13TH CANADIAN MASONRY SYMPOSIUM
HALIFAX, CANADA
JUNE 4TH – JUNE 7TH 2017

EVALUATION OF A REINFORCED BLOCK WALL SLENDERNESS LIMIT BASED ON RADIUS OF GYRATION

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ABSTRACT

In the design of steel, wood, and reinforced concrete walls the slenderness limit is based on the radius of gyration (*r*) of the cross-section rather than thickness (*t*). For concrete, the sister material to concrete masonry, the slenderness limit is $(kh/r) \leq 100$ with *r* for a rectangular cross-section rounded to 0.3*t* yields the same slenderness ratio limit in the current CSA S304.1-14 design standard for masonry structures. This paper proposes the use of the two-dimensional radius of gyration to better capture the geometry of partially grouted walls when compared to using the wall thickness. This results in an increase in the height that walls can be designed and constructed before the conditions for slender wall design are imposed. Elastic and inelastic analysis presented on two tall walls supports the recommendation to establish a new limit based on the radius of gyration.

KEYWORDS: *partially grouted walls, slenderness limit, radius of gyration.*

INTRODUCTION

Slender wall design in North America has evolved immensely over the last 40 years. In the 1980s, research by Amrheim (1981) [1] demonstrated masonry walls with heights exceeding 30 times their thickness could perform adequately under lateral and axial loads when suitable design procedures were used. Further research conducted throughout the 1980s and 1990s on tall masonry walls led to revisions to the CSA S304.1-84 [2] design standard, introducing two new classifications for walls as "slender" and "very slender" [3-6]. Walls with a slenderness ratio that exceeded 30 times the wall depth or $kh/t \geq 30$ could now be designed and constructed as "very slender reinforced walls under low axial load", provided that they met three specific requirements: (1) the walls must be reinforced, (2) support low axial loads, and (3) exhibit ductile failure [2]. However, for steel, wood, and reinforced concrete the slenderness limit is

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based on the radius of gyration (r) rather than thickness (t). For reinforced concrete, the slenderness limit was $kh/r \leq 100$ with r for a rectangular cross-section rounded to $0.3h$. This paper explores the possibility that this limit was the basis for the original $kh/t \leq 30$ limit in the design standard for masonry structures and explores the impact on the height and safety associated with a return to a slenderness limit based on radius of gyration rather than depth of a structural masonry element in compression.

SLENDERNESS LIMIT FOR CONCRETE ELEMENTS IN COMPRESSION

For over three decades, more than 4 editions of the Canadian design standard for reinforced concrete structures (CSA A23.3) [7-10] has defined the ultimate slenderness limit for a concrete element in compression as:

$$\frac{kl_u}{r} \leq 100 \qquad [1]$$

where r is the radius of gyration, l_u is the unsupported length of the structural element in compression, and k adjusts the element's length to account for support conditions according to a modified Euler's Buckling theory. Relevant excerpts pertaining to slenderness limit and radius of gyration from CSA A23.3-14 include clause 10.14.2, which states that the radius of gyration can be taken as $0.30h$ for a rectangular compression member, $0.25D$ for a circular compression members, or computed using the *gross concrete section* for all other shapes. From mechanics of materials, the radius of gyration, r, is calculated as:

$$r = \sqrt{\frac{I}{A}} \qquad [2]$$

For a solid rectangular cross section of length "b" and depth "h" this results in:

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{bh^3/12}{bh}} = \frac{h}{\sqrt{12}} = 0.289h \approx 0.3h \qquad [3]$$

A similar exercise to Eq. 3 for a circular cross-section produces $r = 0.25D$.

EVOLUTION OF THE SLENDERNESS LIMIT FOR MASONRY IN COMPRESSION

According to a source who participated in the development of the CSA-S304.1 design standard in the versions released between 1979 and 2004, the CSA-S304.1 design standards borrowed heavily from the CSA A23.3 design standard for concrete structures [7-10]. The similarities between the structural behaviour of reinforced concrete and reinforced concrete block masonry made this a rationale path for the CSA-S304.1's development. For masonry, the common gravity load resisting structural elements typically include walls. Clause 10.3.5 of the CSA S304.1-84 [2] design standard restricted the maximum slenderness of these elements to:

$$\frac{kh}{t} \leq 10\left(3 - \frac{e_1}{e_2}\right) \qquad [4]$$

where e_1 and e_2 are the eccentricities of the applied gravity loads at the top of the member. Walls exceeding this limit were simply not permitted to be designed or constructed. However, research conducted by Amerheim [1] demonstrated that walls with heights exceeding 30 times their depth could be built and support combined axial and lateral loads where lateral loads exceeded 3.6 *kPa* (75 *psf*) without collapse [1]. Further research by other authors over the next decade would confirm these results [3-6]. In the 1994 edition of the CSA S304.1 [11], the standard introduced new slenderness limits and established two new classes of walls; "slender walls" and "very slender walls". Clause 11.2.4.3.2 established that slender walls were those with slenderness ratios that were greater than those shown in Eq. 4, but not greater than 30. Walls that had slenderness ratios exceeding 30 were considered very slender walls with conditions imposed on the design of these walls for increased safety. These conditions included: a low axial load limit, eccentric pin end conditions to ensure symmetrical single curvature, and the requirement of area of reinforcement steel to be 80% of the steel required at balanced conditions. In CSA S304.1-04 [12], the standard refined the name of walls with a slenderness greater than 30 from "very slender walls" to simply "slender reinforced walls under low axial load", with the similar conditions for increased safety. This terminology and the conditions for "slender reinforced walls under low axial load" remain in effect in the CSA-S304-14 [13] edition of the design standard.

It is very possible that the basis of the slenderness limit $kh/t \leq 30$ introduced in the CSA S304.1-94 [11] found its basis in the CSA A23.3-94 design standard [8]. With this assumption and using masonry terminology, representing unsupported length as "h" and depth "t", the derivation is a straight forward substitution of $r = 0.3t$ and $l_u = h$ into Eq. 1. For a solid rectangular shape this is:

$$\frac{kh}{r} \leq 100 = \frac{kh}{0.3t} \leq 100 \rightarrow \frac{kh}{t} \leq 30 \qquad [5]$$

However, Eq. 5 assumes that the thickness of the wall is a good representation of the radius of gyration of the cross-section, which is not the case for all masonry walls, and in particular partially grouted concrete block masonry walls.

EFFECT ON SLENDERNESS OF A LIMIT BASED ON THE RADIUS OF GYRATION

Slender walls are often encountered in tall low-rise structures, such as warehouses and school gymnasiums and the majority of these walls are partially grouted. The restrictive conditions of low axial load and steel yielding at locations on the axial load-moment interaction diagram other than pure bending translates to wall designs that are more safe, because these conditions ensure that a ductile failure occurs at locations other than pure bending and reduces secondary moment effects. However, these requirements often restrict practical wall designs because they lie 1 mm above the slenderness limit. In these cases, the wall would require only one vertical bar spaced at 1000mm on center if the wall height was exactly at the slenderness limit. This translates into wall designs that require higher strength masonry units or simply can't be designed for the load combinations.

The use of $kh/t \leq 30$ is more conservative than $kh/r \leq 100$, and much easier to calculate by hand. This may have been the rationale for its initial adoption. However, it is also less accurate and more restrictive for non-solid rectangular cross-sections, such as partially grouted walls. Introduction of a limit based on radius of gyration simply recognizes the two dimensional dependency of partially grouted concrete block walls. Partially grouted reinforced concrete block walls are rectangular but have an essentially I-shaped cross-section when aggregated. This cross-section cannot be adequately described by a single dimension. If the existing slenderness limit $kh/t \leq 30$ is replaced with the $kh/r \leq 100$ limit used in the concrete design standard, partially grouted walls could be installed to greater heights without the slenderness restrictions. Introduction of the new slenderness limit based on the radius of gyration does not alter the current *30t* limit as this limit appears to have been derived from the $kh/r \leq 100$ limit found in CSA-A23.3, and is identical for a fully grouted block wall when the radius of gyration is approximated to be *0.3t* as it is in concrete element design. Table 1 shows that the maximum height for a 20 *cm* block wall reinforced with steel reinforcement spaced at 1000 *mm* on centre based on the *kh/r* limit is 6.95 *m* (22.8 *ft*), compared to a maximum height of 5.7 *m* (18.7 *ft*) when based on the *kh/t* limit.

Table 1: Comparison of Maximum Wall Heights Before Considered Slender (CSA S304.1-14 Clause 10.7.3.3) for 20cm and 25cm Units Reinforced Block Walls

Nominal Block Size *(cm)*	Bar Spacing *(mm)*	From MASS Software		r *(mm/m)*	Max. Height (*kh/r≤100*) *(mm)*	Max. Height (*kh/t ≤ 30*) *(mm)*	kh/t using kh/r height
		A *(10^3 mm/m)*	I *(10^6 mm^4/m)*				
20*	200	190.0	571	54.8	5485	5700	28.9
	400	131.2	503	62.0	6197	5700	32.6
	600	111.6	481	65.7	6567	5700	34.6
	800	101.8	469	67.9	6794	5700	35.8
	1000	95.9	463	**69.5**	**6949**	5700	**36.6**
	1200	92.0	458	70.6	7061	5700	37.2
	1400	89.2	455	71.5	7145	5700	37.6
	1600	87.1	452	72.1	7212	5700	38.0
	1800	85.5	451	72.7	7265	5700	38.2
	2400	82.2	447	73.8	7377	5700	38.8
25*	200	240.0	1152	69.3	6928	7200	28.9
	400	158.6	972	78.3	7829	7200	32.6
	600	131.5	912	83.3	8330	7200	34.7
	800	117.9	882	86.5	8651	7200	36.0
	1000	**109.8**	**864**	**88.7**	**8874**	7200	**37.0**
	1200	104.3	852	90.4	9039	7200	37.7
	1400	100.5	843	91.6	9165	7200	38.2
	1600	97.6	837	92.7	9265	7200	38.6

*Note: 20 *cm* and 25 *cm* masonry block units have actual block sizes of 190 *mm* and 240 *mm respectively*.

NMERICAL MODELLING OF SLENDER MASONRY WALLS

To study the applicability of using radius of gyration in place of block thickness as a basis for establishing the slenderness of masonry walls, two reinforced partially grouted masonry walls were modelled using two different approaches: (1) a simplified linear elastic finite element model using SAP2000 and (2) a more rigorous nonlinear finite element model using OpenSees. Figure 1 shows the two partially grouted walls modelled in this study. Each wall measures 1190mm wide and is reinforced with two 15M bars spaced at 1000 *mm* on centre. To determine the feasibility in using $kh/r \leq 100$ when compared with $kh/t \leq 30$, wall heights of 5600 *mm* and 6800 *mm* were selected such that they represent the current and proposed slenderness limits for a 20 *cm* block wall with steel reinforcement placed at 1000 *mm* on centre.

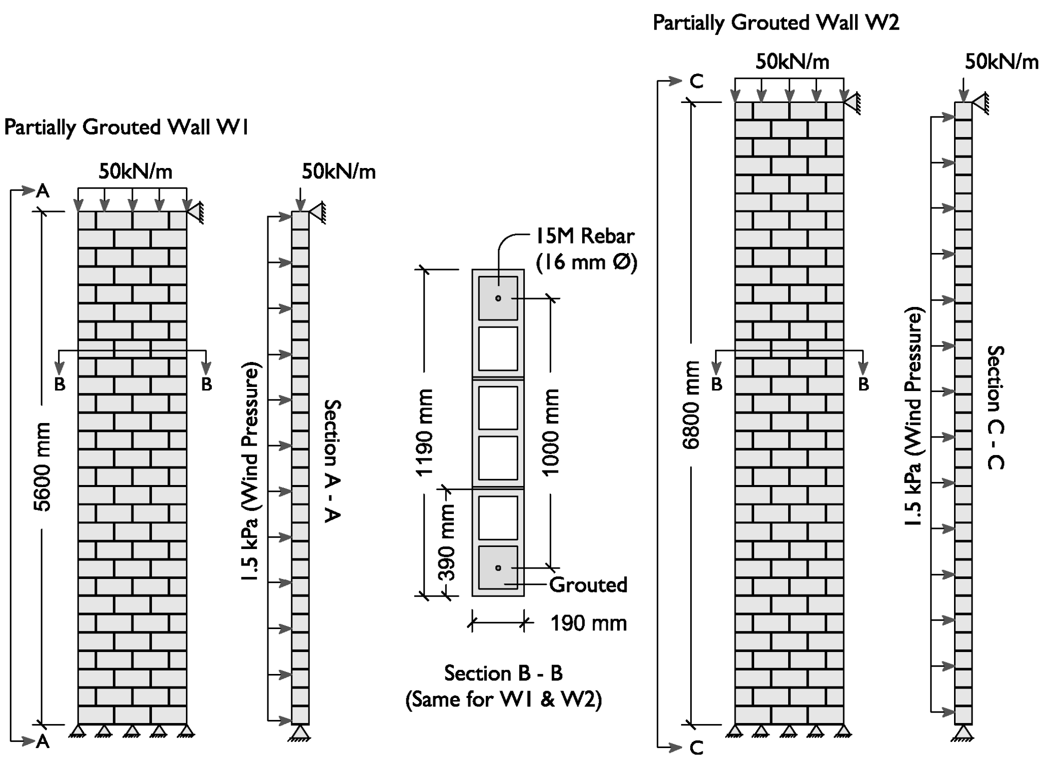
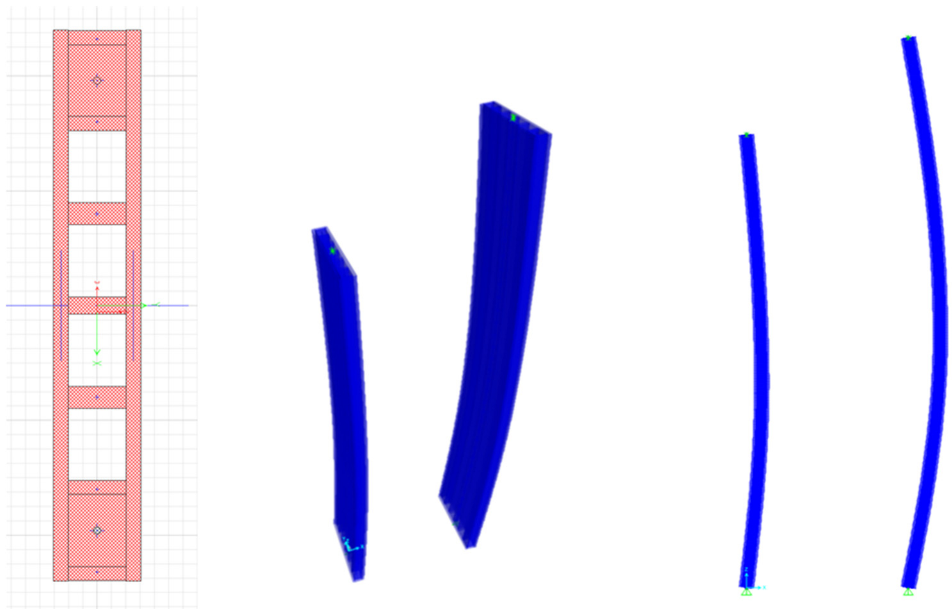


Figure 1: Partially Grouted Masonry Walls W1 (left) and W2 (right)

Elastic SAP2000 Model

The first modelling approach utilized to study the applicability of using a *kh/r* slenderness limit for tall masonry walls was a simplified linear elastic model developed in the finite element software SAP2000 [14]. Figure 2 shows the cross section and profile views for the elastic frame elements used to model the 5600 *mm* and 6800 *mm* walls. An axial load of 62.5 *kN* (translating to a 50 *kN/m* vertical line load), a lateral line load of 1.8 *kN/m* (translating to a 1.5 *kPa* lateral pressure), and a moment at mid-height of 6 *kN-m* (translating to 5 *kN-m/m*) to simulate P-delta effects were applied to both walls. The compressive strength and Young's modulus of the masonry blocks and grouted cores were assumed to be 10 *MPa* and 8500 *MPa*, respectively. The steel reinforcement was assumed to have a yield strength of 400 *MPa*.



(a) (b) (c)

Figure 2: SAP2000 Model: (a) Cross-section; (b) Isometric View; (c) Side View

Nonlinear OpenSees Model

To study the influence of nonlinear material behaviour in addition to second order effects (P-delta) in the slender wall model, the two prototype partially grouted reinforced masonry walls were also modelled using the Open System for Earthquake Engineering Simulation (OpenSees) software [15]. This program is an open-source object-oriented software framework for simulation in which the modeling approach is flexible, allowing the selection and various combinations of a number of different element and material formulations. Previous experience has shown that the OpenSees program is capable of predicting the response of the highly nonlinear structural systems with reasonable accuracy.

The OpenSees model subdivided the two tall masonry walls into thirteen nonlinear beam column elements connected in a two-dimensional assemblage of nodes. The nonlinear beam column element is a force-based element with distributed plasticity. The model accounts for second order effects (P-delta) through the use of a co-rotational coordinate transformation. This is particularly important in this model because of the slenderness of both walls. The top and bottom of the walls are assumed to be pinned, as required by the CSA S304.1-14 design standard [13]. Figure 3 shows an illustration of the two-dimensional OpenSees model. At discrete locations along the length of the nonlinear beam column elements, the cross-section of the wall is subdivided into longitudinal fibres that are assigned uniaxial constitutive relationships for masonry and steel reinforcement. The behaviour of the section is determined implicitly through integration of the fibres, assuming that plane sections remain plane. Figure 4 illustrates the stress-strain relationships adopted for the steel and masonry materials. OpenSees "*Steel01*" uniaxial material was used to model the steel reinforcement in the wall. This material assumes a bilinear stress-strain relationship in which the steel behaves linearly elastic up to the yield load and is then assumed to be perfectly plastic.

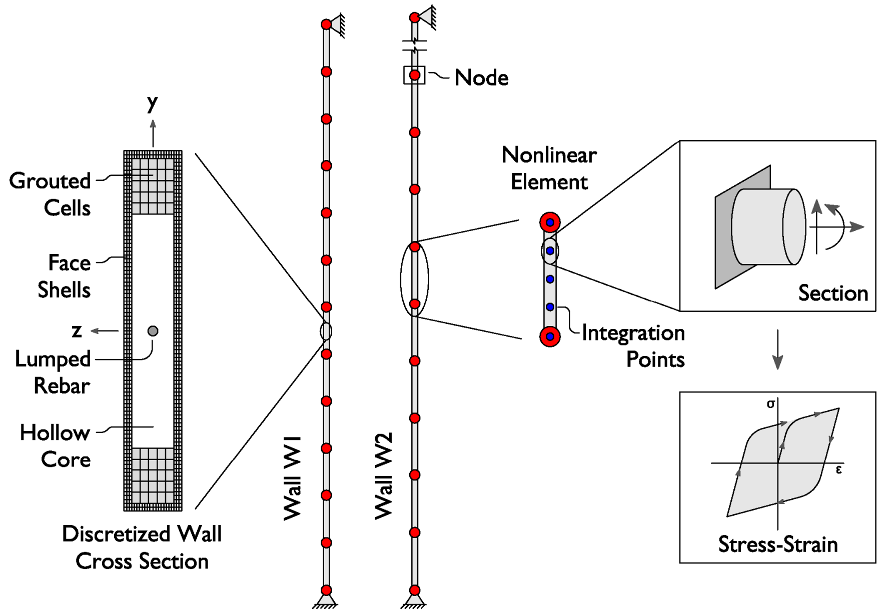


Figure 3: OpenSees Analytical Model and Fibre Section

The steel is assumed to have a Young's modulus of 200 *GPa* and a yield stress of 400 *MPa*. OpenSees "*Concrete02*" uniaxial material was used for modelling the face shells and grouted cells of the masonry block. This model assumes a parabolic stress-strain relationship up to the maximum stress for the masonry in compression, followed by a linear softening branch and a final horizontal plateau at the crushing strain of the masonry. In tension, the material behaves linearly up to the tensile strength of the masonry and then includes a linear tension softening branch to failure.

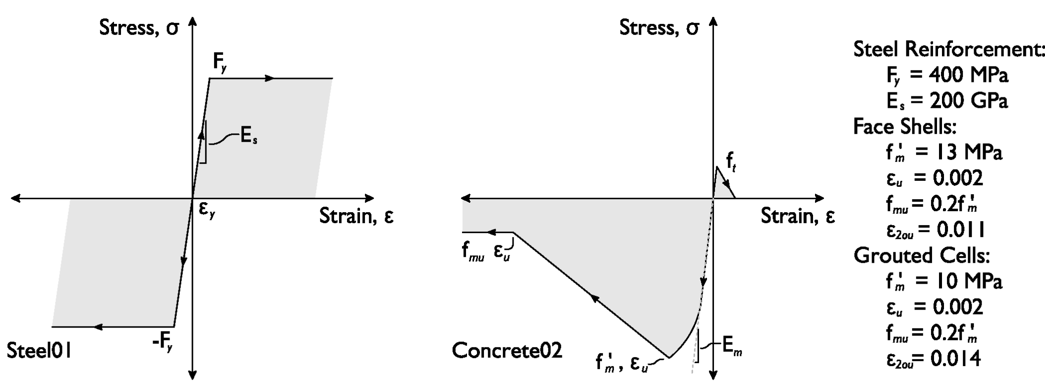


Figure 4: Material Constitutive Relationships for Steel (left) and Masonry (right) [Adapted from Priestley and Elder (1983)]

To calculate the crushing and ultimate stress and strain, the constitutive relationships for masonry proposed by Priestley and Elder (1983) [16] were adopted for this study, which have been shown to produce good results when compared with laboratory results on partially grouted wall specimens. In this model, it is assumed that the maximum stress in the masonry occurs at a strain of 0.002. The material behaviour is shown in Figure 4 and can also be expressed as:

$$\sigma = \begin{cases} f'_m\left[\left(\frac{2\epsilon}{0.002}\right) - \left(\frac{\epsilon}{0.002}\right)^2\right], \\ f'_m[1 - Z(\epsilon - 0.002)], \\ 0.2f'_m \end{cases} \quad [6]$$

where,

$$Z = \frac{0.5}{\left(\frac{3+0.29f'_m}{145f'_m-1000}\right) - 0.002} \quad [7]$$

A maximum stress of 13 *MPa* is assigned to the face shells of the masonry block, while a maximum stress of 10 *MPa* is assigned to the grouted cells in accordance with CSA S304.1-14. The crushing stress for the face shells and grouted cells is assumed to occur at a stress equal to 20% of the maximum stress (f'_m). The flexural tensile strength of the masonry material was assumed to be 0.65 *MPa* for grouted hollow block in accordance with Table 5 of CSA S304.1-14.

NUMERICAL RESULTS

To study the applicability of using $kh/r \leq 100$ when compared with $kh/t \leq 30$ for determining the slenderness of masonry walls, two partially grouted reinforced masonry walls described in the previous section were analyzed using an elastic modelling approach using SAP2000 and a nonlinear finite element model using OpenSees. Table 2 shows important structural parameters for the response of the walls using both modelling approaches. Results demonstrate that both modelling approaches agree that under design levels axial and lateral loads, deflections remain below the lateral deflection limit of *h/360* according to CSA S304.1-14. In addition, reasonable agreement between the SAP2000 and OpenSees models suggest that the wall specimens remain within the elastic range under the design loads.

Table 2: Structural Response Parameters for Tall Masonry Walls

Wall I.D.	Deflection Under Design Loads (mm)		Deflection Limit (*h*/360) (mm)	Yield Pressure (kPa)	Yield Displacement (mm)	Ultimate Deflection (mm)
	SAP2000	OpenSees				
W1	4.74	3.4	15.6	2.32	8.05	125
W2	9.90	10.1	18.9	1.53	6.02	174

To analyze the nonlinear response behaviour of both walls, Figure 5 shows the lateral pressure versus displacement curves from OpenSees for both walls. Figure 5b clarifies the response within the elastic range of the wall specimens, which shows that both walls remain nearly elastic. However, the results show that wall W2, which has an unsupported length of 6800 *mm* is on the verge of yielding under a lateral wind pressure of 1.5 *kPa* and any further increase in pressure results in a large increase in lateral deformation. Figure 6 shows the stress-strain response for the reinforcing steel, the edge of the masonry face shell under compression, and the edge of the grouted core on the compression side for both prototype tall walls. The results confirm that at a lateral pressure of 1.5 *kPa*, the steel in specimen W2 is on the verge of yielding, while the steel in wall W1 remains in the elastic range. Results also show crushing in the extreme masonry face shell fiber in compression at the ultimate displacement. Because the neutral axis remains within the face shell of the cross-section, the grouted cell on the compression side fails in tension after reaching the maximum tensile stress (0.65 *MPa*) for both walls.

DISCUSSION AND CONCLUSIONS

Numerical modelling results confirm that the introduction of a revised slenderness limit for masonry elements in compression based upon the $kh/r \leq 100$ slenderness limit still results in a safe wall design, even at the maximum slenderness. The adoption of this limit, which is in-line with what has been adopted for many years in the CSA A23.3 design standard would permit reinforced masonry walls to be designed and installed to greater heights without the conditions that often prevents their design when considered slender. Partially grouted walls with a height in the order of 36 times their depth would be permissible for walls with steel reinforcement at 1000 *mm* on centre with the revised slenderness limit. This change simply recognizes that two dimensions are required to accurately describe the slenderness of partially grouted block walls. Numerical results have demonstrated that it is overly conservative to assume partially grouted walls adhere to the same slenderness limits as a rectangular cross-section described by a single dimension. In addition, a revised slenderness ratio of $kh/r \leq 100$ does not alter the current $30t$ limit, because the two limits are identical for a fully grouted wall (solid rectangular cross-section). The recommendation was further justified using linear elastic and nonlinear finite element analyses of two prototype walls with height to thickness ratios of 29.4 and 35.7 respectively. The results from both analyses illustrated that under the same loading, the deflection of the 6800 mm tall (h/t = 35.7) wall was 200% to 300% larger than the 5600 *mm* tall wall (h/t = 29.4). The initial reaction is that this is excessive and unsafe however, the 10.1 *mm* maximum deflection over the 6800 *mm* height in the nonlinear model while subjected to *factored loads* produced a deflection limit of $h/673$ which is almost half the required limit of $h/360$ typically required for structural element deflections under services loads. It is interesting to note that while the reinforcing steel was yielding in the 6800 *mm* wall in the nonlinear model, analytical modelling results demonstrated that an increase in bar size to 20M could result in a higher factor of safety under the design pressure. Figure 5 and 6 show the results of increasing the bar size to two 20M bars spaced at 1000 *mm* on centre for the 6800 *mm* tall wall. Results show that the increase in bar size does not decrease the lateral displacement at the design pressure, but it does delay the onset of yielding and reduces the ductility demand on the steel. Ultimate this is shown to be a viable solution and provide a better margin of safety against significant displacement as the wall enters the nonlinear range under the design pressure.

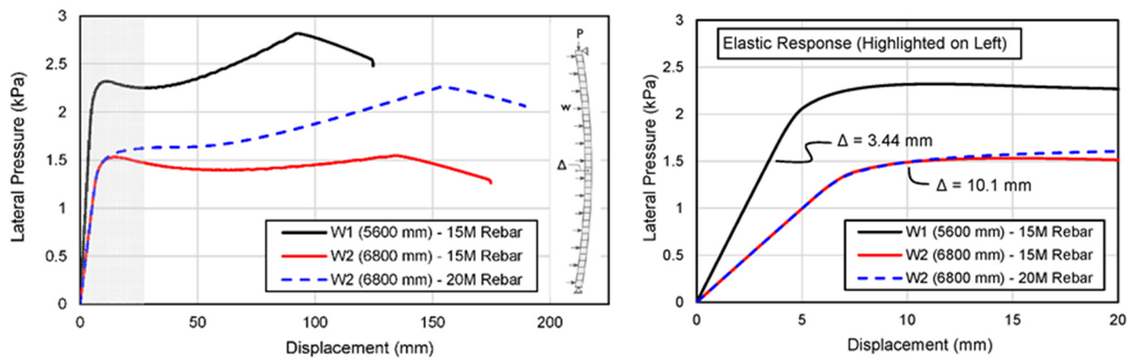


Figure 5: Lateral Pressure versus Top Displacement Response

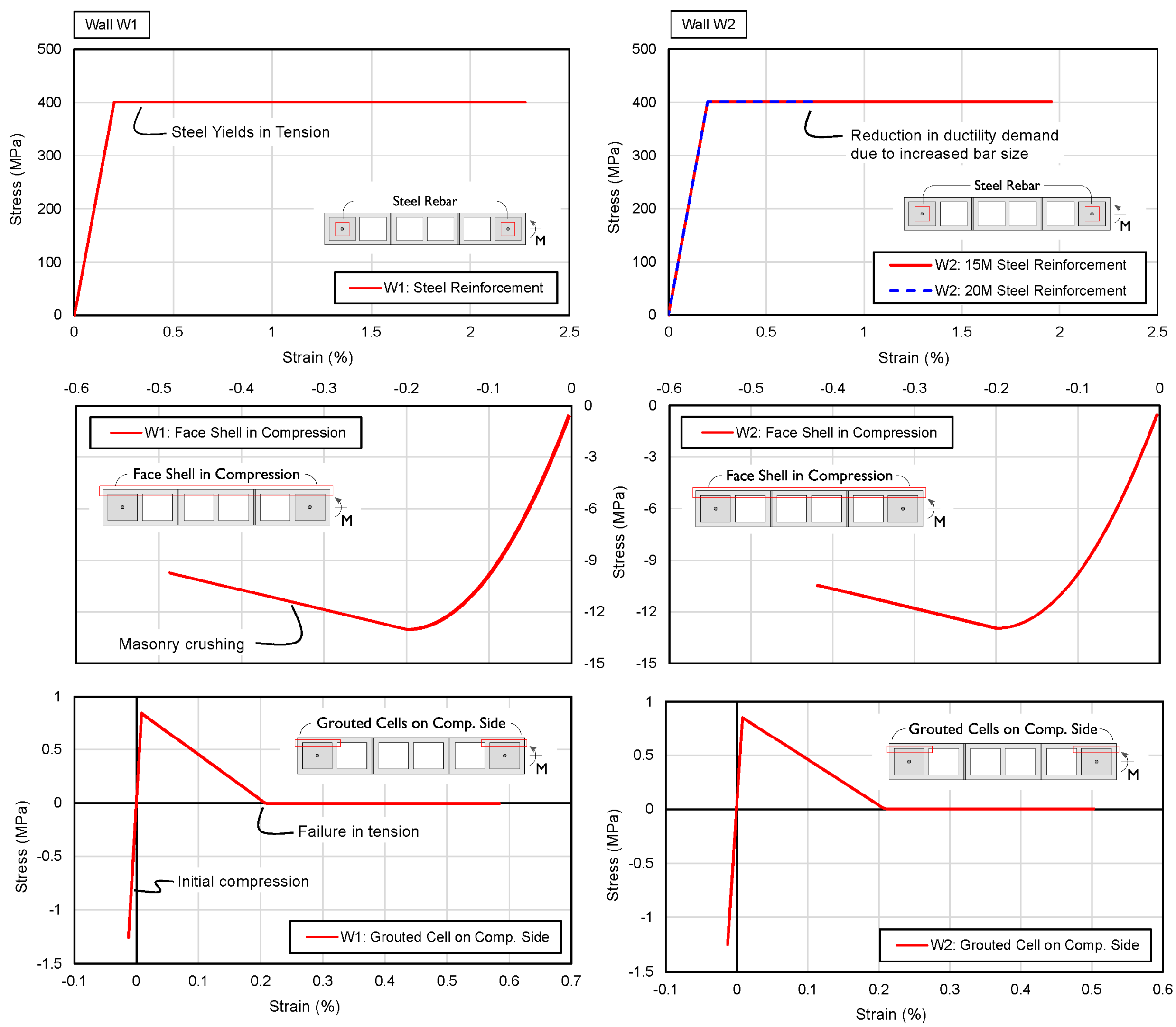


Figure 6: Stress-strain Response for Walls W1 and W2

The analytical models and theoretical basis introduced in this paper indicate that the adoption of a new slenderness limit based on the radius of gyration would be sufficiently safe and of great benefit to the masonry industry in Canada when designing and constructing warehouse and school gymnasium walls whose heights frequently exceed 30 times their depth.

ACKNOWLEGEMENTS

The authors would like to thank the University of Alberta and the Alberta Masonry Council for their support.

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