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DISPLACEMENT BASED MODELLING OF OUT-OF-PLANE BEHAVIOUR OF VERTICALLY SPANNING UNREINFORCED MASONRY WALLS

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ABSTRACT

A semi-empirical tri-linear displacement-based model is developed using the database of 38 experiments to simulate the out-of-plane behaviour of vertically (one-way) spanning unreinforced solid brick masonry walls subjected to uniformly distributed load. This database considers various parameters identified to control the out-of-plane behaviour of masonry walls. These parameters are: aspect ratio, slenderness ratio, axial load ratio, crack height, and material properties. The analytical equations for maximum lateral force and instability displacement are derived using kinematic approach, taking into account the stress block parameters. Using the experimental database, two calibration factors, λ_1 and λ_2 are proposed to modify the analytical relationships. These factors are used to obtain idealized maximum force and two displacement limits which represent the tri-linear force-displacement relationship for out-of-plane loaded URM walls supported by rigid diaphragms at top and bottom. The developed tri-linear model is compared with an existing model using experimental test results.

KEYWORDS: *out-of-plane action, unreinforced masonry, one-way bending, vertical spanning walls, displacement-based model*

INTRODUCTION

Past earthquakes have repeatedly highlighted and identified the out-of-plane bending action of walls as one of the predominant modes of failure for unreinforced masonry (URM) buildings. However, the out-of-plane seismic response of URM walls is still not well understood. In the majority of the cases, this issue has been considered in terms of the capacity of the wall to resist lateral static force. However, several studies have demonstrated that, on one hand, URM walls subjected to earthquake loads tend to behave as rigid bodies subject to rocking, and on the other

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hand, they can resist accelerations much higher than their static strength [1, 2]. This explains why these elements are more sensitive to displacements rather than accelerations. Abrams et al. [3] have also shown that the magnitude of maximum displacement is the key response which governs the stability of a wall under out-of-plane loading. This fact suggests that displacement-based modelling could provide a better way of determining seismic design actions for URM walls compared to the traditional force-based approaches.

In displacement-based approach, the behaviour of a wall subjected to out-of-plane forces is usually modelled as a generalized single-degree-of-freedom (SDOF) system [4-6]. Griffith et al. [7] evaluated the response of URM walls subjected to out-of-plane excitation by means of a tri-linear force-displacement curve suggested by Doherty et al. [4] and demonstrated that the collapse of the wall is mainly conditioned by its maximum strength and ultimate displacement capacity and not by its initial stiffness. Melis [8] observed that the lateral static strength and ultimate displacement of a wall subjected to out-of-plane action are not significantly affected by material properties, whereas, geometry, boundary conditions and applied vertical force are relatively more influential parameters. Derakshan et al. [6] investigated the behaviour of three full-scale URM walls subjected to out-of-plane loading and varying the slenderness ratio. They indicated that wall strength may be overestimated, if bi-linear models are used for analysis. They also found that slenderness ratio (h/t) and overburden ratio (ψ) affect the shape of the tri-linear model. Ferriera et al. [9] developed a relationship based on the rocking behaviour of URM cantilever walls and proposed tri-linear models based on energy-based criteria. Later, Derakhshan et al. [10] proposed an analytical model that describes the out-of-plane response of one-way spanning URM walls having simply supported boundary conditions. They investigated the effects of various parameters, viz. horizontal crack height, masonry compressive strength, and diaphragm support. In the same line, Ferriera et al. [11] proposed a linearized four-branch model for cantilever unreinforced stone masonry walls, tested under one-way bending. The model is presented in terms of an overturning moment-rotation relationship. Recently, Beyer and Lucca [12] proposed a model for URM walls subjected to different out-of-plane excitations at the top and bottom by considering trapezoidal as well as uniform distribution of load.

In contrast to the single-degree-of-freedom approaches which have been extensively studied, only a few works on the analysis of the rocking behaviour of URM walls modelled as multi-degree-of-freedom systems are available in the literature. Psycharis and Spanos et al. [13-14] have studied the dynamic behaviour of systems consisting of two stacked rigid blocks without sliding. D'Ayala and Shi [15] presented a relatively simple dynamic model considering three main patterns of relative rotation of the two rocking blocks, which were further sub-divided into two sub-patterns based on the different reference points of rocking with respect to each other. Recently, Gabellieri et al. [16] proposed a 2-DOF model for analyzing the dynamic out-of-plane seismic behaviour of a single wall considering the hypothesis of a flexible diaphragm with an intermediate hinge and an elastic spring at the top.

The aim of the present study is to develop a simplified tri-linearized displacement-based model for out-of-plane behaviour of vertically spanning URM walls. The walls are assumed to be resting on a flat rigid floor at the base and simply supported at the top. The previously developed analytical model by Derakhshan et al. [10] is calibrated with a database of the test results available in the literature. The proposed model is compared with the model suggested by Derakhshan et al. [10].

ANALYTICAL MODELLING

In this study, the out-of-plane behaviour of a solid URM wall has been studied assuming it to be spanning in vertical direction. The wall is typically assumed to be resting on a flat base (the rotational restraint depends on the adhesion between the masonry and base slab and vertical force in the wall) and laterally restrained (but free to rotate) at the top, as shown in Figure. 1a. The wall, when subjected to uniformly distributed lateral load (w), representing the seismic inertia forces [17], develops the first crack at the base and the vertical reaction at the base shifts to the leeward edge as shown in Figure. 1b. The wall has a height, h; thickness, t; and total weight, W. The wall is subjected to pre-compression load, O. The maximum bending induced tension stress per unit length of wall section is obtained using mechanics, assuming homogenous section properties and elastic behaviour.

$$\sigma_t(x) = \frac{6M(x)}{t^2} - \frac{P(x)}{t} \tag{1}$$

where, $M(x)$ and $P(x)$ are the bending moment and the axial load as function of the height, x. Once, substituted in Equation 1, the maximum tensile stress at a height, x can be obtained as

$$\sigma_t(x) = \frac{3w(hx - x^2)}{t^2} - \frac{O + W(h-x)}{t} \tag{2}$$

The height of the wall crack, x_{cr} measured from the wall base, is the height where σ_t is the maximum. This can be determined by differentiating Equation 2 with respect to 'x' and equating it to zero.

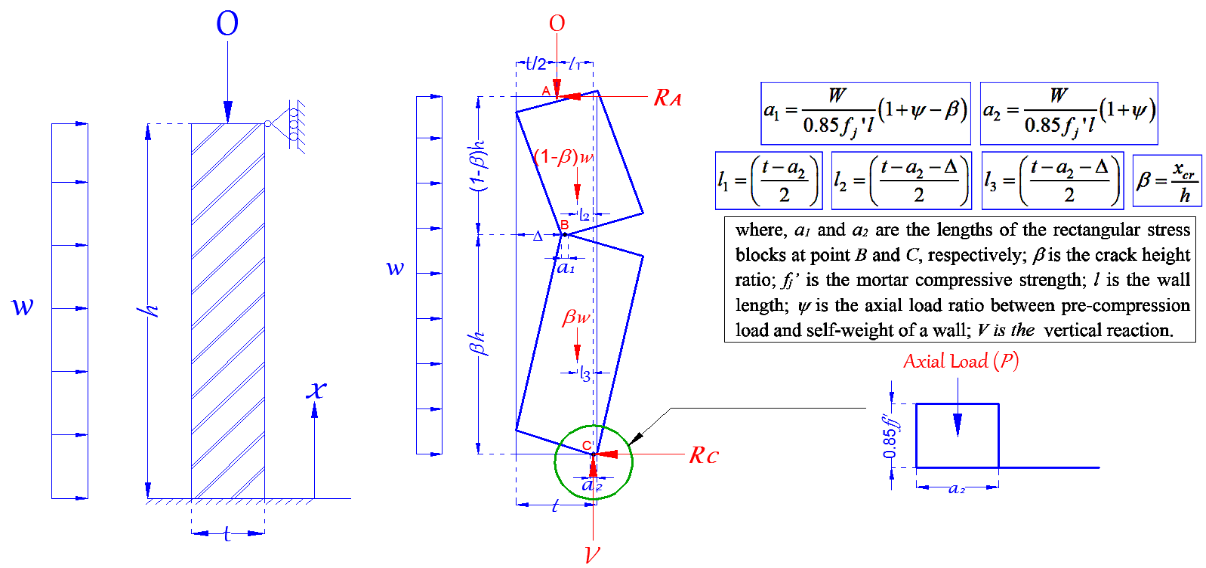
$$x_{cr} = \frac{h}{2} + \frac{Wt}{6w_{cr}} \tag{3}$$

where, w_{cr} is the uniformly applied lateral load corresponding to cracking.

Substituting $\sigma_t(x=x_{cr})$ equal to the masonry bond strength, f'_{fb}, in Equation 2, the uniformly distributed load corresponding to cracking can be obtained as

$$w_{cr} = \frac{\left(f'_{fb} + \frac{O}{t} + \frac{hW}{2t}\right) + \sqrt{\left(f'_{fb} + \frac{O}{t}\right)\left(f'_{fb} + \frac{O}{t} + \frac{hW}{2t}\right)}}{1.5\left(\frac{h}{t}\right)^2} \tag{4}$$

The total force, F_{cr} acting on the wall at cracking, can be obtained as a product of w_{cr} and h. Equation 4 shows that the out-of-plane strength of URM wall is mainly dependent on self-weight of wall and pre-compression load applied at the top of the wall. Figure 1b shows deformed shape of the wall after application of uniformly distributed lateral load, and different forces with their points of application. A cracked URM wall rocking with horizontal displacement, Δ is modelled as two rigid blocks separated by a fully cracked section. It is assumed that the self-weight of the wall and pre-compression load is applied at the centerline of the wall.



(a) (b)

Figure 1: (a) Wall Cross-Section Subjected to Out-of-Plane Force; (b) Deformed Shape and Reactions on URM Rigid Wall with Stress Block Parameters (Derakhshan et al. 2013)

The cracked wall rocks in the out-of-plane direction, about the pivotal points at the top, at cracked joint, and at bottom of the wall segments (Figure. 1b). High compressive stresses develop at the pivots due to line contacts. Since masonry has a finite compressive strength, stress blocks (Figure 1b) are formed at the pivot points B and C, and the reactions at these points slowly get displaced from their original position. A range of maximum stress, $\alpha f_m'$ has been recommended [18-20] by different researchers, where α is a factor used to convert the peak flexural compressive stress to an equivalent uniform compressive stress, which usually varies from 0.7 to 0.86 (α = 0.85 used in the present study). The restoring moment gradually reduces, due to decrease in the eccentricity between the vertical loads and reaction, as the displacement, Δ increases.

The horizontal reactions top and bottom (R_A and R_C) and lateral force, w corresponding to a given displacement, Δ can be obtained considering equilibrium.

$$w = \frac{2}{\beta h^2}\left[\left\{W\left(t-\Delta-\frac{a_1+a_2}{2}\right)\right\}+\left\{\frac{O}{1-\beta}\left(t(1-0.5\beta)-\Delta-0.5(a_1+a_2-a_2\beta)\right)\right\}\right] \tag{5}$$

The maximum out-of-plane lateral force, w_o and instability displacement, Δ_{ins} can be obtained by substituting zero displacement and zero lateral force, respectively in Equation 5. The total force, $(F_o)_{Anal}$ acting on the wall, is obtained as a product of w_o and h.

$$w_o = \frac{2}{\beta h^2}\left[\left\{W\left(t - \frac{a_1 + a_2}{2}\right)\right\} + \left\{\frac{O}{1-\beta}\left(t(1-0.5\beta) - 0.5(a_1 + a_2 - a_2\beta)\right)\right\}\right] \tag{6}$$

$$\Delta_{ins} = \frac{t\left[\left\{1 - \left(\frac{a_1 + a_2}{2t}\right)\right\} + \psi\left[\left(\frac{1-0.5\beta}{1-\beta}\right) - \left(\frac{a_1 + a_2 - a_2\beta}{2(1-\beta)t}\right)\right]\right]}{1 + \left(\frac{\psi}{1-\beta}\right)} \tag{7}$$

The theoretical maximum lateral load, w'_o and maximum instability displacement, Δ'_{ins} for the case of infinite masonry strength, can be obtained by setting a_1 and a_2 (refer to Figure 1b) equal to zero, in Equations 6 and 7. The total force, F'_o acting on the wall, is obtained as a product of w'_o and h.

$$w'_o = \frac{2}{\beta h^2}\left[Wt + \frac{O}{1-\beta}t(1-0.5\beta)\right] \tag{8}$$

$$\Delta'_{ins} = \frac{t\left[1 + \psi\left(\frac{1-0.5\beta}{1-\beta}\right)\right]}{1 + \left(\frac{\psi}{1-\beta}\right)} \tag{9}$$

Equation 8 indicates that the initial rigid threshold resistance, w'_o is function of the geometry (i.e. height and thickness), the self-weight, and the pre-compression load. Similarly, the displacement, Δ'_{ins} at which the resistance of the wall to overturning tends to zero, relates to the axial load ratio, crack height ratio, and the thickness of wall (Equation 9).

CALIBRATION OF THE ANALYTICAL MODEL

Experimental results have been used to calibrate the analytically developed equations. For this purpose, the results from a set of 38 out-of-plane tests, conducted in laboratory and in-situ, on URM walls [6], [21-29] have been used. The details of the test results are shown in Table 1. In the present study, only the tests conducted on simply supported, vertically spanning walls (i.e. the top and bottom of the wall specimens are restrained against translation, whereas the vertical edges of the wall are free) subjected to uniform lateral loading (using airbags) have been considered. In the database, slenderness ratio, h/t of specimens varies from 7.71 to 22, aspect ratio, l/h varies from 0.28 to 0.50, and the axial load ratio, ψ varies between 0 and 1.36. The mortar compressive strength, f_j' of the specimens varies from 0.7 to 22.4 N/mm^2; the masonry compressive strength, f_m' varies from 3.2 to 24.5 N/mm^2; and the masonry bond strength, f'_{fb} varies from 0.04 to 0.5 N/mm^2. The modulus of elasticity, E_m of the masonry in the specimens varies from 400 to 13475 N/mm^2. The density of masonry, γ_m has not been provided in most of the articles, and it has been assumed as 18 kN/m^3, in all the specimens, for the purpose of analytical estimation of lateral load capacity. The experimental force-displacement curves are idealized as equivalent tri-linear models as shown in Figure 2. This figure also shows the analytical rigid-rocking curve (a line joining F'_o with Δ'_{ins}) which is used as a bi-linear envelope to the tri-linear idealization.

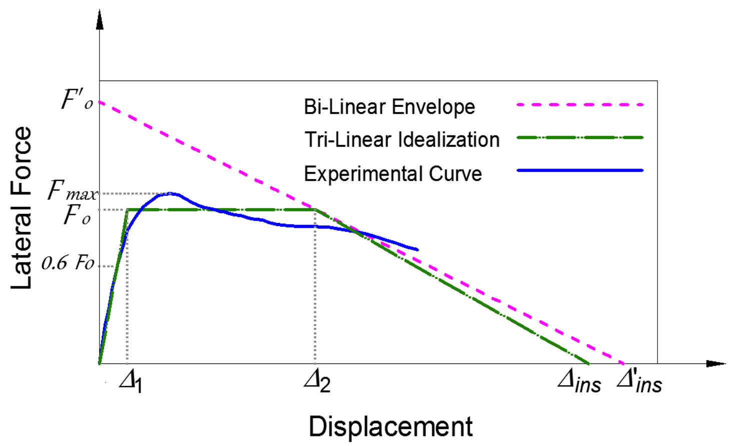


Figure 2: Idealized Behaviour of Wall under Out-of-Plane Action

The energy-based criterion as recommended by Ferriera et al. and ASCE 41-2013 [9 and 30] is used for tri-linear idealization. The initial stiffness is obtained as secant stiffness at 60% of the idealized maximum strength value. The value of idealized maximum force (corresponding to the plastic plateau), F_o is defined considering the energy balance corresponding to 20% loss of strength (i.e. $0.8F_{max}$) after the maximum experimental force value is reached. Δ_2 is the point corresponding to the idealized maximum force, F_o on the rigid rocking curve (bi-linear envelope). The instability displacement, Δ_{ins} is obtained from Equation 7. The analytical value of the displacement, $(\Delta_1)_{anal}$ can be obtained considering the un-cracked (gross) moment of inertia, I_g of the wall.

$$(\Delta_1)_{anal} = \frac{5(F_o)_{Anal} h^3}{384 E I_g} \tag{10}$$

where, $$I_g = \frac{lt^3}{12} \tag{11}$$

To calibrate the analytical model with the experimental results, two calibration factors, λ_1 and λ_2 are defined as

$$\lambda_1 = \frac{(F_o)_{\text{Exp}}}{(F_o)_{Anal}} \tag{12}$$

$$\lambda_2 = \frac{I_{eff}}{I_g} = \frac{(\Delta_1)_{Anal}}{(\Delta_1)_{Exp}} \tag{13}$$

For each test specimen considered in the present study, the experimental values of the parameters $(F_o)_{Exp}$ and $(\Delta_1)_{Exp}$ are obtained from the energy balance, whereas the analytical values are calculated using Equations 6 and 10, respectively, considering the parameters reported in the corresponding studies. In the present study, the mean value of the ratio of the idealized maximum force, F_o to the peak force F_{max} in experimental force-displacement curve (Figure 2) has been found as 0.92 (Table 1). This value is very close to 0.90 as suggested by Derakhshan et al. [10]. The average crack height ratio, β for the considered specimens was found to be 0.593 (Table 1), which is smaller than 0.677 suggested by Derakhshan et al. [10].

Table 1: Details of Experimental Tests Considered in the Present Study

Sr. No.	Author	Year	Specimen ID	Axial Load Ratio, ψ (O/W)	F_{cr} (kN)	β	$(F_o)_{Anal}$ (kN)	F'_o (kN)	Δ_{ins} (m)	Δ'_{ins} (m)	F_{Max} (kN)	$(F_o)_{Exp}$ (kN)	$(F_o)_{exp}/F_{Max}$	λ_1	$(\Delta_1)_{Exp}$ (mm)	$(\Delta_1)_{Anal}$ (mm)	λ_2
1	Derakhshan et al.	2008	T1	0.00	10.04	0.55	3.75	3.80	0.217	0.220	3.39	3.10	0.914	0.828	13.0	0.58	0.044
2	Derakhshan et al.	2009	1A	0.00	10.04	0.55	3.59	3.64	0.217	0.220	2.86	2.60	0.909	0.724	12.0	0.58	0.048
3	Derakhshan et al.	2009	2A	0.00	15.34	0.52	3.83	3.86	0.218	0.220	3.45	3.20	0.926	0.835	3.0	0.11	0.038
4	Derakhshan et al.	2009	3A	0.00	6.90	0.59	3.39	3.42	0.218	0.220	2.67	2.45	0.918	0.724	5.0	0.78	0.157
5	Ismail et al.	2009	-	0.00	4.25	0.63	3.11	3.22	0.212	0.220	2.20	1.95	0.886	0.628	3.0	0.83	0.275
6	Dizhur et al.	2010	AS-SS	0.00	33.91	0.56	12.90	12.94	0.399	0.400	16.03	14.50	0.904	1.124	2.0	0.30	0.151
7	Ismail et al.	2010	AC01	0.00	6.94	0.58	5.01	5.10	0.261	0.265	8.13	7.20	0.886	1.436	5.0	0.23	0.045
8	Ismail et al.	2010	AC02	0.00	2.30	0.62	1.71	1.75	0.157	0.160	0.57	0.50	0.877	0.292	6.0	0.82	0.137
9	Ismail et al.	2012	ABO1_1	0.00	4.22	0.64	3.05	3.18	0.211	0.220	3.26	3.10	0.950	1.018	3.0	0.45	0.149
10	Ismail et al.	2012	ABO2_1	0.00	4.29	0.62	3.14	3.27	0.212	0.220	3.50	3.20	0.913	1.019	5.0	0.33	0.066
11	Ismail et al.	2012	ABO1_2	0.00	2.18	0.61	1.68	1.70	0.153	0.155	0.79	0.75	0.947	0.447	4.0	5.39	1.347
12	Ismail et al.	2012	ABO2_2	0.00	5.69	0.58	4.24	4.29	0.237	0.240	7.28	6.40	0.879	1.510	20.0	1.84	0.092
13	Ismail et al.	2012	ABO3_2	0.00	4.27	0.63	3.07	3.33	0.203	0.220	3.48	3.10	0.892	1.010	4.0	1.17	0.294
14	Derakhshan et al.	2013	D1A	0.00	7.54	0.59	3.70	3.74	0.228	0.230	2.68	2.45	0.913	0.662	3.0	0.75	0.250
15	Derakhshan et al.	2013	D2A	0.00	10.98	0.55	3.92	3.98	0.227	0.230	2.83	2.50	0.884	0.637	12.0	0.63	0.053
16	Derakhshan et al.	2013	D3A	0.00	16.76	0.52	4.19	4.22	0.228	0.230	3.44	3.15	0.917	0.752	3.0	0.13	0.042
17	Derakhshan et al.	2013	T1A	0.00	11.19	0.63	7.77	7.99	0.340	0.350	9.07	8.50	0.938	1.095	4.0	0.17	0.042
18	Derakhshan et al.	2013	T2A	0.00	12.05	0.63	8.06	8.11	0.348	0.350	9.15	8.30	0.907	1.030	5.0	0.13	0.026
19	Derakhshan et al.	2013	T3A	0.00	13.97	0.57	8.84	8.88	0.348	0.350	11.89	10.70	0.900	1.210	3.0	0.04	0.014
20	Dizhur et al.	2014	W1	0.00	4.83	0.63	3.39	3.45	0.226	0.230	3.38	3.10	0.917	0.914	3.0	0.20	0.068
21	Dizhur et al.	2014	W3	0.00	4.87	0.63	3.43	3.49	0.226	0.230	3.34	3.00	0.898	0.873	2.5	0.18	0.071
22	Dizhur et al.	2014	W4	0.00	11.82	0.58	7.70	7.73	0.328	0.330	8.40	8.00	0.952	1.040	7.0	0.06	0.008
23	Dizhur et al.	2014	W5	0.00	9.95	0.63	6.90	7.10	0.321	0.330	9.86	9.20	0.933	1.333	3.0	0.18	0.060
24	Ismail et al.	2016	ABO-4	0.00	4.29	0.62	3.22	3.35	0.212	0.220	3.39	3.10	0.915	0.962	7.0	0.54	0.078
25	Lin et al.	2016	-	0.00	4.61	0.64	3.19	3.42	0.215	0.230	3.92	3.70	0.945	1.159	5.0	0.74	0.149
26	Derakhshan et al.	2013	D2B	0.30	11.36	0.55	5.78	5.89	0.201	0.205	3.98	3.70	0.930	0.640	5.0	0.93	0.186
27	Derakhshan et al.	2008	T2	0.31	10.42	0.55	5.65	5.76	0.192	0.195	4.02	3.80	0.945	0.673	4.0	0.87	0.217
28	Derakhshan et al.	2009	1B	0.32	10.42	0.55	5.41	5.52	0.191	0.195	4.06	3.75	0.925	0.693	4.0	0.87	0.217
Mean					-	-	-	-	-	-	-	-	-	-	-	-	**0.154**
29	Derakhshan et al.	2008	T3	0.63	10.79	0.55	7.52	7.71	0.180	0.185	5.38	5.10	0.949	0.678	3.5	1.16	0.331
30	Derakhshan et al.	2009	1C	0.63	10.79	0.55	7.21	7.39	0.180	0.185	5.41	5.10	0.942	0.707	3.0	1.16	0.386
31	Derakhshan et al.	2013	D2C	0.70	11.83	0.55	8.05	8.26	0.187	0.192	5.30	5.00	0.944	0.621	4.0	1.29	0.324
32	Derakhshan et al.	2013	T1B	0.70	13.30	0.61	17.26	18.26	0.266	0.281	14.54	13.20	0.908	0.765	5.0	0.37	0.075
33	Derakhshan et al.	2013	T2B	0.70	14.14	0.61	18.10	18.33	0.278	0.282	13.34	12.80	0.959	0.707	4.0	0.29	0.073
34	Derakhshan et al.	2013	D1B	1.00	8.87	0.57	10.14	10.37	0.180	0.184	8.02	7.30	0.910	0.720	4.0	2.05	0.513
35	Derakhshan et al.	2009	3B	1.02	8.11	0.57	9.22	9.43	0.172	0.176	7.96	7.30	0.917	0.792	4.5	2.13	0.474
36	Derakhshan et al.	2014	AH1A	1.02	1.07	0.71	1.29	1.34	0.105	0.109	2.05	1.90	0.925	1.472	2.5	1.04	0.417
37	Derakhshan et al.	2014	ATA	1.02	3.12	0.67	11.02	11.22	0.177	0.180	16.62	16.30	0.981	1.479	11.0	0.53	0.048
38	Dizhur et al.	2014	W2	1.36	6.65	0.60	11.70	12.32	0.168	0.177	8.19	7.40	0.904	0.633	5.0	0.71	0.141
Mean					-	**0.593**	-	-	-	-	-	-	**0.920**	**0.891**	-	-	**0.278**

Table 1 presents the values of λ_1 and λ_2 for all the specimens considered in the present study. The mean value of λ_1 has been found to be 0.891 (COV=0.326) (Table 1). Ideally, the value of λ_1 should be expected equal to unity. The estimated value suggests a reduction in the experimentally observed strength in comparison with the analytical estimation, which is attributed to the roundness of the wall corners, prior crushing of the masonry at pivot points, and the wall deformations that were neglected in the analytical study. In a similar study, Derakshan et al. [10] have defined two empirical correction factors ($F_{max}/(F_o)_{Anal}$) and ($(F_o)_{Exp}/F_{max}$). The product of the two factors is equivalent to λ_1 in the present study, which is equal to 0.747, based on the reported values (0.83 and 0.9, respectively). The difference in the two studies is due to the larger dataset (results of 38 specimens against 10) being used in the present study.

It has been observed that the calibration factor λ_2 is sensitive to the axial load ratio, ψ. However, no clear trend could be observed (Figure 3) between λ_2 and ψ to enable an expression; accordingly, the results for λ_2 have been considered separately for $\psi \leq 0.50$ and $\psi >$ 0.50. The average estimated value of λ_2 was found to be 0.154 (COV=1.576) for axial load ratio, ψ between 0 to 0.50, whereas it is 0.278 (COV=0.606) for axial load ratio, ψ between 0.50 to 1.36 (Table 1). These values are close to those reported by Derakhshan et al. [10] ($(0.18\psi+0.04)I_g$), and compared in Figure 3.

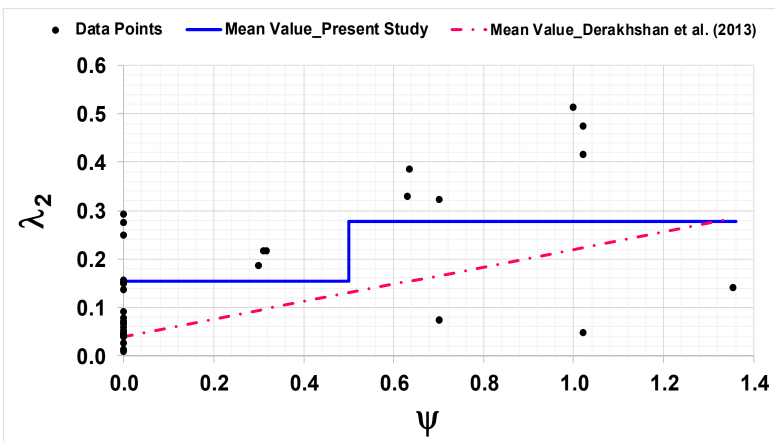


Figure 3: Variation of Effective Stiffness with Axial Load Ratio

PROPOSED MODEL

Based on the estimated values of the calibration factors, λ_1 and λ_2, the following semi-empirical tri-linear model (refer to Figure 2) is proposed:

$$F_o = \frac{1.782}{\beta h}\left[\left\{W\left(t-\frac{a_1+a_2}{2}\right)\right\}+\left\{\frac{O}{1-\beta}\left(t(1-0.5\beta)-0.5(a_1+a_2-a_2\beta)\right)\right\}\right] \tag{14}$$

$$\Delta_1 = \frac{5F_o h^3}{384E(0.154I_g)} \quad \text{.......... For axial load ratio, } \psi \leq 0.50 \tag{15}$$

$$\Delta_1 = \frac{5F_o h^3}{384E(0.278I_g)} \quad .... \text{ For axial load ratio, } \psi > 0.50 \tag{16}$$

$$\Delta_2 = \left(1 - \frac{F_o}{F'_0}\right)\Delta_{ins} \tag{17}$$

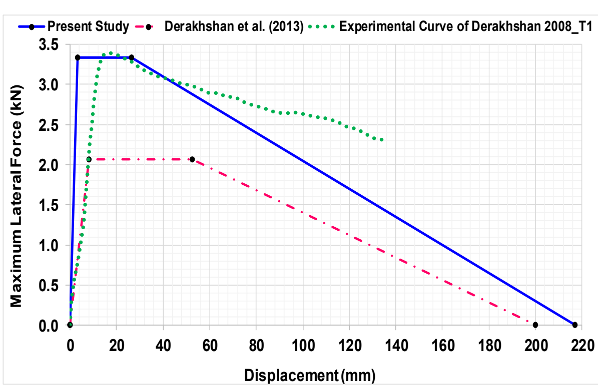
where, Δ_{ins} is estimated using Equation 7.

COMPARISON OF THE PROPOSED MODEL WITH DERKAKSHAN ET AL. (2013)

The proposed model has been compared with the model proposed by Derakhshan et al. (2013) for two representative walls from the database, with details shown in Table 2. The constructed models are shown in Figure 4. It can be observed from the table that the value of maximum theoretical lateral load, F'_o and instability displacement, Δ'_{ins} obtained from the present study are different than Derakhshan et al. (2013). This difference is because in Derakshan et al. model, the crack height ratio, β was assumed to be constant as 0.677, whereas, in the present study, the value of β has been estimated for each case separately. The idealized maximum force, F_o for both the walls is found to be lower in Derakhshan et al. model, compared to the present study (Figure 4), due to the lower value of the mean empirical correction factor (0.747) compared to that (0.891) estimated in the present study. The Derakhshan et al. model predicts higher values of Δ_I in comparison to the present study, because in Derakshan et al. model, the value of Δ_I has been fixed at 4% of Δ_{ins}, whereas, in the present study, this value is obtained using the wall parameters and axial load ratio. Similar comparison has also been performed for other walls, but the results are not presented here for brevity. The comparison shows that the proposed model is in a better agreement with the experimental load-displacement curves compared to Derakhshan et al. model.

Table 2: Details of the Walls Used for Comparison

Sr. No.	Author	Year	Specimen ID	l (m)	h (m)	t (m)	f_j' (N/mm²)	f'_{fb} (N/mm²)	E_m (N/mm²)	Present Study						Derakhshan (2013) Model					
										F'_o (kN)	Δ_{ins} (m)	Δ'_{ins} (m)	F_o (kN)	Δ_I (mm)	Δ_2(mm)	F'_o (kN)	Δ_{ins} (m)	Δ'_{ins} (m)	F_o (kN)	Δ_I (mm)	Δ_2(mm)
1	Derakhshan et al.	2008	T1	1.20	3.50	0.22	3.95	0.44	3410	3.80	217.01	220.00	3.34	3.33	26.28	2.80	200.00	220.00	2.06	8.00	52.45
2	Dizhur et al.	2014	W1	1.15	4.10	0.23	3.40	0.15	12760	3.45	225.99	230.00	3.02	1.18	28.14	2.94	210.00	230.00	2.16	8.40	55.77



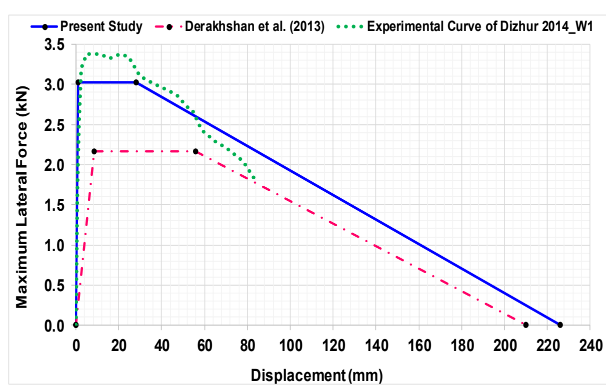


Figure 4: Comparison of the Proposed Model with Derakhshan et al. (2013)

CONCLUSIONS

The literature on out-of-plane behaviour of one-way spanning URM walls has been reviewed and various governing parameters have been identified. Analytical expressions for estimating the force-displacement response have been derived and calibrated with experimental results reported in the literature. Two calibration factors representing the ratio of experimental and analytical values of strength and yield displacement have been obtained. These factors have been used for developing a semi-empirical tri-linear model. The obtained tri-linear force displacement model can be used directly for fragility analysis of one-way spanning URM walls connected to rigid diaphragms, under out-of-plane seismic action.

In the present study, the mean crack height ratio for simply supported walls has been found to be 0.59, in comparison to the 0.67 suggested by Derakhshan et al. (2013). It has been observed that for the range of the axial load ratio for which the experimental results are available in literature, the effect of strength of the mortar joint on estimated lateral load capacity is very small. It results only around 5% difference when the crushing of masonry (mortar joint) at the pivot points is ignored. However, in multi-storey URM buildings, the axial load ratio may be much larger and the mortar joint strength may have a more significant effect. For the dataset considered in the present study, the instability displacement varies from close to the thickness, t of the wall specimen (for $\psi = 0$) to $0.75t$ (for $\psi = 1.36$). The calibration factor, λ_1 equal to 0.891 has been estimated to find the maximum idealized wall lateral load resistance. The sensitivity of the calibration factor λ_2 to the axial load ratio, ψ has been observed, but a clear trend enabling a relationship between λ_2 and the axial load ratio, ψ could not be found. The average estimated value of λ_2 has been observed to be 0.154 for axial load ratio, ψ between 0 to 0.50, whereas it is 0.278 for axial load ratio, ψ between 0.50 to 1.36. These values are close to those reported by Derakhshan et al. (2013). The proposed model based on these parameters has been shown to predict the load-displacement curve closer to the experimental results.

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