INTERACTION OF SHEAR AND FLEXURAL COLLAPSE MODES IN THE ASSESSMENT OF IN-PLANE CAPACITY OF MASONRY WALLS

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ABSTRACT
Assessment of the load displacement curve for masonry walls is a key ingredient in all the safety verification formats for seismic analysis. Available numerical methods for the largest part use a spatial discretization based on a finite element representation in terms of beam, membrane or shell elements. In order to include the basic features of the masonry material, nonlinear plastic or damage constitutive models can be selected. It is however well known that rich constitutive models require an increasing number of constitutive parameters, which usually are hardly assessed in practice. Moreover, heterogeneous masonry textures do not allow experimental testing on small specimens so that data can be only inferred from nondestructive techniques.

In fact, the real safety analyses performed in engineering offices rarely exit from the standard Mohr-Coulomb plasticity model [1, 2]. If the biaxial crushing failure is not included in the model through a cap closure, some problems could emerge from the interaction of the flexural and shear collapse modes. In particular, the formation of a diagonal resisting strut in panels acted on by overturning forces could produce a shear over strength due to a biaxial compression state.

By using standard rules evaluating the shear resistance of cracked sections it is however possible to check nonlinear load displacement curves in order to detect premature shear failures along the flexural response curve. By this way it is possible to build up response curves which include shear failure and reduced ductility even if the finite element software has only basic plasticity options. The comparison of various wall models allow a better understanding of the limits of advanced nonlinear packages in the field of masonry behaviour under combined loading.

KEYWORDS: masonry wall, in plane loading, shear resistance, plasticity

INTRODUCTION
Textured brick masonry walls can be viewed as a composite material encompassing brick inclusions dispersed in a mortar matrix. It is however very difficult to tackle masonry body problems with a micro structural level analysis, unless only a representative small volume is investigated.

In general, in order to grasp the main features of the masonry nonlinear behaviour, an "*elasto-plastic in compression and brittle in tension*" constitutive law must be adopted. A suitable choice is to consider a Mohr-Coulomb plasticity for the homogenized masonry material, although perfect plasticity deviates from real behaviour in combined stress states [3]. If the wall is composed of slender columns and lintel beams, the 2D problem can be rearranged in terms of stress resultants of given sections. In this way, a true no tension material is considered, but the link between normal stress and shear stress states is weakened, since, in terms of stress resultants, the two verification procedures have no link in terms of combination of limit stress states [4, 5]. Therefore, if a non-linear stress combination is available from the analysis, the constitutive law itself results in a combined verification format, else if we compute non-linear bending and shear resistances in terms of stress resultants, the two verifications decouple.

However, even the very basic Mohr-Coulomb plasticity without cap closure has deficiencies when shear is acting on heavily compressed sections, since the biaxial compression at the toe of inclined thrust lines results in an incorrect bearing resistance. This can happen when the breadth of a wall is large in comparison with its vertical shear arm.

In the paper a more precise verification procedure is set up by linking bending resistance and shear resistance by means of the effective compressed area. In particular, the wall force displacement curves leading to limit bending and shear resistances are plotted against the top section displacement, directly identifying if the critical verification is for overturning or sliding mechanisms. Finally, by equating the limit resistances of a generic wall, the characteristic aspect ratio of the wall is identified, which separates the flexural collapse zone and the shear collapse zone. It turns out that this parameter is dependent even on the type of verification formula adopted for shear forces, so that no unique critical height can be defined.

FORCE DISPLACEMENT CURVE FOR A CANTILEVER WALL

In experimental test results, it is evident that there is a continuous stiffness reduction of masonry panels caused by a concurrent application of constant normal force and an increasing shear force. This fact points out a strong geometrical effect due to the deformation component, i.e. a more than linear increase of the local curvature with respect to the external action.

The key point of the represented problem is the definition of the boundary between the compressed and the inactive zones. As it is well known [6, 7], the volume of the structure is only the container of the real structure which is the subset in equilibrium with compressive stresses only in the assumption of a no-tension material.

With reference to a rectangular wall of width L, thickness t and height H, clamped at its base and acted on by shear and pressure at its top, we consider the different limit states in the main sections of the panel during the shear increase.

The constitutive relationship for the masonry has been studied extensively on wall specimens and columns [5, 8-10]; for simplicity reasons and in agreement with experimental results [11], in this analysis a simple no-tension and elasto-plastic in compression law is assumed. In practice, it means fixing three parameters: the limit strength f_k, the plastic strain ε_y, and the ductility index $D = \varepsilon_u / \varepsilon_y$ (figure 1).

The analysis of the cantilever wall is fully developed in [8]. Since the bending moment is known as a function of shear force, and the compressed zone results from equilibrium, the curvature diagram of the panel can be obtained in an analytical form. Finally, by direct indefinite integration, the horizontal displacement is obtained as an analytical function of the applied shear.

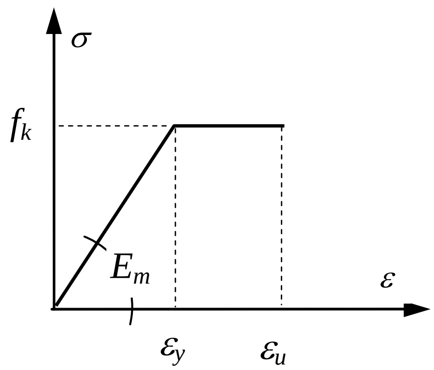

Figure 1: Elastic plastic representation of compression in masonry

In what follows, a very simple treatment is reported only for documentation purpose. Let us define:

$$M_e = N\frac{L}{6},\ \ V_e = \frac{M_e}{H},\ \ \delta_e = \frac{M_e}{t\cdot L}\cdot\left(\frac{4\cdot H^2}{E\cdot L^2}+\frac{6}{5\cdot G}\right) \qquad (1.a,\ 1.b,\ 1.c)$$

Where L is the panel length and N is the total normal force corresponding to the applied pressure summed up with the panel weight.

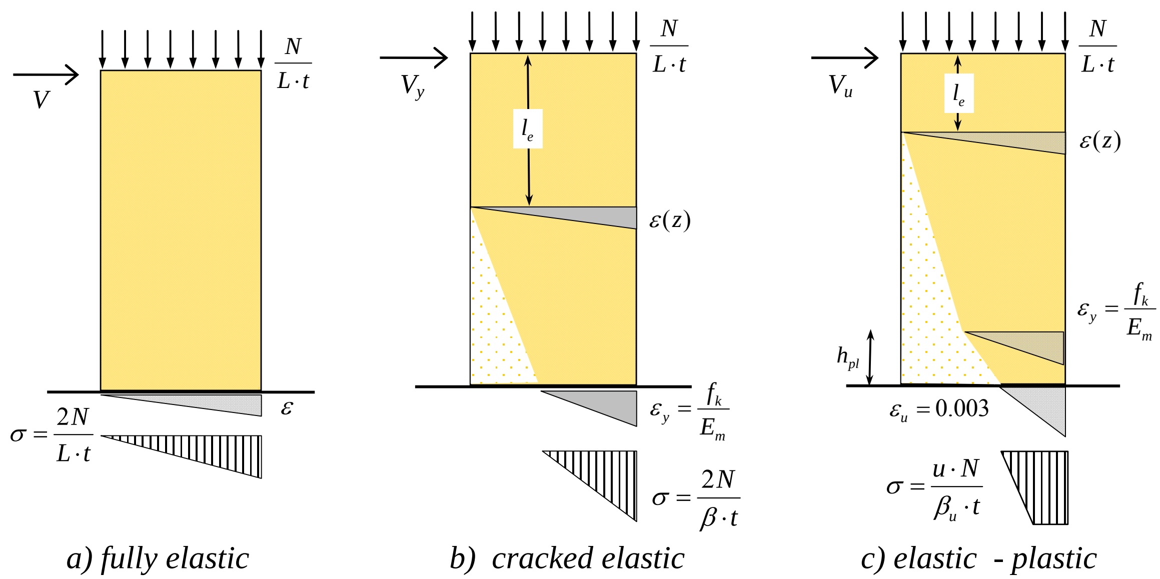

Figure 2: Equilibrium configurations of the masonry panel under compression and shear

In the deformation process after passing the elastic range, the active zone of the panel is a subset of the width L in the strip beneath the section where the moment is raised up to the limit decompression value (figure 2.b). At the onset of plastic situation, by indicating with t the wall thickness, the moment and the shear hold:

$$M_y = N\left(\frac{L}{2} - \frac{2}{3}\cdot\frac{N}{t\cdot f_k}\right), \quad V_y = \frac{M_y}{H}. \tag{2.a, 2.b}$$

By integrating the curvature, the displacement and the rotation at the boundary between the cracked and elastic zones in the wall are obtained. Finally the displacement for $z = 0$ is computed by kinematical composition of the elastic and cracked terms. Given:

$$\mu = \frac{L\cdot N}{L\cdot N - 2H\cdot V}, \quad \eta = \frac{H\cdot V}{L\cdot N - 2H\cdot V}, \tag{3.a, 3.b}$$

the displacement is expressed in simplified form as a function of μ and η:

$$\psi(V) = -\frac{N^3(\mu - 6\eta)}{6EtV^2}. \tag{4.a, 4.b}$$

The top displacement is evaluated as a function of the top shear:

$$\delta(V) = \delta_e \frac{V_e}{V} + \psi(V). \tag{5}$$

At the end of the loading process, the base section reaches the highest normal force eccentricity until the outer fibre deformation arrives at the ultimate strain of the constitutive law (figure 2.c), which can, for instance, be set as $\varepsilon_u = 0.003$ with a ductility index around 1.5 [11].

Equilibrium and compatibility requirements fix unambiguously the shape of the stress block; thus, we can compute exactly the width of the compressed zone β_u and the ultimate curvature:

$$\chi_u = \frac{\varepsilon_u}{\beta_u} = \varepsilon_u \frac{t\,f_k}{N}\left(1 - 0.5D^{-1}\right), \quad h_{pl} = H\left(1 - \frac{V_y}{V_u}\right). \tag{6}$$

By considering the curvature increment in the basement strip h_{pl} as a concentrated plastic rotation, the contribution at failure of the plastic hinge to the top displacement is:

$$\Delta_u \cong \frac{1}{2}\left(\chi_u - \chi_y\right)\cdot h_{pl}\cdot\left(H - h_{pl}\right). \tag{7}$$

The shear displacement path in the plastic range ($V_y \le V \le V_u$) is given by the formula:

$$\delta(V)=\delta_e\frac{V_e}{V}+\psi(V)+\Delta_u\frac{(V-V_y)}{(V_u-V_y)}. \tag{8}$$

EVALUATION OF SHEAR LIMIT DUE TO SHEAR FAILURE

The shear resisted by flexure can be incompatible with the shear resistance of the compressed zone; in that case collapse by shear sliding is taking place. In particular the resisting shear stress can be expressed as a function of the compressed zone and the vertical mean stress; the most widely used formulas are defined as a consequence of the Coulomb hypothesis [5, 6]:

$$\tau_{MC}=f_{vk}+\sigma\cdot\tan\varphi\ ,\ \tau_{TC}=f_{vk}\sqrt{1+\frac{\sigma}{f_{vk}}}\ , \tag{9}$$

Where the second formula is due to [12]. The resisting shear force linked to a reduced compressed zone coming out from a given axial force and external shear force combination, is easily calculated as:

$$V_{R,V}=\tau(N)\cdot\beta(V_S)\cdot t\ . \tag{9}$$

The flexural shear resistance $V_{R,M}$ is steadily increasing up until the flexural failure, but for given height to length ratio of the wall, it can be greater than the shear sliding force $V_{R,V}$. In this case the load displacement curve must be cut at that last value.

As an example, assuming that a cantilever wall with the following data is considered: N = 300 kN, t = 0.3 m, L = 4.0 m, H = 3.0 m, E=1800000.0 kPa, f_{mk} = 6000 kPa, f_{mt} = f_{vk} = 400 kPa, the flexural and sliding shear resistance as a function of wall displacement is plotted in figure 3, where the grey line is relative to the Coulomb shear and the black one to the Turnsek Cacovic one.

More precisely, once a compressed zone length is selected, the overturning equilibrium defines the flexural external shear; the shear value is used in the top displacement calculation, and a shear sliding resistance is evaluated on the basis of the compressed area.

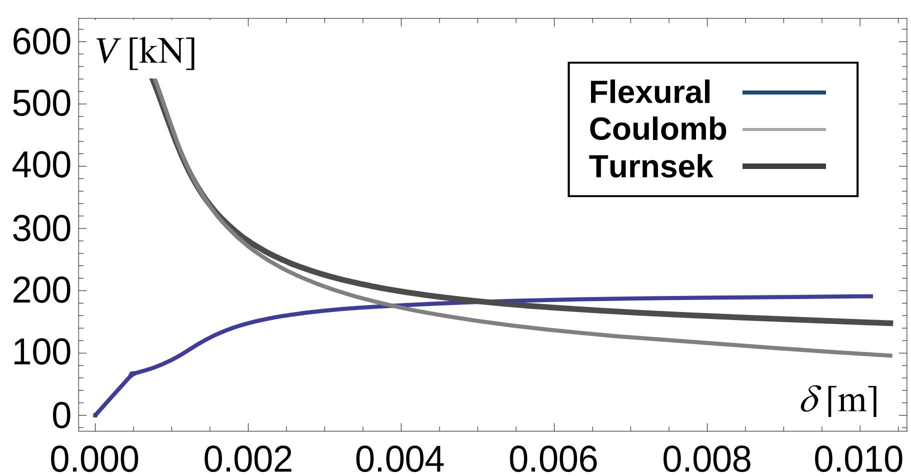

Figure 3: Resisting shear forces as a function of the wall displacement – stocky wall

As it is clearly visible, the limit displacement is approximately 0.3% of the height, well below the considered wall limit deformation. However, the consideration of the sliding shear reduces this limit by halving the allowed displacement, although the reduction in resisting shear force is limited.
If a wall of width L = 2.0 is considered, the previous picture would look very different and the flexural limit would apply, as highlighted in figure 4.

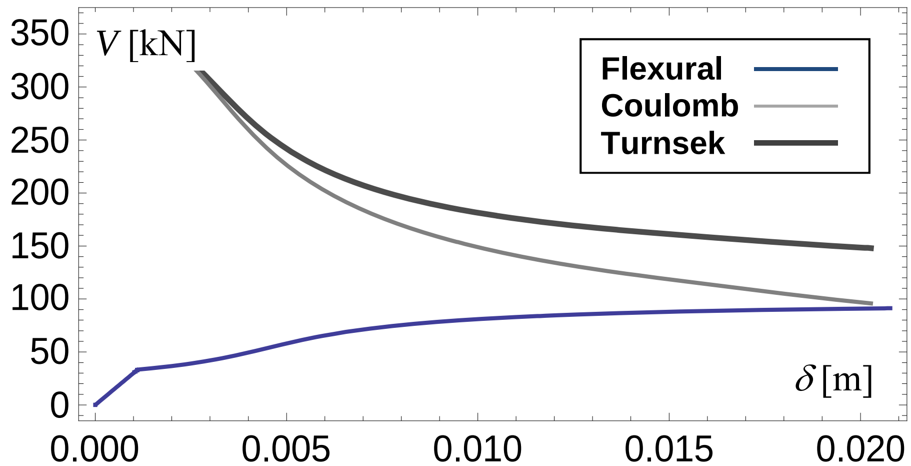

Figure 4: Resisting shear force as a function of wall displacement – slender wall

In order to define the condition which delimits the two types of failure, we can equate the values of the combined ultimate limit shear computed by flexural and sliding conditions, and derive the height for which the behaviour is changing from ductile to brittle.

If we consider relevant the intersection of the two curves at the fully plastic stage, it is possible to compute the height separating the two fields of flexural and shear predominance. The two equations are:

$$\frac{N\left(L-\beta_u\right)}{2H} = \beta_u t \cdot f_{vk0} + \phi N\,; \qquad (10.a)$$

$$\frac{N\left(L-\beta_u\right)}{2H} = \beta_u t \cdot f_{vk0}\sqrt{1+\frac{f_{mk}}{f_{vk0}}}\,. \qquad (10.b)$$

It has to be pointed out that the smallest compressed zone is defined by:

$$\beta_u = \frac{N}{f_{mk}t}\cdot\frac{1}{1-0.5D^{-1}} = \frac{\rho N}{f_{mk}t}\,. \qquad (10.b)$$

By solving the presented equations the critical height is obtained:

$$H_{MC} = \frac{f_{mk} Lt - \rho N}{2t \cdot \left(\rho f_{vk0} + \phi f_{mk} \right)} \; , \; H_{TC} = \frac{f_{mk} Lt - \rho N}{2t\rho \cdot \sqrt{f_{vk0} \left(f_{vk0} + f_{mk} \right)}} \; . \qquad (11.a, 11.b)$$

In the presented case, the use of the above mentioned formulas leads to an evaluation of the limit height of 3.75 m in case Coulomb shear stress is assumed, or 4.5 m in case Turnšek and Cacovic shear stress is selected.

It is evident that by the presented procedure is always possible to understand if a given wall will fail in flexure or shear in order to plan the nonlinear strategy for the push over curve.

CONCLUSIONS

The presented analysis is a very simple one but allows understanding if a compressed masonry panel loaded in shear will fail by in plane overturning or sliding shear.

It is very clear that the main parameter discriminating between the two fields is the height to length ratio. It is however apparent from the derived formulas that even the normal and shear strength enter in the critical height formula.

The presented example illustrates cases in which the shear resistance cuts or does not cut the flexural load displacement curve.

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REFERENCES

1. Pelà L., Aprile A., Benedetti A. (2009) “Seismic assessment of masonry arch bridges”, Engineering Structures, vol. 31, pp.1777-1788.

2. Roca P., Gariup G., Pelà L., Cervera M. (2010) “Structural Analysis of Masonry Historical Constructions”, Classical and Advanced Approaches, Archives of Computational Methods in Engineering, vol.17, pp.299–325.

3. Benedetti A., Pelà L., (2012) “Experimental characterization of mortar by testing on small specimens”, Proc. 15th International Brick and Block Masonry Conference, Florianopolis, Brazil, June 2012.

4. Hilsdorf H.K., “Investigation into the failure mechanism of brick masonry loaded in axial compression” Designing, engineering and constructing with masonry products, Gulf Publishing Company, 1969, pp. 34-41.

5. Magenes G., Calvi, (1997), "In plane seismic response of brick masonry walls", Earthquake Engineering and Structural Dynamics, vol. 26, pp. 1091-1112.

6. Heyman, J., (1962), "The stone skeleton", Cambridge Univ. Press, Cambridge, UK.

7. Di Pasqale S., (1992), "New Trends in the Analysis of Masonry Structures", Meccanica, vol. 27-3, pp. 173-184.

8. Benedetti A., Steli E., "Analytical models for shear–displacement curves of unreinforced and FRP reinforced masonry panels", Construction and Building Materials, Volume 22, Issue 3, March 2008, pp. 175-185.

9. Roca P., (2006), "Assessment of masonry shear-walls by simple equilibrium models", Construction and Building Materials, vol. 20, pp. 229–238.

10. Heyman J., (1992), "Leaning Towers", Meccanica, vol. 27-3, pp. 153-159.

11. Aprile A., Benedetti A., Grassucci F., (2001), "Assessment of Cracking And Collapse For Old Brick Masonry Columns" ASCE Jour. of Struct. Engnrg, , vol. 127-12, pp. 1427–1435.

12. Turnšek V., Cacovic F., (1971): "Some experimental results on the strength of brick masonry walls", Proc. of the 2nd Intern. Brick Masonry Conference, Stoke-on-Trent, pp. 149-156.