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SEQUENTIALLY LINEAR ANALYSIS ON MASONRY WALLS – NEW CRACK CLOSURE ALGORITHM

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ABSTRACT

Finite element models, in use for simulation of materials characterised by brittle failure like masonry, often encounter problems related to snap back, bifurcation points, divergence or material softening leading to negative tangent stiffness and the consequent ill conditioning of the formulation. Serious convergence issues led to quite inaccurate results due to large deviation from tolerance norms. This gave rise to several non-iterative total approaches one of which is the Sequentially Linear Analysis (SLA), which has been in development from the early 2000s. The SLA is an event-by-event strategy, where a sequence of scaled linear analyses is performed, coupled with decreasing secant stiffness and strength at the critical integration point in the model. The use of positive secant branches, damage increments, and multiple integration points not entering into failure, makes this method robust and devoid of convergence troubles. Originally implemented within the scope of the smeared crack approach in a plane stress context, the method has undergone several developments over the years and an overview is presented in this article. But there are impending issues/areas to be addressed within the framework to make it a serious practical alternative to popular non-linear incremental-iterative methods like the Newton Raphson method. In SLA, an integration point softening in tension or compression, when subject to a reversal of stress states as in the case of a closing crack reloaded in compression, does not regain the original stiffness upon stress reversal. Stress reversal is quite common in cyclic loading but is also observed in monotonic cases when stress redistribution occurs. Currently, applications of SLA are limited to monotonic loading and this article illustrates the stress reversal problem with a monotonic analysis on a Calcium silicate masonry wall tested at TU Delft under the research program in relation to induced seismicity in Groningen, The Netherlands. A solution strategy to overcome the issue is proposed and outlined, with validation studies to follow in the near future.

KEYWORDS: *crack closure algorithm*, *masonry shear walls, sequentially linear analysis.*

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INTRODUCTION

In the realm of finite element analysis concerning materials typified by quasi-brittle to extremely brittle nature, the obtained solution is not always fully converged. In a more simplistic sense, the solution obtained traces through regions that suffer deviation from the true equilibrium path into alternative equilibrium paths. This has been observed in different instances. Firstly, the adopted incremental iterative procedure fails to handle material softening which leads to negative tangent stiffness causing the ill conditioning of stiffness matrices in the finite element formulation. Secondly, in the incremental iterative method the load is applied in finite steps which could push multiple integration points simultaneously into damaged state resulting in a bifurcation point which in turn leads to alternative equilibrium states. Thirdly, the snap through, snap back and divergence situations are issues that can be dealt in the incremental-iterative concept only by using enhanced path following techniques procedures like the Arc-length control method or the crack mouth opening displacement (CMOD) but requires extreme care from the user to steer the analysis (also knowledge of crack locations apriori) and don't always guarantee obtaining the true response.

Thus, there was need to address this issue and develop robust algorithms to trace the equilibrium path correctly. Initially focus was laid on other path following constraints to avoid knowledge of cracking locations apriori or when the failure zone switches during the failure process. One such approach was by Gutierrez [1] where the constraint was formulated depending on the energy release.

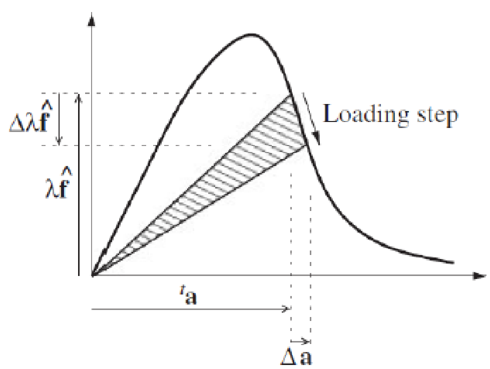


Figure 1: Geometric representation of the finite increment Δτ, the path parameter in the case of Energy release control. The shaded area represents the energy released during load step [1]

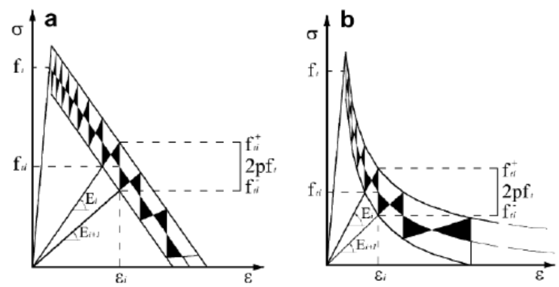
Extensions were also made for implementation of an Arc-length control on the basis of energy release rate by Gutierrez, Verhoosen et al [2]. Unique energy release constraints were applied to the cases of geometrically linear damage, geometrically linear plasticity and geometrically non-linear damage using a flexible algorithm which switches from force control until damage starts to energy release control during damage and back. Nevertheless, the FEM framework in the context of nonlinear studies for brittle materials provided an interesting topic for researchers to confront.

REVIEW OF THE SLA APPROACH & ADVANCEMENTS

Sequentially linear analysis (SLA) has been a proven alternative to capture sharp snap backs and responses characterized by negative stiffness since it operates on the positive secant stiffness idea. It is a sequence of scaled linear analysis coupled with decreasing positive secant stiffness and strength parameters in the most critical integration point. The procedure [3] is applied in the context of a Total strain based smearing cracking model and specifically a plane stress formulation. The concept was then extended to truss elements and interface elements as well. Instead of incrementing the load or the prescribed displacement in steps, the constitutive law is discretized into a series of the so-called saw teeth (see Fig 2) with reducing positive secant stiffness to overcome the issue of negative tangent stiffness. The procedure involves finding critical integration points one at a time where the limiting strength (the current) has been violated, and the strength and stiffness of this integration point are reduced stepwise based on the saw-tooth law. Thus, the method circumvents the problem associated with the regular incremental-iterative procedures with respect to convergence as it traces through every event i.e a jump or snap back that may occur in the response of the structure.

When the saw tooth discretization was done either by constant stiffness reductions or constant strength reductions, problems were encountered with regards to mesh objectivity. This was overcome by updating the tensile strength or the ultimate strain or even both to keep the energy invariant with the latter being reported as the most effective [4]. Eventually moving towards a more general approach to achieve mesh objective results, a ripple bandwidth concept was introduced by Rots et al. [5]. In this approach, a strength range set is defined which is a percentage (p) of the maximum tensile strength. Consequently, a band is introduced into the softening part of the stress strain relation delimited by two parallel curves at an equal distance from the parent curve. This is shown in Figure 2. An extension to a compression failure criterion in the form a parabolic softening relation, with a crush bandwidth model that regularizes compressive fracture energy, was made by Kabos [6].

Initially, the method was developed only for a proportional loading scheme. SLA was then extended to non-proportional loading using different ideas by DeJong [7], Elias [8], and Van de Graaf [9] but the topic is still being debated upon. When there are non-proportional loads, which are constant over the structure unlike loads which vary over time, problems arise due to considerable stress rotations. Hence, the original SLA procedure was modified initially by DeJong [7] using the superposition of stresses caused by proportional and non-proportional loads and finding the critical load multiplier using the principal stress criterion. De Jong's idea neglects the possibility of triggering of new events i.e an avalanche of ruptures caused by stress redistributions when a damage increment is made. The sequentially linear method unloads and reloads the structure after every damage increment; thereby in principle neglecting the inappropriate stress fields generated at every new step which could cause rupture in wrong elements. This aspect was later questioned by Elias leading to the proposal of a Gradual redistribution strategy also known as the Force-release method [8] but it missed the inherent ability of SLA to trace snap backs. Van de Graaf [9] suggested a constrained maximization analogy in conjunction with a double load multiplier strategy (one for the constant/non-proportional load and the variable/proportional load) which yielded qualitatively good results.



$$f_{ti}^{-} = f_{ti}^{+} - 2pf_t$$

$$E_{i+1} = f_{ti}^{-}/\varepsilon_i = E_i/a_{i+1}$$

$$a_{i+1} = (E_i/f_{ti}^{-})\varepsilon_i = f_{ti}^{+}/f_{ti}^{-} = (f_{ti}^{+}/f_{ti}^{+}) - 2pf_t$$

$$E_{i+1} = E_i/a_{i+1}$$

$$f_{ti}^{+} = \varepsilon_u^{+} E_i(\frac{D}{E_i + D})\ where\ \varepsilon_u^{+} = \varepsilon_u + pf_t/D$$

Figure 2: Ripple bandwidth concept shown for (a) linear tension softening (b) exponential softening; where 'N' is the Number of teeth required, 'p' is the strength range, and E_i and f_{ti} are the Young's moduli and the tensile strength at the i^{th} damage level. [5]

The original implementation of SLA was done in a smeared crack formulation for a plane stress context. However, modelling of 3D specimens using SLA would only be possible with implementations for shell and solid elements. Therefore extension to shell elements of SLA was proposed by DeJong [10] and the one to solid elements (only within the proportional loading scheme) by Voormeeren [11]. Saw tooth law extensions were also made to model coulomb friction laws with or without dilatancy by Van de Graaf [9] and for materials with snap back at constitutive law level, like Glass, by Invernizzi et al [12].
Several studies have been performed in the past using SLA. A concrete beam externally reinforced with pre-stressed CFRP was analysed by Alfaiate [13]. Seismic assessment of a slender masonry tower like the Qutab Minar in India was performed by Mariani et al. [14]. Small scale slender beams made of engineered cementitious composites (a class of high performance fiber-reinforced cement based composite, HPFRCC) was analysed by Billington [15]. Shear critical reinforced beams which are characterised by very brittle failure were analysed by Slobbe [16]. A masonry facade scaled to 1/10 th of the original magnitude subjected to self-weight, vertical compression loads and settlements (by a controlled hogging deformation at the bottom) was investigated with NLFEA and SLA by Giardina [17].

Approaches combining the incremental method and SLA were attempted by researchers with the aim of improving over SLA in the context of performance issues. SLA inherently required more computational memory and time [9] since theoretically the procedure can continue until all integration points in the specimen are completely pushed to full damage along the saw tooth law. Two combined incremental and total (Non-iterative) approaches were first presented by Graca-e-Costa et al., and were called the Automatic Method and the NIEM (Non-Iterative Energy based method)[18]. These methods were able to properly track the material loading history as against a total method like SLA and also account for the non-linear incremental material behaviour caused by the principal stress rotations encountered in the non-proportional loading [19]. The method was such that no material parameters were to be regularized to ensure mesh objectivity and also ensured that the stress path was on the limit surface when the stress state was on the surface i.e a so-called consistency. The idea of both these methods it to employ a non-iterative incremental solution until non-convergence appears and then to shift to the total approach intermediately due to the fact that no unique admissible path can be traced even using advanced path tracing methods. Another method generalizing the SLA (load-unload method) and the gradual redistribution strategy (Force-Release method) was presented by Elias [20]. CITA method was proposed by Laefer et al. [21] to improve upon SLA in the context of performance (speed) by using a piecewise linear stress strain curve in tension. The tangent elasticity modulus was used to calculate the structural stiffness in this method despite some elements having negative stiffness but resulting stiffness equation were solved using Methods based on LDL^T decomposition like Bunch and Kaufman Method [22] or Aasen's method [23].

NEED FOR A CRACK CLOSURE ALGORITHM IN SLA

The procedure of SLA [3] and the strategy to realise non-proportional loading proposed by Van de Graaf [9] form the basis of the SLA framework currently in use. In principle, sequence of linear analysis are performed with a scaled combination of constant and variable loads (referred to as non-proportional and proportional loads respectively in literature thus far). The scaling procedure was rather simplistic in a variable loading scheme when the identification of the critical integration point was determined based on the ratio of the stress level to the current tensile strength being maximum upon application of a unit elastic load. The inverse of this ratio resulted in the global scaling factor, λ_{crit}, to scale the solution obtained for the unit elastic load. Then the critical integration point is subject to reduction of tensile strength or young's modulus based on the saw-tooth constitutive law. The procedure to find the critical load multiplier in real-life loading situations, where combination of constant and variable load occurs, becomes complicated as cited by DeJong [7]. The procedure of Van de Graaf [9] introduces a constrained optimisation idea to deduce load multiplier sets that result in constitutively admissible stresses. The critical load multiplier is determined by the maximum of the common subset of all load multiplier sets from different integration points. In situations of conflict, the double load multiplier (with λ_{con} for constant loads and λ_{var} for variable loads for each analysis step 'j') as shown in equation (1) is utilised and the last "successful" load combination ($F_{crit}^{(j-1)}$) in scaled in a proportional way.

$$F_{crit}{}^{(j)} = \lambda_{con}{}^{(j)} F_{con} + \lambda_{\mathrm{var}}{}^{(j)} F_{\mathrm{var}} \tag{1}$$

The fact that SLA method is based on secant stiffness as against the traditionally used tangent stiffness forces the unloading mode to be secant as well. This in turn causes a specific problem when an integration point, after a few damage steps on the saw tooth, starts to unload indicating crack closure and a possible reloading in compression. Upon reaching the origin of the constitutive model combining the tension and compressive behaviour, the damage history is carried over. That is, the reduced stiffness present in the tensile history of the integration point is carried over to the compressive regime. This is not acceptable as the reduced stiffness will lead to higher strains in compression indicating softening much earlier than anticipated and the problem hereon is referred to as the "stress reversal problem". Reversal of stress states are observed when masonry structures or components initially loaded by constant loads (for e.g., Dead loads) exhibit nonlinear behaviour leading to formation of cracks and the following application of variable loads (for e.g., seismic load) leads to the closing of cracks and subsequent reloading in compression. The reverse also is possible when an integration point in compression saw tooth is reloaded in tension leading to erroneous carrying over of the reduced stiffness. Hence, there is need to incorporate this aspect into the SLA workflow as it is relevant also in the monotonic loading cases where reversal of stress occurs due to stress redistributions.

Single Element Test
Several single element tests [22] were performed to understand the aforementioned limitation of the constitutive model employed in the SLA framework. One such is where a quadrilateral plane stress element with a single integration point and appropriate boundary conditions as shown in the Figure 3, is loaded in tension. Subsequent to 4 damage steps in tension it is reloaded in compression thereby simulating the crack closure phenomenon.

It is clearly observed that after four steps into damage in tension, reversal of load into compression leads to crushing. However, contrary to tracing the first crushing secant branch and continuing with second branch in compression, the procedure follows the secant branch from the tensile regime along which the unloading happens and therefore results in compressive softening to the 5th damage level as shown in Figure 3. Thus, there is transfer of stress history and leads to strain values in compression higher than the expected values.

Shear Wall test
Several in-plane quasi-static cyclic tests were performed on calcium-silicate and clay brick walls at TU Delft and is detailed upon in [25]. Monotonic SLA is done on one of the walls, with the existing constitutive relationship combining the tension and compression behaviour in one direction. The short wall, 2.75m x 1.1m, with double clamped boundary conditions (but free to move at the top vertically in the direction of overburden) is subject to an overburden pressure of 0.7 MPa and a lateral load subsequently in cyclic fashion. The experimental setup is shown in Figure 4 and the end stage crack pattern depicts a combination of flexure, toe crushing and sliding failures. Although the test is cyclic in nature, the test could be used as a benchmark in a monotonic approach to investigate the stress reversal from tension to compression softening that occurs due to redistribution of stresses during the rocking phenomenon.

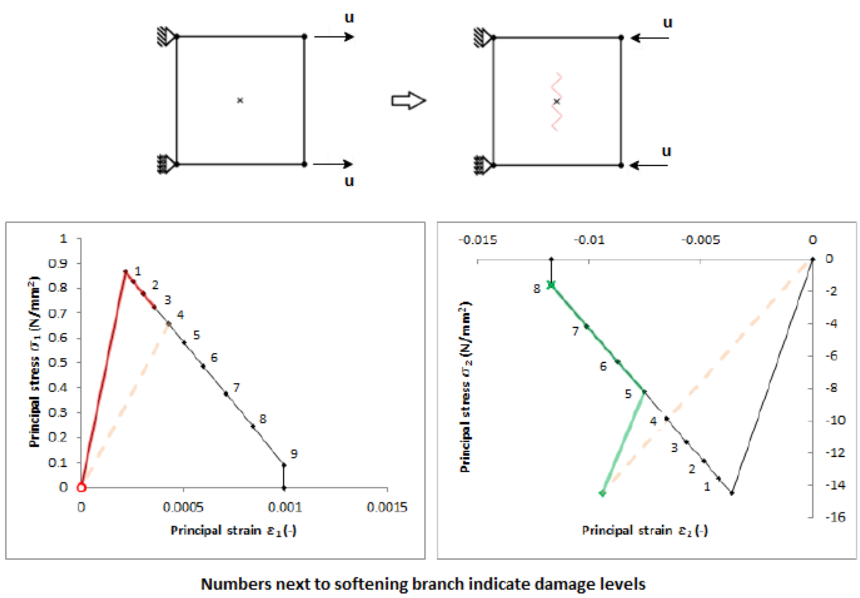


Figure 3: Single plane stress element subjected to 4 steps in tension followed by load reversal in compression simulating the effect of crack closure showing the incorrect transfer of damage stiffness from one stress state to the other [22]

The mesh is of quadrilateral plane stress elements of size 0.1m x 0.098m (crack/crush bandwidth of 0.099m) with linear interpolation and integration scheme of 2x2 in plane of the element. The parameters used for the simulations are listed in Table 1 which are deduced from the material tests also performed as a part of the campaign. The entire test set-up is shown in Figure 4 and the details of the experiment can be found in [25]. The specimen is subjected to over 20000 linear analysis steps and from the evolution of the constant load multiplier, λ_{con} it is observed that beyond 6000 steps the overburden load is not recovered anymore signifying the inability of the wall to carry vertical loads. This could be treated as the structural failure and the end point of the analysis. Two variation studies are performed, firstly with all elements in the mesh having uniform non-linear properties in tension and compression as cited in Table 1 and with a saw tooth formulation. The second variation is performed by providing higher tensile strength (0.45 MPa while keeping all other parameters the same) in all elements except in the bottom and top two rows of elements to enforce the rocking type behaviour, instead of the diagonal crack pattern observed in the first case which does not conform to the experimental observations. The global force displacement curve in Figure 5 shows the ability of the method to obtain severe snap backs in the response of the structure and also a clear post peak behaviour which is often missed out in the traditional NLFEA or is plagued by convergence issues.

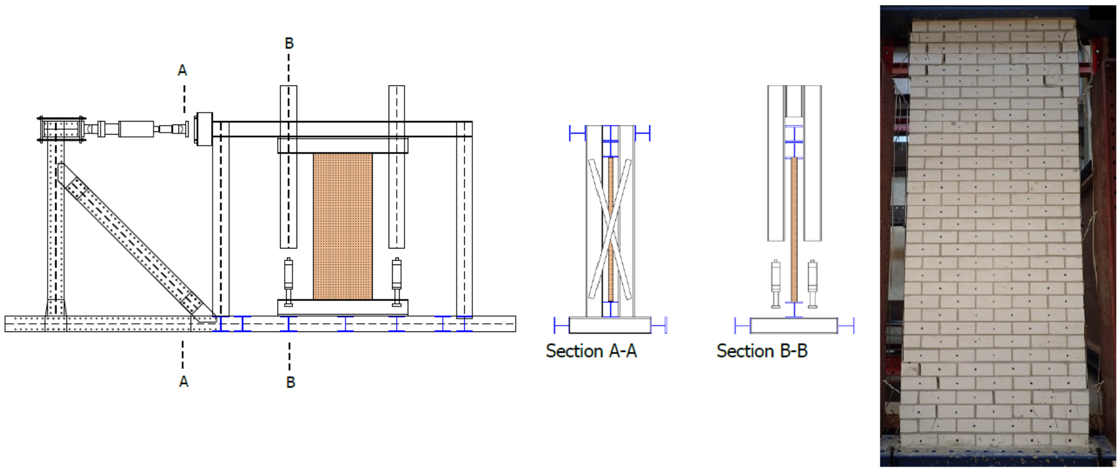


Figure 4: Experimental scheme for cyclic tests and the crack pattern at end stage

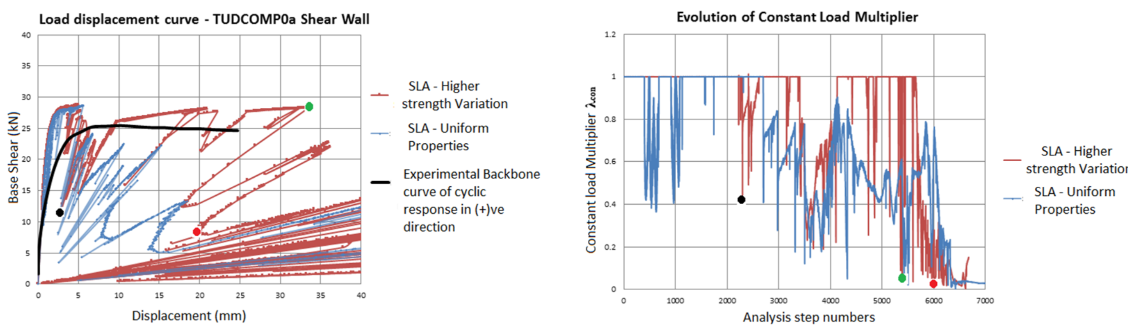


Figure 5: Comparison of SLA results against the experimental backbone curve (positive direction) of the cyclic response of the shear wall TUDCOMP0a tested in TU Delft [25]. The analysis steps 2300, 5400 and 6000 are marked in black, green and red spots respectively and the evolution of the constant load multiplier λ_{con} is also shown.

Table 1: Numerical Analysis parameters

Material	Young's Modulus	Tensile Strength (f_t)	Compressive Strength (f_c)	Fracture energy (G_f) & ($G_{f\text{-}c}$)
Ca-Si Masonry	5091 MPa	0.14 MPa	5.91 MPa	0.015 N/mm & 43.4 N/mm

Figure 6 corroborates the drop in capacity as seen in the load displacement curve and the reduction of constant load multiplier towards step 6000. Toe Crushing is clearly seen in principal strain E2 plots which eventually leads to loss of capacity. Simultaneously, through cracking in the top and bottom layer substantiates the rocking behaviour observed in the experiment. Sliding failure observed in the experiment is not captured here since SLA currently accounts only for the tensile and compressive failure in a total strain based smeared approach. The aforementioned stress reversal problem, in this monotonic pushover study, is clear from Figure 7 in two different instances. First instance is when integration point no. 1 of element no. 300, highlighted in Figure 6, is critical at step 3436 in compression in the tangential direction to the primary crack face at

Damage level 5. The next damage induced in this point is at step 6619 when the integration point is reloaded in tension but with same reduced young's modulus as in compression while the stiffness, in principle, should return to the original stiffness. Subsequently, the point undergoes further cracking in 7 more damage steps to reach step 6629 before turning critical again at step 8603 (which is well beyond step 6000 but is considered only for an academic purpose as the SLA process can technically proceed until each available integration point is completely damaged). This time the crack closes to carry over the reduced stiffness to reloading in compression. Figure 7 is just an illustration to show the carryover of reduced stiffness and does not show the actual strain of integration point 1 of element 300 at the mentioned step numbers.

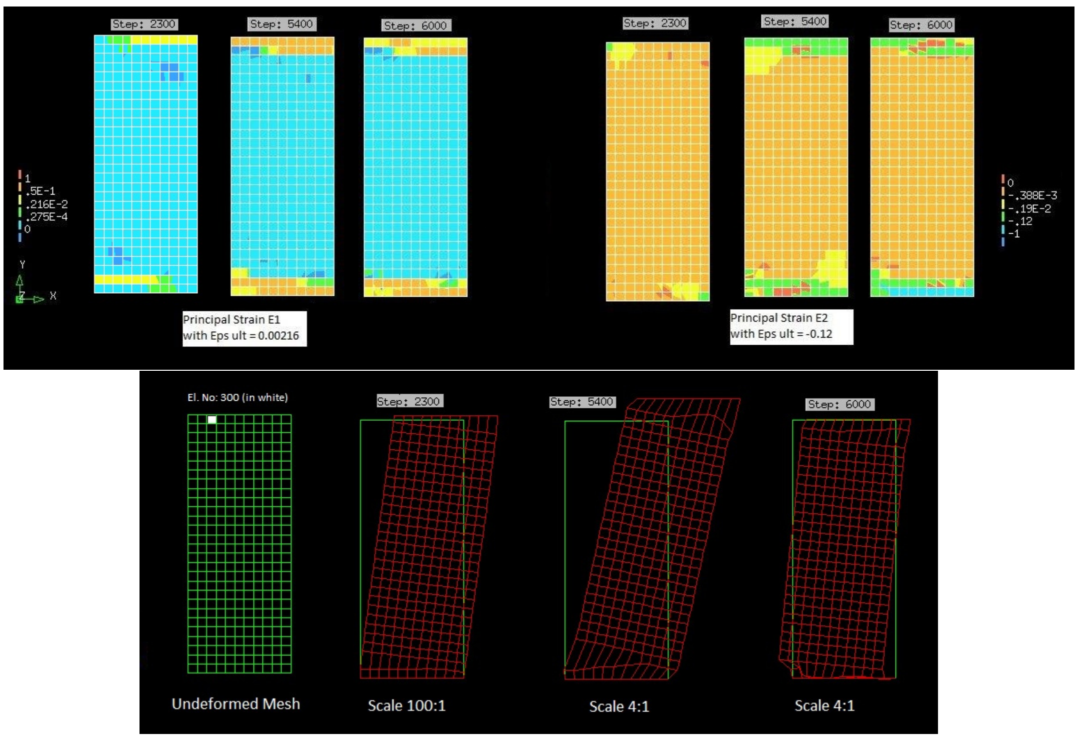


Figure 6: Principal strain plots (E_1 and E_2) showing evolution of cracking and crushing, and the deformed profiles showing the rocking behaviour and eventual toe crushing.

Concept for the New Crack Closure Algorithm

The coupling of the constitutive model for tension and compression could be done by incorporating status parameters which check for the previous stress state of the critical integration point. In case of conflict indicating a possible stress reversal, the stiffness has to be set to initial undamaged stiffness of the material, thereby introducing a local increase in stiffness in an element. This requires the need for additional status parameters and the need to store the stress history of each integration point in the element which is computationally intensive. Also, there is a possibility of an infinite loop of crack closure and opening which brings to fore also the need for a new termination strategy to be devised. Nevertheless, the problem could be solved and is currently being investigated with implementation and validation studies to follow. The workflow proposed is shown in Figure 8 as a flowchart.

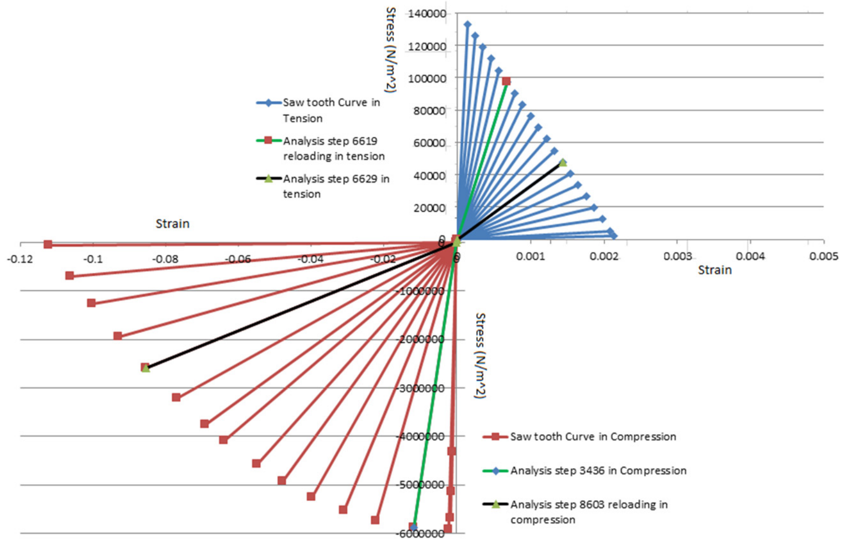


Figure 7: Graph showing the saw tooth curves in tension and compression (not to same scale) and the stress reversal problem for integration point number 1 of element 300 (shown in Figure 4) at two instances, steps 6619 and 8603, is presented.

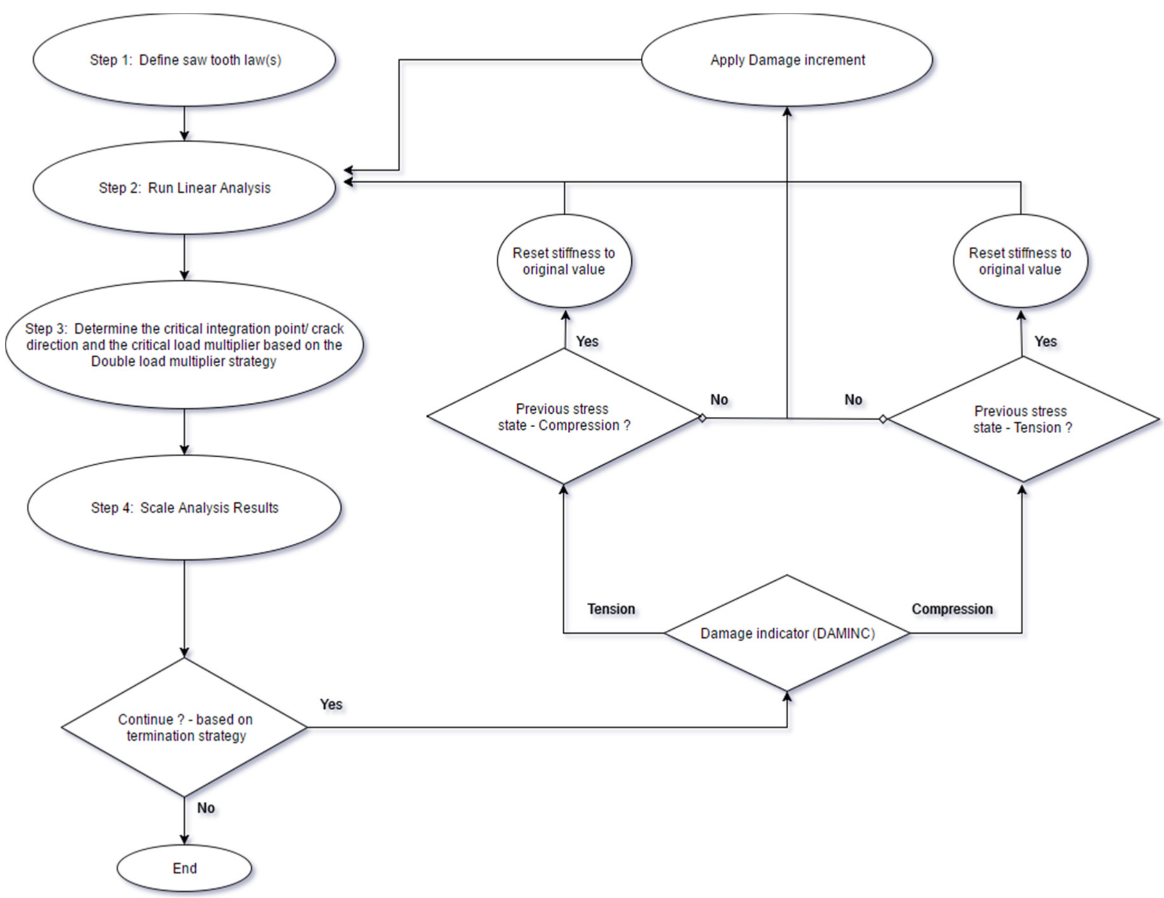


Figure 8: Proposed Crack Closure algorithm

CONCLUSIONS

This article presents a review on SLA and the recent advancements in this topic, and the concept for a crack closure algorithm to overcome the incorrect transfer of damage history during change of stress states. A shear wall in-plane cyclic test is studied using SLA in a monotonic perspective with comparison against the experimental backbone curve. The need for a new crack closure algorithm to avoid the stress reversal problem, during stress redistributions in a monotonic loading case, has been elucidated using this example. The proposed crack closure algorithm addresses the appropriate coupling of the tension and compression saw tooth laws, with validation studies to follow in the near future. This would also enable application to cyclic loading regimes where the reversal of stresses becomes more relevant, but has not been dealt with in this study. The extension of SLA to cyclic loading is currently being investigated.

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