



# SEISMIC COLLAPSE EVALUATION OF REINFORCED MASONRY CORE WALL Systems

# Hosseinzadeh, Shadman<sup>1</sup>; Aly, Nader<sup>2</sup>; Ashour, Ahmed<sup>3</sup> and Galal, Khaled<sup>4</sup>

### ABSTRACT

Reinforced Masonry Shear Walls (RMSW) are commonly used in low- to medium-rise buildings as the seismic force resisting system. Since the 20<sup>th</sup> century, several experimental and analytical studies have been conducted to evaluate the seismic response of RMSW component and system (i.e. building consisting of planar walls). However, research on RM core wall systems is scarce, especially concerning the torsional effects on the system's seismic inelastic response. A core is a structural element with a cellular section that is typically closed on three sides and is either open or partially closed by coupling beams on the forth side. In the current study, a five story building, designed according to the National Building Code of Canada (NBCC 2015) and the Canadian Standards Association (CSA) S304-14, was considered to assess the seismic performance and collapse capacity of RM core wall systems. SeismoStruct software package was used to model the building numerically. The nonlinear model is validated against experimental data of an asymmetric RM building. Collapse risk evaluation has been conducted for reliable design of RM cores within the context of FEMA P-695 by subjecting the mentioned building's numerical model to various ground motions scaled at different intensity levels. This study is proposing RM Core wall system as a reliable alternative seismic force resisting system. It also contributes to better understanding and quantification of its seismic response and collapse capacity.

**KEYWORDS:** reinforced masonry, core wall, torsional effects, collapse evaluation, risk assessment

<sup>&</sup>lt;sup>1</sup> PhD student, Dept. of Building, Civil & Environmental Engineering, Concordia University, Montréal, Québec H3G 1M8, Canada. E-mail: shad\_hos@encs.concordia.ca

<sup>&</sup>lt;sup>2</sup> PhD student, Dept. of Building, Civil & Environmental Engineering, Concordia University, Montréal, Québec H3G 1M8, Canada. E-mail: n\_aly@encs.concordia.ca

<sup>&</sup>lt;sup>3</sup> Post-Doctoral Fellow, Dept. of Building, Civil & Environmental Engineering, Concordia University, Montréal, Québec H3G 1M8, Canada. E-mail: eng.ahmed3ashour@gmail.com

<sup>&</sup>lt;sup>4</sup> Professor, Dept. of Building, Civil & Environmental Engineering, Concordia University, Montréal, Québec H3G 1M8, Canada. E-mail: galal@bcee.concordia.ca

### **INTRODUCTION**

Reinforced Masonry Shear Walls (RMSW) are widely used in low- and medium-rise masonry buildings as the seismic force resisting system (SFRS) to provide the lateral strength, stiffness and energy dissipation capacity required to resist lateral loads arising from earthquake or wind loads. In the past few decades, there has been considerable advancement in understanding the RMSW response under lateral loading. Several experimental and analytical studies focused on studying the seismic performance of RMSW elements (e.g. [1], [2], [3]). However, previous research studies addressed planar shear walls and, based on the literature, no research considered the RMSW core system. More recently, RMSW were introduced in high-rise buildings which reached up to 28 stories in China [2]. The available space around the elevator service area is an optimum region to place the core wall system similar to the common practice in reinforced concrete structures. The need of access to the lift and other services requires an open section to be applied, (i.e. a C-shaped element). This opening is partially closed by spandrel or slabs connected along each floor level.

Although reinforced concrete core walls are very popular in practice, their inelastic behaviour under seismic loading has not been examined in detail. There are limited number of experimental studies on the cyclic response of C-shaped walls e.g. [3]. Unlike rectangular walls, which only resist lateral loads in one direction, C-shaped walls provide lateral stiffness in both loading directions and possess torsional stiffness.

In this study, a comparison was made between the seismic response of three individual rectangular RMSW connected together using the floor rigid diaphragm and a C-shaped wall having similar overall dimensions. A 5-story building is used to simulate a mid-rise building with RMSW. The rectangular walls were assembled using rigid links with the master nodes being at the center of the wall. In addition, different torsional sensitivities [4] has been taken into account, to investigate the effect of torsion on the seismic response of the core system.

The nonlinear numerical model was first validated against the experimental results of the 2-story building tested by Heerema et al. [5]. The building had rectangular and flanged shape RMSW. Furthermore, quantification of seismic performance of RM cores have been conducted within the context of FEMA P-695 [6]. This is achieved by subjecting the reference building to various ground motions scaled at different levels. Finally, seismic collapse risk of the RM core system was assessed in terms of fragility curves considering different torsional sensitivities.

# **BUILDING LAYOUT**

The archetype building selected in this study, shown in Figure 1, is a 5-story RMSW building. The overall building height is 19.45 m, with first floor height of 4.85 m and typical floor height of 3.65 m. The floor is18 m x18 m and is built using 200mm thick pre-stressed hollow core concrete slab. Two C-shaped walls are located close to the building's center of mass. Each wall consists of two 3m flanges and a 6m web with thickness of 250 mm. The columns are made up of reinforced masonry blocks with the dimension of 400 mm x 400 mm. The dimension, spacing, and cross section details of the walls and columns are summarized in Table 1



Figure 1: Archetype Building with C-shaped Walls: (a) Typical Floor Plan View; (b) 3-D Layout

Element type	Element ID	Туре	Typical height (mm)	Web length (mm)	Flange length (mm)	Aspect ratio (shorter wall)	Vertical reinf.		Horizontal reinf.	
							$\varphi_v$ (mm)	$\left  \begin{array}{c} \rho_v \\ (\%) \end{array} \right $	$\varphi_h$ (mm)	$egin{array}{c}  ho_h \ (\%) \end{array}$
Wall	W1,2	C- shaped	3650	6000	3000	1.22	15	0.15	15	0.4
Column	C1	RM Column	3650	400	400	-	20	1.00	10	0.063

Table 1: Walls' and Columns' Design Details

# TORSIONAL SENSITIVITY

The structural system response to earthquake loads can include torsional forces due to the eccentricity between Center of Mass (CM) and Center of Rigidity (CR). In NBCC 2015 [4], this concept is defined as inherent torsion. The uncertainties related to the position of CM and CR induce an additional seismic moment called accidental torsion. NBCC 2015 [4] recommends a limit for classifying the torsional sensitivity of structures. The limit is based on the torsional sensitivity parameter (*B*) which is defined at each level x (*B<sub>x</sub>*) as  $\delta_{max}/\delta_{ave}$ . In the aforementioned expression,  $\delta_{max}$  is the maximum story displacement at the extreme point of the structure at level x induced by the equivalent static forces with accidental eccentricity and  $\delta_{ave}$  is the average displacement at the extreme points of the structure at level x produced by the above forces. According to NBCC 2015 [4], as the value of *B* exceeds 1.7 and IEFaSa (0.2)>0.35, the building is torsional sensitive. In this case, a 3-D dynamic analysis is mandatory for evaluating the response of structure.

In this study, the torsional sensitivity was defined based on NBCC 2015 [4]. Three levels of torsional sensitivity (i.e. B = 1.0, 2.1 and 2.5) were considered to cover the expected range of eccentricities in buildings with core walls. This allows having a comprehensive and realistic assessment of the proposed structural system with core walls for RM buildings. The torsional sensitivity levels were achieved by shifting the CM from the CR by a certain eccentricity. The amount of eccentricity was estimated based on an iterative procedure by analyzing the structure against the design earthquake lateral forces based on NBCC 2015 [4]. The nonlinear dynamic analyses and collapse fragility assessment of the archetype building were performed at the three torsional sensitivity levels.

#### MASONRY WALL MODEL

SeismoStruct [7] package was used to conduct all the analysis for the RMSW systems. Inelastic displacement based beam-column fiber elements were used to model the RMSW. Different material models were assigned to the fibers to resemble the reinforcement and masonry cyclic response. Based on sensitivity analysis, it was found that the number of fibers needed in the cross section discretization is 300 for walls and 200 for the columns. All walls are assumed to be fixed at the base and the soil-structure interaction was neglected as per NIST [8] recommendations.

Two uniaxial stress-strain relations were utilized to model the behavior of the RMSW; one for the masonry and another for the reinforcing steel. The nonlinear steel model of Menegotto and Pinto [9] is implemented with the modification on isotropic strain hardening presented by Filippou et al [10]. A Young's modulus ( $E_s$ ) of 200 GPa, a yielding strength ( $f_y$ ) of 495 MPa, and a strain hardening ratio of 0.01 has been considered in modeling the reinforcement. It should be noted that, these material properties are as reported by Hereema et al. [5] and were only used in the nonlinear modeling validation. A yielding strength of 400 MPa was used in the design (i.e. as per the design code CSA S304-14[11] requirements) and analyses of the archetype building. The masonry behavior was modeled based on the constitutive model proposed by Mander et al. [12]. The specified masonry compressive strength ( $f_m$ ) was assumed to be18 MPa and the elastic modulus ( $E_m$ ) was calculated according to CSA-S304-14 [11] (i.e. 850 $f_m$ ).

### MODEL VALIDATION AND CALIBRATION

Verification of the model predictions with a two-story asymmetric RMSW building tested by Heerema et al. [5] is presented in Figure 2. The same loading protocol used for the experimental program was implemented to compare the numerical and experimental hysteretic response. Figure 2 shows a comparison between the experimental and numerical hysteresis loops for the building. As shown in Figure 2, the model captures the experimental response with an acceptable accuracy with a maximum error of 25% in prediction of lateral force. In general, the model was capable of capturing both the elastic and inelastic response of the RM shear walls.



**Figure 2: Numerical Model Validation** 

# SEISMIC COLLAPSE EVALUATION OF RM CORE WALLS

The seismic behavior of the archetype building was investigated in moderate seismic zone of eastern Canada [13], Montréal, Quebec. The walls are founded on site class C which represents very dense soil.

The seismic collapse evaluation methodology is based on FEMA P695 [6] which consists of first establishing the seismic performance of the studied building using nonlinear static pushover and dynamic analyses. Afterwards, adjusting the nonlinear dynamic analyses results to explicitly consider uncertainties in ground motion, design, test data, and modeling. Finally, using the adjusted results, seismic collapse fragilities are established to assess the collapse risk of the archetype building.

### Static Pushover analysis

The calibrated nonlinear model was utilized to perform pushover analysis in order to estimate the lateral force and deformation capacity of the archetype building. Pushover analysis was conducted according to FEMA P695 [6] using a lateral load distribution proportional to the fundamental mode shape. The results were used to establish the capacity curves depicted in Figure 3 for the archetype building utilizing three rectangular walls and C-shaped wall. The predicted lateral force capacity was found in good agreement with design capacity based on CSA S304-14 [11].

Comparing the capacity curves shown in Figure 3, it can be seen that the initial stiffness values were almost identical for the C-shaped and rectangular walls. However, utilizing C-shaped walls significantly increased the lateral load capacity. In addition, the results showed an increase in the ultimate displacement and ductility capacity of the C-shaped walls compared to that of the rectangular walls.



Figure 3: Pushover Curves: (a) E-W Direction; (b) N-S Direction

Spectral shape factors (SSF) are introduced by FEMA-P695 [6] to adjust seismic collapse margin ratios and account for the elongation of natural period prior to collapse. SSFs can be found in Table 7-1 of FEMA-P695 [6] depending on the fundamental period and period-based ductility. Pushover analysis results were used to estimate the archetype building period-based ductility. The period-based ductility factor for a given structure ( $\mu_T$ ) is defined, as shown in Eq. (1), as the ratio of ultimate roof drift displacement ( $\delta_u$ ) to the effective yield roof drift displacement ( $\delta_{\gamma,eff}$ ).

$$\mu_{\rm T} = \frac{\delta_u}{\delta_{y,eff}} \tag{1}$$

The effective yield roof drift displacement is calculated using Eq. (2) according to FEMA-P695 [6].

$$\delta_{y,eff} = C_0 \frac{V_{\text{max}}}{W} \left[ \frac{g}{4\pi^2} \right] \left[ \max(T, T_1) \right]^2 \tag{2}$$

Where  $C_0$  relates fundamental-mode (single degree of freedom) displacement to roof displacement,  $V_{max}/W$  is the maximum base shear normalized by the building weight, g is the gravity constant, T is the fundamental period of the structure based on empirical formula and  $T_1$  is the fundamental period of the model computed using eigenvalue analysis.

The values of Period-based ductility are tabulated in Table 2, ( $\mu T Rect.$ ) for the archetype building using three individual rectangular walls, and ( $\mu T C-Shaped$ ) for the C-shaped configuration. It is evident that there is a clear increase in ductility as a result of using the C-shaped walls.

**Table 2 Period based Ductility Factors** 

$\mu_{T  \mathrm{Rect.}}$	4.76
$\mu_{TC-Shaped.}$	12.15

#### Incremental Dynamic Analysis

In this study, the dynamic response of the RM core wall building was determined using Incremental Dynamic Analysis (IDA) analysis. This dynamic analysis method was introduced by Vamvatsikos and Cornell [14]. In this method, the structure is imposed to a series of Nonlinear Response History Analyses (NLRHA) with increasing the intensity of the ground motions until the collapse of the structure occurs. IDA allows a complete assessment of the structure's response and properly captures the higher modes of vibration contribution to the buildings' dynamic behaviour. The intensity measure (IM) chosen in this study is the spectral acceleration (5% damping) at the fundamental period of the structure (T) specified as  $S_a$  (T, 5%). T is calculated based on the empirical formula provided in NBCC 2015 [4]. The analysis results are used to construct the IDA curves which relates the IM and the Engineering Demand Parameter (EDP). The chosen EDP in this study is the inter-story drift ratio (IDR<sub>max</sub>).

Several ground motions were selected and scaled to perform the NLRHA. A set of 22 simulated records from Assatourians and Atkinson [13] were used to obtain Canadian design response spectrum-consistent motions for the considered region (i.e. Montréal, Quebec). The characteristics of the selected ground motions reflected the seismicity in the region of eastern Canada as described by Assatourians and Atkinson [13]. The selected time histories had magnitudes ranging between 6 and 7 for site class C and epicentral distance ranging between 15 and 100 km. The ground motion records are scaled based on the ratio between the design spectral acceleration  $S_a(T)$  and the geometric mean spectral acceleration ( $S_{gm}$ ) of each pair. The geometric mean is calculated using Eq. (3).

$$S_{gm}(T) = \sqrt{S_x(T)S_y(T)}$$
<sup>(3)</sup>

 $S_x$  and  $S_y$  are the orthogonal components of spectral acceleration at period *T*. The scale factor is calculated at T= 0.56 s which is the average fundamental period of the orthogonal directions of the building. The scaled ground motions are then used in the IDA.

#### SEISMIC COLLAPSE RISK EVALUATION

To establish the collapse fragility curves, IDA results were used to determine the intensity of each ground motion which causes the structure to collapse. The collapse probability at each IM was estimated as the number of records causing collapse divided by the total number of records. In this study, the collapse criteria of the RM core walls is defined based on the interstory drift limits. According to FEMA P695 [6] the collapse of the structure occurs when a certain limit of drift is exceeded, and is referred to as EDP-based rule. ASCE41-13 [15] adopts the maximum interstory drift ratio to assess the level of damage in structural elements and SFRS. To be consistent with the ASCE41-13 [15] guidelines, the collapse point is set to 2.5% interstory drift ratio which corresponds to the collapse prevention performance level of NBCC 2015 [4]. The calculated collapse probabilities were then used to construct the collapse fragility curve which is a plot that relates the probability of collapse to the ground motion intensity.

A cumulative distribution function has been recommended by the FEMA P695 [6] to generate fragility functions based on a log-normal probability distribution. Eq. (4) illustrates the probability distribution function.

$$P(C \mid IM = x) = \phi(\frac{\ln(x/\theta)}{\beta})$$
(4)

Where P(C|IM = x) is the probability of collapse at a given intensity,  $\phi$  is the standard normal cumulative distribution function,  $\theta$  is the collapse median intensity or the intensity level with 50% probability of collapse, and  $\beta$  is the standard deviation (dispersion) of IM. The fragility function parameters were found using the maximum likelihood method by Baker [16].

The collapse fragility curves for the archetype building in N-S direction utilizing three rectangular walls, and C-shaped walls with the three torsional sensitivity levels is shown in Figure 4.



Figure 4: Collapse Fragility Curves: (a) C-shaped vs Three Rectangular Walls (b) Three Rectangular Walls with Different Torsional Sensitivity Levels (c) C-Shaped Walls with Different Torsional Sensitivity Levels

Figure 4a shows that the use of C-shaped walls resulted in a higher collapse spectral intensity ( $\hat{S}_{CT}$ ) compared to the use of rectangular walls. The justification of this finding is because of the contribution of the effective flange width on enhancing the ultimate curvature of the walls. The larger ultimate curvature of the walls with the C-shaped configuration delayed the collapse and thus resulted in reducing the collapse risk under high seismic loads. Figure 4b and Figure 4c compares the fragility curves of different torsional sensitivity levels. As shown, the torsional

sensitivity can significantly affect the collapse fragility curves. The median collapse intensity ( $\hat{S}_{CT}$ ) of the RM core system decreased by approximately 20% when torsional sensitivity parameter (*B*) is equal to 2.1. The reduction in median collapse capacity was more than 75% when *B* is 2.5, in comparison with the one with no torsional sensitivity (i.e. *B* = 1).

Using the collapse fragility curves, the median collapse capacity  $(\hat{S}_{CT})$  is evaluated as the 5% damped spectral acceleration corresponding to 50% probability of collapse. Then, the Collapse Margin Ratio (CMR) is calculated using Eq. (5) as the ratio between the median collapse intensity  $(\hat{S}_{CT})$  and the Maximum Considered Earthquake (MCE) intensity  $(S_{MT})$ . CMR is a principal parameter used to characterize the collapse safety of the structure.

$$CMR = \frac{\widehat{S}_{CT}}{S_{MT}}$$
(5)

The estimated SSFs are multiplied by the calculated CMRs to account for the elongation in natural period prior collapse and calculate the Adjusted Collapse Margin Ratio (ACMR). The calculated ACMR values are then compared with the allowable values proposed by FEMA-P695 which are introduced in terms of the total system uncertainty ( $\beta_{TOT}$ ). The total system collapse uncertainty is function of record-to-record uncertainty ( $\beta_{RTR}$ ), design requirements related uncertainty ( $\beta_{DR}$ ), test data related uncertainty ( $\beta_{TD}$ ) and modelling uncertainty ( $\beta_{MDL}$ ). This value is computed using Eq. (6).

$$\beta_{TOT} = \sqrt{\beta_{RTR}^2 + \beta_{DR}^2 + \beta_{TD}^2 + \beta_{MDL}^2} \tag{6}$$

Table 7-3 of FEMA-P695 provides acceptable values of Adjusted Collapse Margin Ratio, ACMR<sub>10%</sub> and ACMR<sub>20%</sub>, based on total system collapse uncertainty, considering values of acceptable collapse probability at MCE to be 10% and 20%, respectively. The value of  $\beta_{TOT}$  was calculated to be 0.725, assuming the values of the different uncertainty parameters as shown in Table 3.

Uncertainty Parameter	Value
$\beta_{RTR}$	0.4
$\beta_{DR}$	0.35
$\beta_{TD}$	0.35
$\beta_{MDL}$	0.35
$\beta_{TOT}$	0.725

**Table 3: Uncertainty Parameters** 

Table 4 summarizes the seismic collapse performance parameters of the archetype building considering the three rectangular walls or the C-shaped wall for the torsional sensitivity of B = 1.0. The ACMR<sub>10%</sub> is shown as an acceptance criteria. Ensuring calculated ACMR is higher than ACMR<sub>10%</sub> reflects that the structure's probability of collapse at MCE is lower than 10%.

Shear Wall Configuration	$\hat{S}_{CT}$	CMR	SSF	ACMR	ACMR10%
Three rectangular walls	1.3	4.44	1.36	6.04	2.53
C-shaped wall	2.0	6.83	1.36	9.29	2.53

Table 4: Summary of Collapse Performance Evaluation of RM Shear Walls

The calculated ACMR values for both rectangular and C-shaped walls are satisfying FEMA P695 acceptance criteria. However, there is more than 50% increase in the ACMR when the C-shaped wall is utilized. This is attributable to the contribution of the effective flange wall width in improving the inelastic response of the shear walls

# CONCLUSION

The current study evaluated the seismic collapse risk of a 5-story RM core wall building. It demonstrated the significant enhancement in overall response when utilizing C-shaped walls compared to three individual rectangular walls. Nonlinear pushover analysis was conducted to evaluate the capacity and period based ductility. The results showed that by utilizing C-shaped walls instead of the individual walls, the ultimate curvature increased due to contribution of the effective length of the flange. Thus, it resulted in greater values of ductility and ultimate capacity for the C-shaped walls. The lateral capacity of the structure increased by 45% in N-S direction and by 55% in E-W direction. Furthermore, the utilization of the C-shaped wall resulted in a 190% increase in the period-based ductility of the building.

The seismic collapse fragilities demonstrated that the C-shaped walls had a greater seismic collapse capacity and a lower probability of collapse compared to rectangular walls. The results showed more than 50% increase in the Adjusted Collapse Margin Ratio when the C-shaped walls were used. This reflects a substantial improvement in the overall seismic performance when utilizing C-shaped walls. Furthermore, it was evident that torsional sensitivity can significantly affect the fragility function of C-shaped and rectangular walls. It resulted in up to 20% and 75% reduction in seismic collapse capacity for torsional sensitivity levels of B=2.1 and B=2.5, respectively. In addition, increasing the torsional sensitivity leads to steeper fragility curves reflecting higher collapse probabilities.

### **ACKNOWLEDGEMENTS**

The authors acknowledge the support from the Natural Science and Engineering Research Council of Canada (NSERC), l'Association des Entrepreneurs en Maçonnerie du Québec (AEMQ), the Canadian Concrete Masonry Producers Association (CCMPA) and the Canadian Masonry Design Centre (CMDC).

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