ARCHING EFFECT IN LATERALLY LOADED URM WALLS

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ABSTRACT

According to Eurocode 6 the load-bearing capacity of laterally loaded unreinforced masonry (URM) walls may be determined assuming arching effect. The rules adopted in EC6 are based on tests performed in the UK at CERAM in 1976 by Hodgkinson, Haseltine and West. Based on the test results a design method was developed in which the depth of the compression zone was assumed to be $1/10^{th}$ of the wall thickness. In the adopted analytical model the allowable compressive strength of the arch was taken as 1.5 times the compressive strength in axial compression neglected. Neglecting the deformation of the wall, the distributed lateral load-bearing capacity of an unreinforced masonry wall was calculated as $f_d = (t/L)^2$, where: t is the thickness of the wall, f_d is the compressive strength of the masonry in the direction of the arch thrust and L is the distance between the rigid supports. In this paper the theory developed in the UK is analysed and compared to the results of preliminary tests which were performed at the Structures Laboratory of Eindhoven University of Technology. Moreover, a new analytical model has been developed which enables a more detailed calculation of the load-bearing capacity taking into account the deformation and initial imperfections of the wall.

KEYWORDS: *URM, arching effect, lateral loading*

INTRODUCTION

In laterally loaded unreinforced masonry walls, built between rigid supports, arching effects may develop, resulting in a higher load-bearing capacity compared to pure flexural bending capacity. This phenomenon is recognized in Eurocode 6 [1]. The design rules which are adopted in this code are based on test results on URM walls performed at CERAM in the UK in 1976 [2]. Recently, some doubts raised about the validity of the formula. In this respect the backgrounds of the EC6-formula have been searched for and the adopted theory has been compared to other theoretical models in literature. Based on this literature review, a new design model has been developed taking

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into account the deformation of the wall. Validation of this model will be done by performing several experiments. In this paper the results of two preliminary experiments will be described.

BACKGROUND EC6 APPROACH

The design method adopted in EC6 is based on lateral loading tests on masonry walls conducted in the UK in 1976 at CERAM by Hodgkinson, Haseltine and West. The tests results were used to develop a theoretical model, assuming the depth of the compression zone equals $1/10^{th}$ of the wall thickness (Figure 1). Furthermore, it was assumed that the compressive strength in bending may be taken as 1.5 times the compressive strength in axial compression.
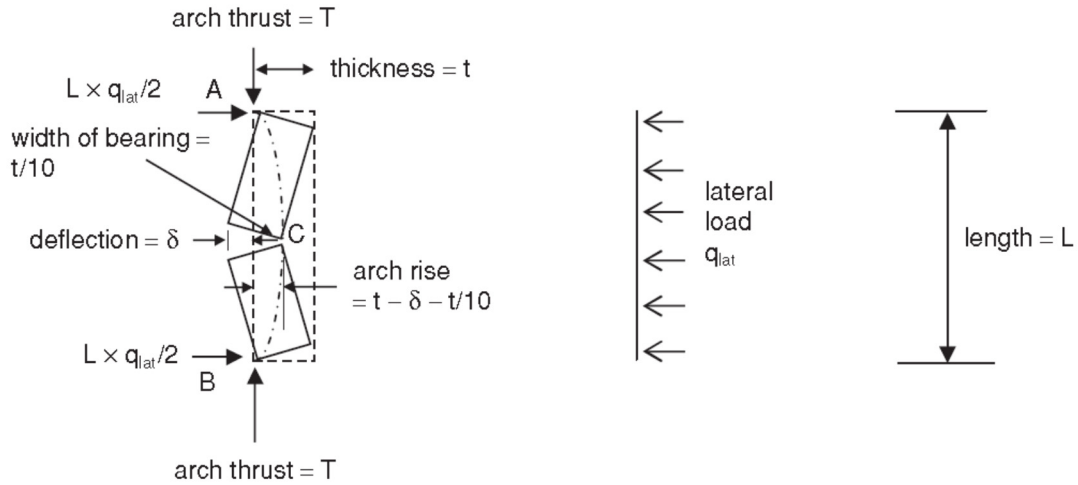

Figure 1: Principle of arching in laterally loaded URM walls [3]

Rotational equilibrium relative to point C in Figure 1, can be calculated as follows:

$$q_{lat}.\frac{L}{2}.\frac{L}{2} = T\left(t - \delta - \frac{t}{10}\right) + q_{lat}.\frac{L}{2}.\frac{L}{4} \quad \text{or} \quad q_{lat}.\frac{L^2}{8} = \left(\frac{9}{10}t - \delta\right)T \tag{1}$$

Where the arch thrust per unit height equals:

$$T = 1.5\frac{f_k}{\gamma_M}.\frac{t}{10} \tag{2}$$

Combining this formula with formula (1) and neglecting the deformation at mid-span δ (for $L/t <$ 25), yields:

$$q_{lat} = \frac{8}{L^2}\frac{9}{10}t\,1.5\frac{f_k}{\gamma_M}\frac{t}{10} \tag{3}$$

Rounding down the value 1.08 to 1 leads to the final formula in EC6:

$$q_{lat} = \frac{f_k}{\gamma_M}\left(\frac{t}{L}\right)^2 \tag{4}$$

LITERATURE REVIEW

The first scientific study dealing with arching effect in unreinforced masonry walls was published in 1956 by McDowell et al. [4]. The theory developed in this paper, which is also described in Drysdale et al. [5], takes into account the influence of mid-span deflection. Design graphs were developed for the determination of the thrust force and the resisting moment (Figure 2).
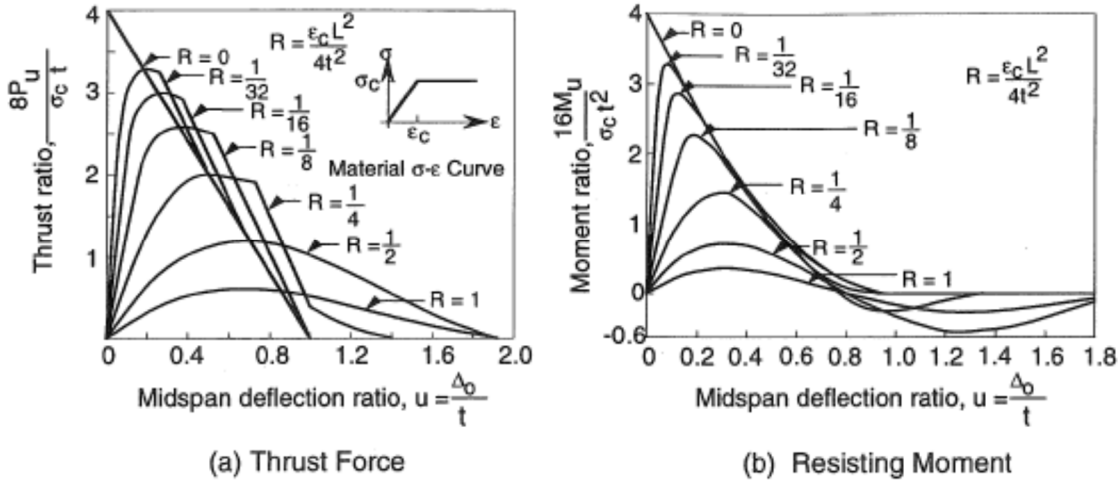

(a) Thrust Force   **(b) Resisting Moment**

Figure 2: design graphs for thrust force and resisting moment [5]

There may be some discussion about the theory and design graphs developed by McDowell et al. According to Figure 2 a mid-span deflection u lager than twice the wall thickness t could occur, while, for unreinforced masonry, this situation can never be reached.

Another aspect to discuss concerns the compressive strength which should be adopted. In [5] the maximum thrust force is calculated as $0.85 f'_j (t/10)$, where f'_j is the compressive strength of the mortar joint at the contact surface, which is taken conservatively as the mortar cube strength. This assumption yields:

$$q_{lat} = 0.61 f'_j \left(\frac{t}{L}\right)^2 \tag{5}$$

On the other hand, the EC6-formula is based on the characteristic value of the compressive strength f_k of the masonry in the direction of the loading, i.e. parallel to the bed joints. As a result a comparison between formula (4) and (5) is not straightforward.

In literature several alternative formulas are derived based on a best fit of experimental test results, but no general theory was found.

NEW ANALYTICAL MODELLING

Since no theory was found in literature for an accurate calculation of the deformation, a new analytical model has been developed based on the geometry and the deformation of the thrust line [6]. For an equally distributed lateral load, the thrust line is a parabola. The horizontal component

of the thrust force N depends on the lateral load q, the rise of the parabola f and the span of the wall L:

$$N = \frac{qL^2}{8f} \qquad (6)$$

The rise of the arch is equal to the distance between the action line of the horizontal components of the stress resultants at mid-span and at the supports respectively. The maximum load-bearing capacity of the wall will be reached if the horizontal thrust force N equals the load-bearing capacity of the cross-section at the supports and at mid-span. The design value of the axial force capacity can be calculated as follows:

$$N_{Rd} = \psi \times f_d \qquad (7)$$

Where: x is the depth of the compression zone of the cracked cross-section (Figure 3), ψ is the ratio of the area under the stress-strain diagram Ω to the area of the circumscribed rectangle ($\varepsilon_u.f_d$) (Figure 4):
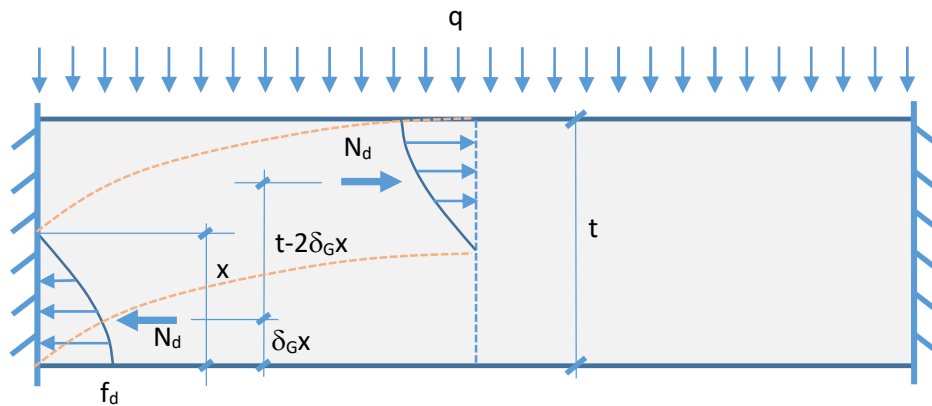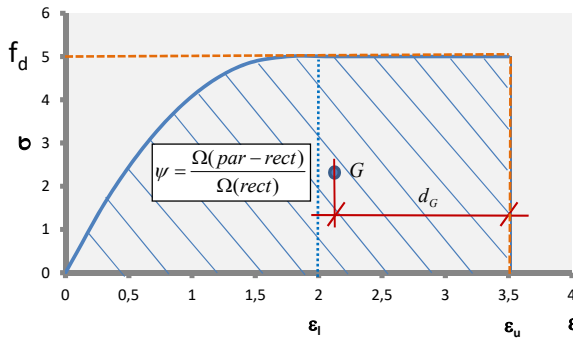
$$\psi = \frac{\Omega}{f_d\,\varepsilon_u} \qquad (8)$$

Figure 3: compressive stresses at cracked cross-section

σ–ε-diagram	ψ	δ_G
Linear	0,500	0,333
Parabolic-rectangular	0,810	0,416
$\varepsilon_l = 2,0‰$ and $\varepsilon_u = 3,5‰$		
Rectangular (stress block)	1,00	0,500

Figure 4: definition of parameters ψ and d_G (G = gravity center and $\delta_G = \dfrac{d_G}{\varepsilon_u}$)

Neglecting deformation, the rise of the arch equals $t - 2\delta_G x$ (Fig 3.). As a result, the design value of the axial force acting at mid-span and at the supports may be calculated using the following formula:

$$N_d = \frac{qL^2}{8(t - 2\delta_G x)} \tag{9}$$

where

$$\delta_G = \frac{d_G}{\varepsilon_u} \tag{10}$$

d_G is the ultimate strain ε_u minus the strain corresponding to the gravity center G of the surface underneath the σ–ε-diagram (Figure 4).

The maximum value of q will be obtained if $N_d = N_{Rd}$:

$$\frac{qL^2}{8 \psi f_d} = x(t - 2\delta_G x) \tag{11}$$

Finding the maximum value of q may be obtained by putting the first derivative related to x equal to zero:

$$t - 4\delta_G x = 0 \tag{12}$$

hence $\quad x = \dfrac{t}{4\delta_G} \tag{13}$

Introducing this value in formula (11) yields:

$$q = \frac{8 \psi f_d}{L^2} \frac{t}{4\delta_G} (t - 2\delta_G \frac{t}{4\delta_G}) \tag{14}$$

$$q = \frac{\psi f_d}{\delta_G} \left(\frac{t}{L}\right)^2 \tag{15}$$

For linear elastic behaviour, parabolic-rectangular and rectangular σ–ε–diagram respectively, the following results may be obtained (neglecting deformation of the wall):

$$q = 1{,}5\, f_d \left(\frac{t}{L}\right)^2 \qquad\qquad q = 1{,}95\, f_d \left(\frac{t}{L}\right)^2 \qquad\qquad q = 2{,}0\, f_d \left(\frac{t}{L}\right)^2$$

Since masonry is not infinitely stiff, contraction of the thrust line will occur due to axial compression by the thrust force. Before loading, the length of half the parabolic thrust line s_0 can be calculated as follows (Figure 5):

$$s_0 = \frac{1}{4a}\left[a\,L\,\sqrt{a^2\,L^2 + 1} + \text{arcsinh}(a\,L)\right] \tag{16}$$

where

$$a = \frac{4f_0}{L^2} \tag{17}$$

f_0: is the rise of the parabolic arch

When the line of thrust is axially loaded by a compressive force, the corresponding contraction $(s_0\text{-}s_1)$ will result in a deflection of the arch or a decrease of the rise $\Delta f = f_0 - f_1$ (Figure 5),

where

$$s_0 = \frac{1}{4a_0}\left[a_0\,L\,\sqrt{a_0^{\,2}\,L^2 + 1} + \text{arcsinh}(a_0\,L)\right] \qquad a_0 = \frac{4f_0}{L^2} \tag{18}$$

$$s_1 = \frac{1}{4a_1}\left[a_1\,L\,\sqrt{a_1^{\,2}\,L^2 + 1} + \text{arcsinh}(a_1\,L)\right] \qquad a_1 = \frac{4f_1}{L^2} \tag{19}$$
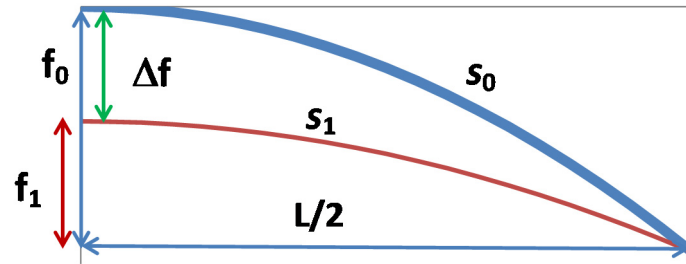

Figure 5: length of parabolic thrust line and deflection due to contraction

Assuming a constant average stress along the parabolic line of thrust, the average strain and stress and the horizontal thrust force are respectively equal to:

$$\varepsilon_{av} = \frac{s_0 - s_1}{s_0} \quad , \qquad \sigma_{av} = E\,\varepsilon_{av} \qquad \text{and} \qquad N_d = \psi\,x\,\sigma_{av} \tag{20}$$

Due to the deformation of the wall, the available wall thickness is reduced to $(t - \Delta f)$. As a result, the optimum value of the depth of the compression zone $x = \dfrac{t - \Delta f}{4\delta_G}$.

Introducing x into formulas (20) yields:

$$N_d = \psi \frac{t - \Delta f}{4\delta_G} E \frac{a_1 \left[a_0 L \sqrt{a_0^2 L^2 + 1} + \text{arcsinh}(a_0 L) - a_1 L \sqrt{a_1^2 L^2 + 1} - \text{arcsinh}(a_1 L) \right]}{a_0 \left[a_0 L \sqrt{a_0^2 L^2 + 1} + \text{arcsinh}(a_0 L) \right]} \qquad (21)$$

where

$$a_0 = \frac{4 f_0}{L^2} \qquad \text{and} \qquad a_1 = \frac{4 (f_0 - \Delta f)}{L^2} \qquad (22)$$

Taking into account the deflection Δf, the load-bearing capacity of the compression zone is equal to:

$$N_{Rd} = \psi \ x \ f_d = \psi \frac{t - \Delta f}{4\delta_G} f_d \qquad (23)$$

Failure will occur if the design value of the horizontal thrust force equals the design value of the axial resistance of the cross-section: $N_d = N_{Rd}$. By equating formulas (21) and (23), the corresponding value of the deflection Δf may be found. Due to the complexity of formula (21) the value of Δf has to be determined numerically or graphically as is illustrated in Figure 6a. If no intersection may be found, snap-through will occur as shown in Figure 6b.
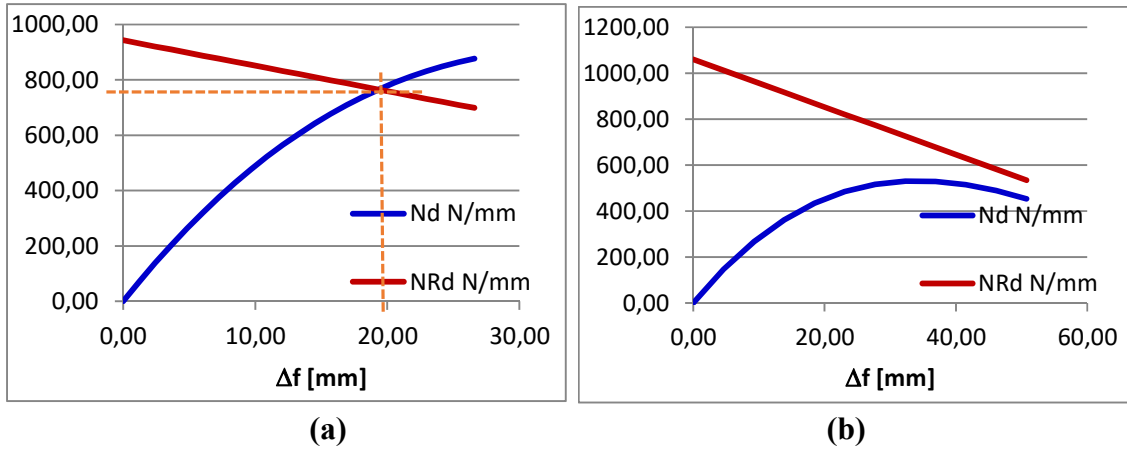

(a)   (b)

Figure 6: determination of the maximum deflection of the wall (a) and snap-through (b)

When Δf has been determined, the value of $N_{Rd} = \psi \frac{t - \Delta f}{4\delta_G} f_d$ can be calculated, which enables to determine the capacity of the masonry wall due to the lateral load q:

$$q = \frac{8 N_{Rd} (t - 2\delta_G x)}{L^2} \qquad (24)$$

In cases where snap-through behaviour is decisive, the load-bearing capacity is determined by the maximum value of q according to formula (24) where N_{Rd} is replaced by N_d.

EXPERIMENTAL VERIFICATION

The new design model will be validated by performing a series of tests at the Structures Laboratory of Eindhoven University of Technology. In this paper only the test set-up and the results of two preliminary tests will be described.

In order to avoid influence of friction at the support, the walls were tested in a horizontal position and loaded by four point loads as shown in Figure 7. The production of the test specimens was done in a traditional vertical position by an experienced mason. The walls were made of calcium silicate blocks (CS20) combined with thin layer mortar. At the edges of the wall, return walls were executed. After hardening, the walls were rotated in horizontal position. During this manipulation the first two specimens were already cracked before testing. It was nevertheless decided to continue the experiments after filling the cracks with mortar. At the supports the return walls were rigidly fixed between HEB-columns as shown in Figure 7.

The first step of the experiment consisted of releasing the temporary supports underneath the wall. During this operation the deformation was measured and visual inspection indicated that cracks occurred at the supports and at mid-span (Figure 8a). In this load case the equally distributed load is equal to the self-weight of the wall, i.e. 0.12 m x 18.5 kN/m³ = 2.22 kN/m².
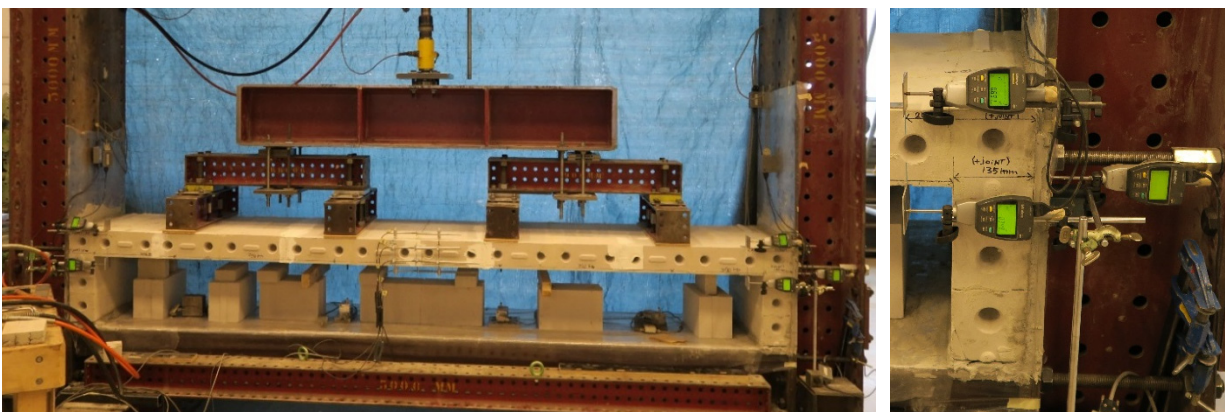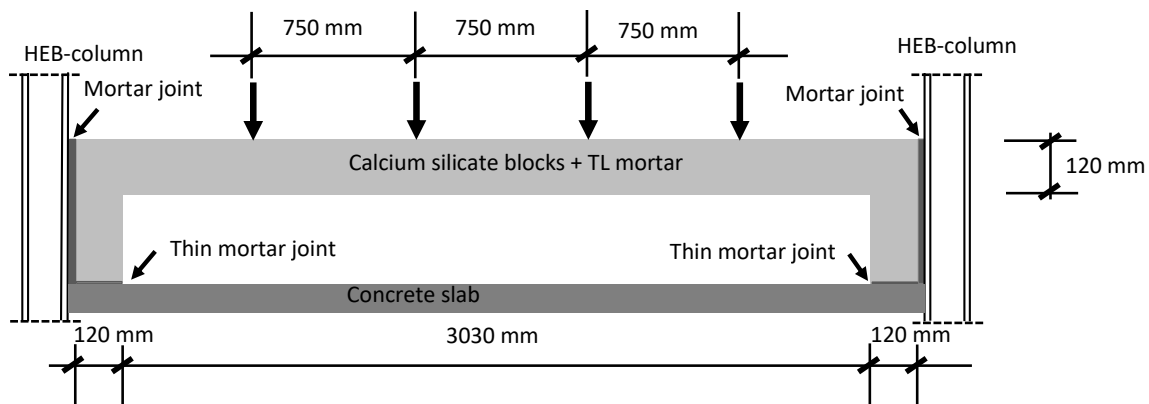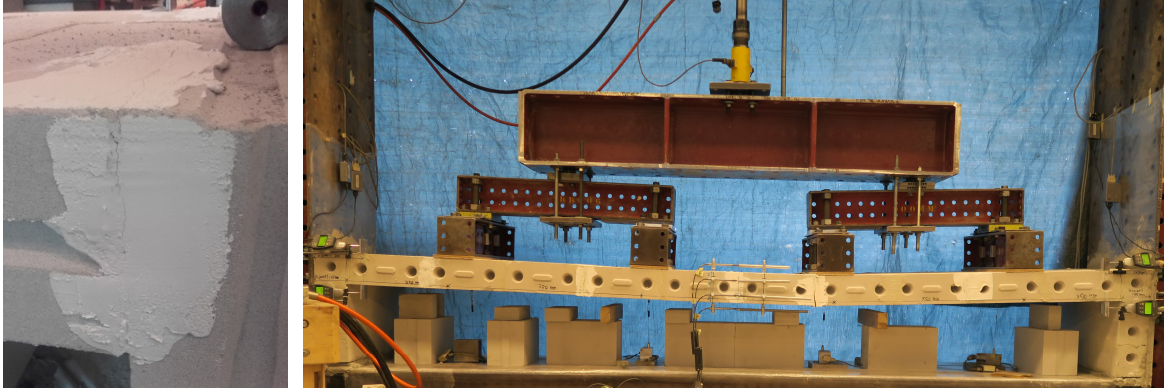

Figure 7: test set-up with four point loads; width of the specimen = 600 mm; slenderness L/t = 25

(a)   **(b)**

Figure 8: crack at return wall (a) and deflection of the wall (b)

The second step of the experiment consisted of introducing a vertical load on the wall through four point loads (Figure 7). The deflection of the wall at near collapse state is shown in Figure 8b.

For the calculation of the load-bearing capacity of the laterally loaded wall, the compressive strength of the masonry has to be determined. This can be done making use of the formula adopted in Eurocode 6 or by performing compression tests parallel to the bed joints (Figure 9).
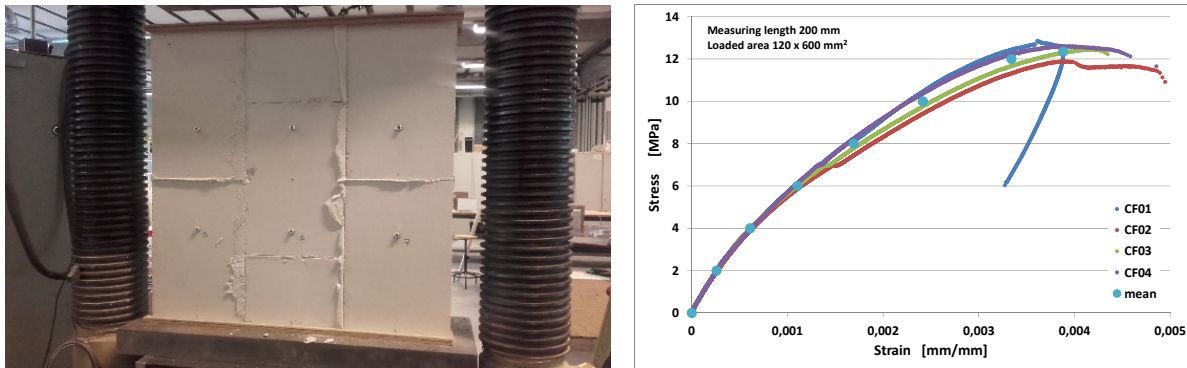

Figure 9: test specimen and results of compression tests parallel to the bed joints

Using EC6 formula the following compressive strength is found: Mean value of the compressive strength of the masonry unit CS20: f_b =20 MPa. Mean value of the compressive strength of the general purpose mortar M10: f_m =10 MPa. Characteristic value of the compressive strength of the masonry: $f_k = 0.6\, f_b^{0.65} f_m^{0.25} = 7.5$ MPa. The mean value of the compressive strength is at least 20% higher than the characteristic value. From compression tests on wallettes parallel to the bed joints (Figure 9) a mean value of the compressive strength of 12 MPa was found. This value was used in the theoretical modelling.

The results of the two preliminary tests are summarized in table 1 and compared to the theoretical values obtained by the EC6-formula and the new design method where the compressive strength of the masonry was assumed to be 12 MPa. In Figure 10 the displacement at mid-span related to the total force on the wall is shown for the two preliminary tests.

Table 1: Experimental test results compared to theoretical values;
Compressive strength f = 12 MPa; E-modulus = 7 200 MPa (= 600 f)
Parabolic-rectangular σ–ε-diagram: ε_l = 3.5 mm/m; ε_u = 4.5 mm/m (see Figure 9)

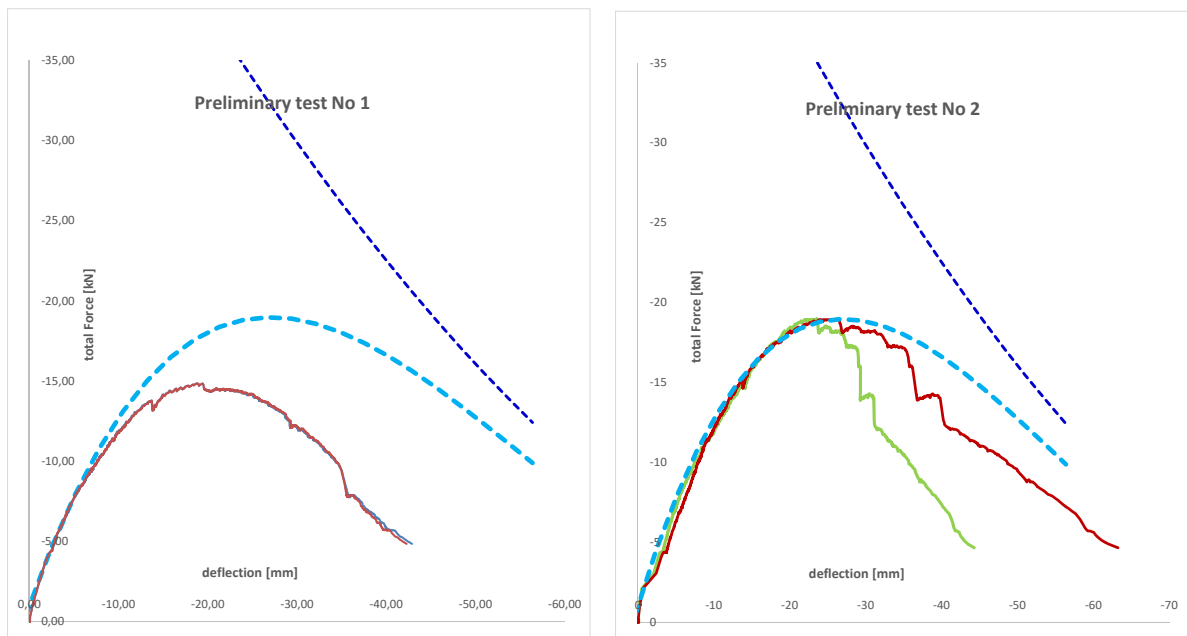| Test number	Experimentally determined load-bearing capacity q_{exp} [kN/m²]	Load-bearing capacity according to EC6-formula q_{EC6} [kN/m²]	Load-bearing capacity according to new design method (parabolic diagram) q_{new} [kN/m²]
1	8.16+2.22 =10.38	11.29	12.66
2	10.45+2.22 =12.67	11.29	12.66

Figure 10: Force-deflection diagram for preliminary tests No 1 and 2 (no self-weight); dotted lines according the theoretical modelling

According to Figure 10 failure occurred due to snap-through and due to material failure. Since both walls were cracked before testing and were repaired with mortar, it is not surprising that the load-bearing capacity is lower than the theoretical value, particularly in the case of snap-through. More tests will be performed with smaller slenderness (L/t = 2000/120 = 17) in order to investigate the wall arching behaviour in case of material failure.

CONCLUSIONS AND FUTURE RESEARCH

Based on the preliminary test results, it may be concluded that the design method according to Eurocode 6 is a conservative approach as far as the span to wall thickness ratio is smaller than 25. The new design method described in this paper [6] gives more accurate results since it takes into account the influence of the deformation of the masonry wall. In the near future more tests will be performed in order to validate the design method more in detail and to perform a parametric study.

The presented design method deals only with solid masonry walls (group 1 units) and is not applicable for masonry made of hollow concrete blocks or perforated clay blocks (group 2 or 3 units). In a future research this types of units will be considered as well.

Since the lateral load capacity of masonry walls is also important for seismic loading, cyclic and dynamic load situations will be investigated in a future research program.

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