when the masonry reached its ultimate compression strain, ε_{mu} = 0.0025 as proposed by [10]. The RMSW ultimate strength, Q_u, can be computed using Eq. 12

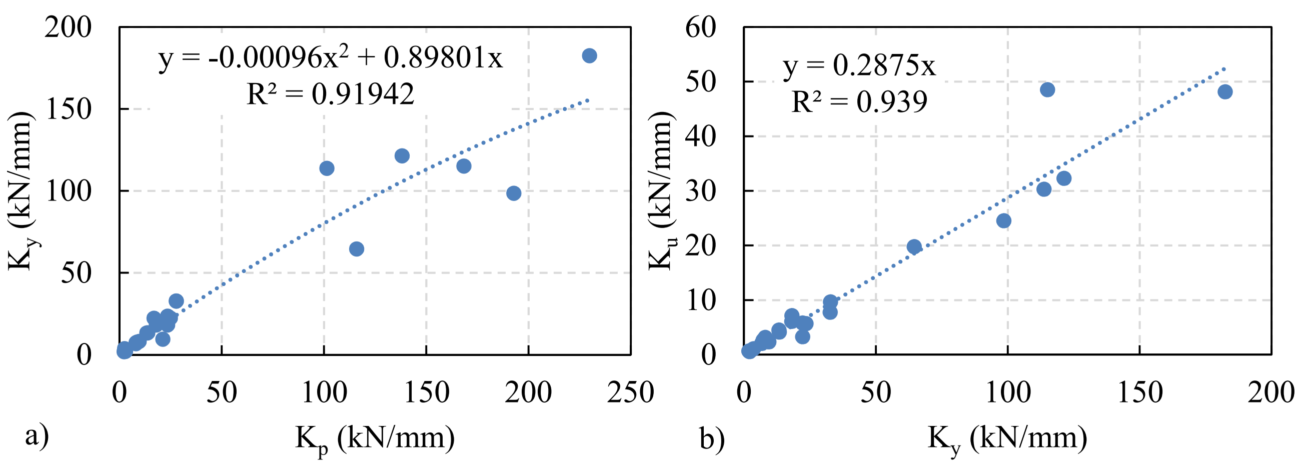
$$Q_u = \frac{M_u}{h_w} \qquad (12)$$

20% strength degradation

Finally, the RMSW strength corresponds to 20% strength degradation, $Q_{0.8u}$, was calculated by simply multiplying Q_u by 0.8 to represent the fourth point. The displacement corresponds to Q_y, Q_u and $Q_{0.8u}$ will be discussed in the following section.

DISPLACEMENT CALCULATIONS

Simple linear regression analysis was first used to validate the capability of K_p proposed by Paulay and Priestly [3] to predict the RMSW stiffness at the yield, K_y. Therefore, the RMSW stiffness corresponding to yielding was extracted from the experimental data of the twenty-five RMSW and compared to the calculated K_p based on Eqs.1-3. It was observed that K_p was capable of predicting K_y for RMSW having low yield stiffness (i.e. less than 50 kN/mm). However, simple linear regression was not valid for RMSW having higher yield stiffness. Consequently, non-linear regression was implemented to correlate the experimental yield stiffness with K_p as shown in Figure 6(a). On the other hand, simple linear regression successfully correlated the experimental K_u with K_y and $K_{0.8u}$ with K_u as shown in Figure 6 (b and c). Consequently, Δ_y, Δ_u, and $\Delta_{0.8u}$ can be calculated according to Eq. 13 to15.



$$K_y = -0.00096K_P^2 + 0.89801K_P$$

$$\text{(i.e } K_P \text{ kN/mm)},\ \Delta_y = \frac{Q_y}{K_y} \tag{13}$$

$$K_u = 0.2875K_y,\ \Delta_u = \frac{Q_u}{K_u} \tag{14}$$

$$K_{0.8u} = 0.5479K_u,\ \Delta_{0.8u} = \frac{Q_{0.8u}}{K_{0.8u}} \tag{15}$$

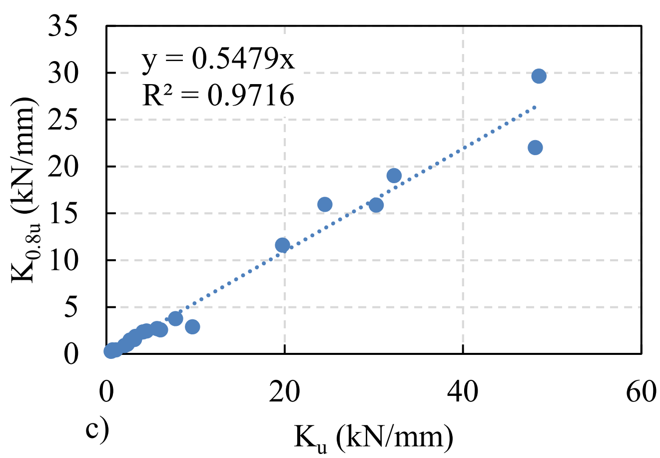


Figure 6: Simple regression relationship between: a) K_y and K_p, b) K_u and K_y, c) $K_{0.8u}$ and K_u.

MODEL VALIDATION

Figure 7 presents the model predictions compared to the experimental results adopted from (Sherman [5], Ahmadi [6], and Kapoi [7]). It can be observed that the model was capable of capturing the overall RMSW lateral response. Generally, the post peak branch was perfectly computed and showed a good agreement with the experimental data (e.g. WSU-Wall C5, and WSU-Wall C6) but this was not necessary the consensus of all (e.g. WSU-Wall C2). In this regard, the ultimate strength was under estimated in Wall WSU-Wall C2 and as a result the displacement corresponding to ultimate strength was also under estimated. This could be one of the limitations of the best fitting models. In addition, it can be observed from Figure 7 that sometimes the model was able to perfectly capture only one of the loading directions (e.g. UT-W-18). However, this observation was reported in walls having unsymmetrical load-displacement envelope. Finally, this preliminary discussion showed a qualitative assessment of the predictive backbone curve, as such more insight quantitative assessment will be presented in future work.

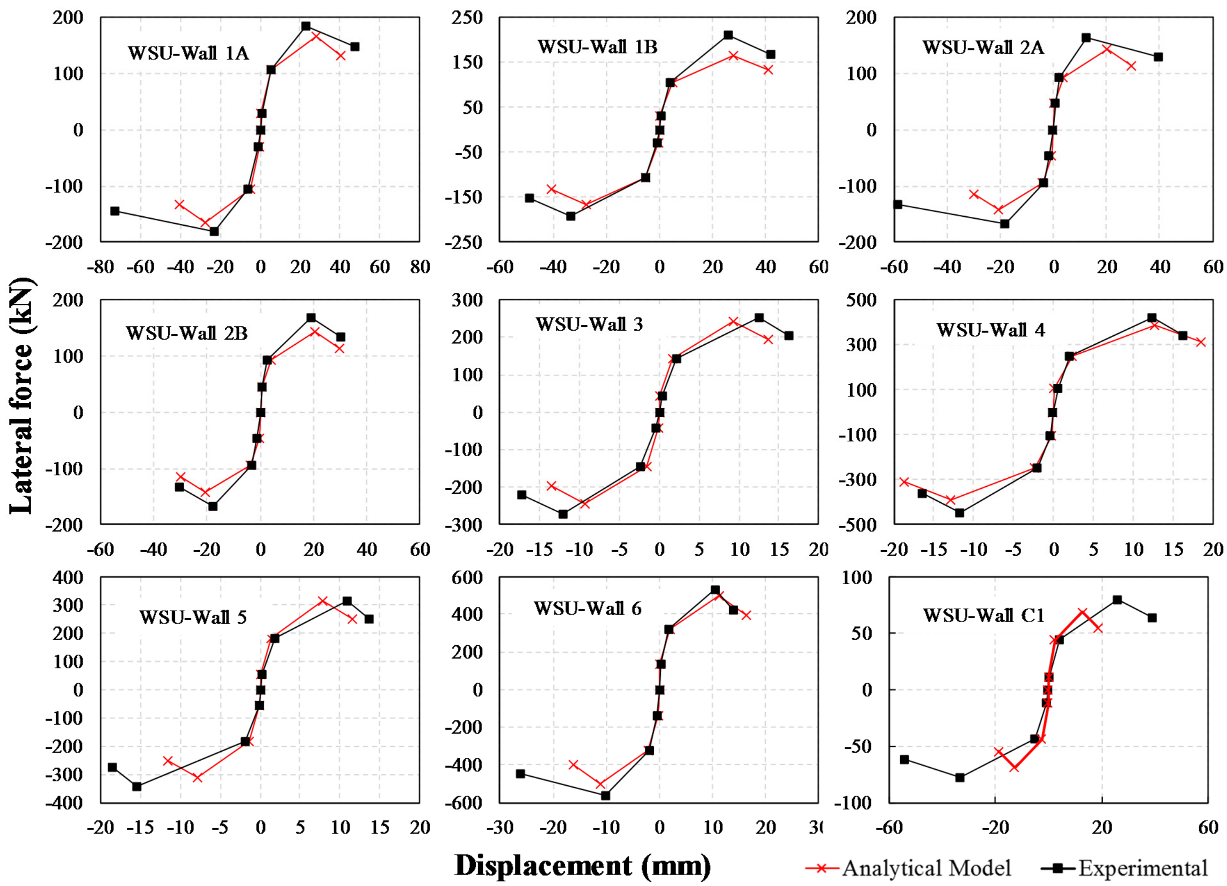


Figure 7: Model predictions versus the experimental data adopted from (Sherman, [5]; Ahmadi, [6]; Kapoi, [7])

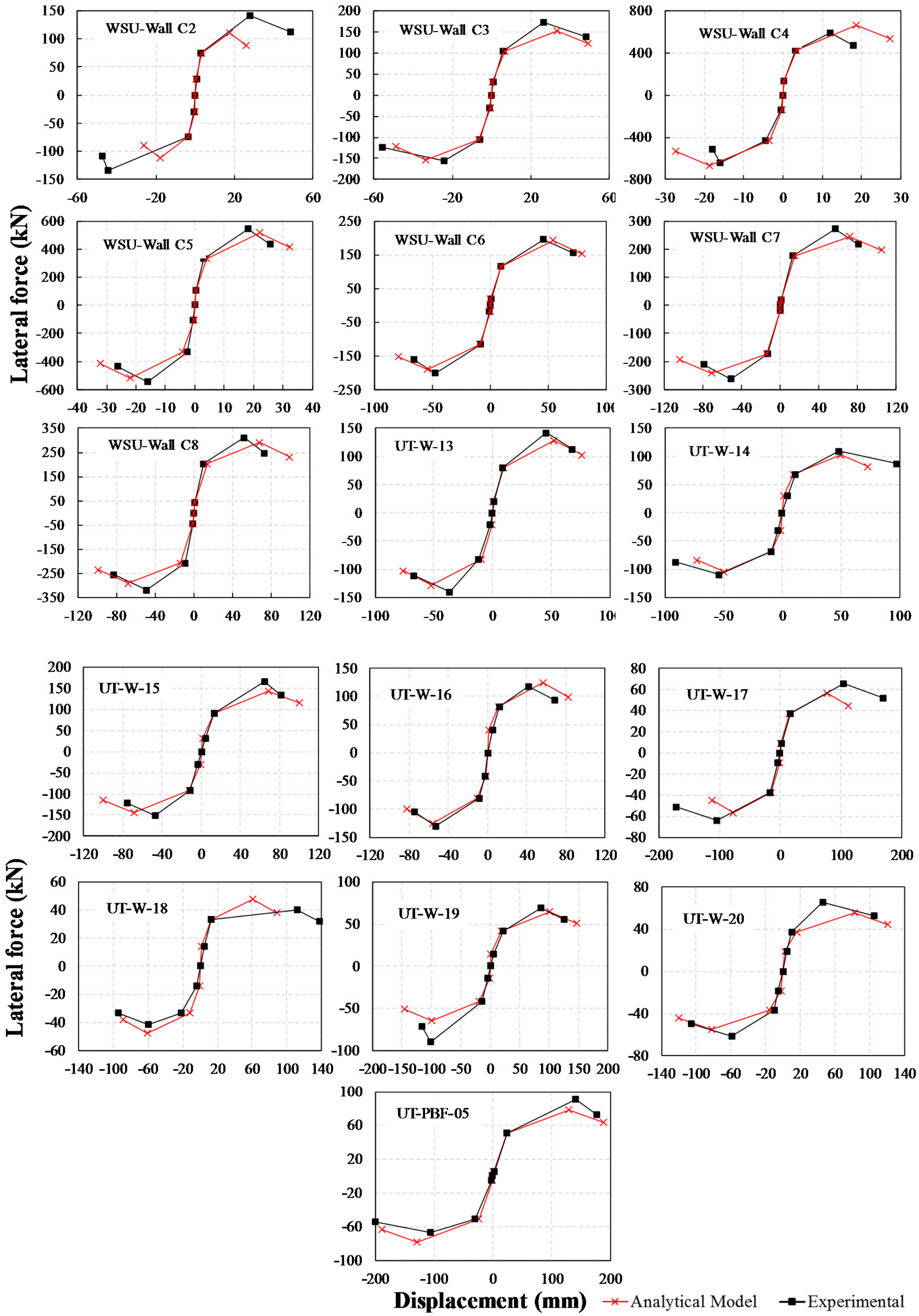


Figure 7 cont.: Model predictions versus the experimental data adopted from (Sherman, [5]; Ahmadi, [6]; Kapoi, [7])

CONCLUSIONS

This paper presents a quad-linear backbone model for flexure-dominated RMSW tested under quasi-static cyclic loading. The model predictions were calibrated/validated against twenty-five RMSWs. The model predictions show a good agreement with the corresponding reported experimental data. In this regard, first principles using force equilibrium and strain compatibility were used to compute the RMSW lateral force corresponding to; crack initiation, yielding, ultimate strength and 20% strength degradation. Moreover, simple regression analysis was utilized to predict the wall stiffness at the same four loading stages. Consequently, the wall displacements corresponding to the four loading stages can be computed. It should be noted that this model is currently validated against another set of flexure dominated RMSW tested under quasi-static cyclic loading in order to investigate if simple regression is sufficient for larger sample or multivariate analysis may present better predictions. The presented model is one of the few available analytical models capable of predicting the RMSW load-displacement response up to 20% strength degradation.

ACKNOWLEDGEMENTS

The authors acknowledge the support of the Natural Science and Engineering Research Council of Canada (NSERC), l'Association des entrepreneurs en maçonnerie du Québec (AEMQ), the Canadian Concrete Masonry Producers Association (CCMPA) and the Canadian Masonry Design Centre (CMDC).

NOTATIONS

Δ_{cr} = Displacement at roof slab center of mass correspond to initiation of cracking in masonry;
Δ_y = Displacement at roof slab center of mass correspond to yield strength;
Δ_u = Displacement at roof slab center of mass correspond to ultimate strength;
ε_y = Reinforcement bar's yield strain;
ε_{mu} = Ultimate compression masonry block's strain;
ε_0 = Strain corresponding to ultimate stress;
A_g = Gross cross-sectional area;
A_e = Effective cross-sectional area;
B_w = Wall width;
C_M = Building roof's center of mass;
C_R = Building roof's center of rigidity;
E_m = Masonry young's modulus;
f_y = Reinforcement bars yield stress;
f'_m = Masonry compressive ultimate stress;
G_m = Masonry shear modulus;
h_w = Wall height;
I_g = Gross cross-section moment of inertia;
I_e = Effective cross-section moment of inertia;
K_P = Cross-section secant stiffness correspond to the yield strength as proposed by Paulay and Priestly [3]
K_y = Cross-section secant stiffness correspond to the yield strength;

K_u = Cross-section secant stiffness correspond to the ultimate strength;
$K_{0.8u}$ = Cross-section secant stiffness correspond to the 20% strength degradation
L_w = Wall length;
M_u = Cross-section moment capacity;
M_y = Cross-section yield moment capacity;
P = Applied axial load;
Q_{cr} = Strength corresponding to initiation of cracking in masonry;
Q_y = Yield strength;
Q_u = Ultimate strength and
$Q_{0.8u}$ = Strength corresponding to 20% strength degradation.

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