SPATIAL VARIABILITY OF BOND STRENGTH AND STOCHASTIC STRENGTH PREDICTION OF UNREINFORCED MASONRY WALLS IN VERTICAL BENDING

J. Li[1], M. J. Masia[2], M. G. Stewart[3], S.J. Lawrence[4] and L. M. Heffler

[1] PhD Student, Centre for Infrastructure Performance and Reliability, The University of Newcastle, NSW, 2308, Australia c3124120@uon.edu.au

[2] Associate Professor, Centre for Infrastructure Performance and Reliability, The University of Newcastle, NSW, 2308, Australia Mark.Masia@newcastle.edu.au

[3] Professor and Director, Centre for Infrastructure Performance and Reliability, The University of Newcastle, NSW, 2308, Australia Mark.Stewart@newcastle.edu.au

[4] Conjoint Professor, Centre for Infrastructure Performance and Reliability, The University of Newcastle, NSW, 2308, Australia spl@bigpond.net.au

ABSTRACT

The flexural bond strength of unreinforced masonry (URM) is a key material property affecting the wall out-of-plane lateral load capacity. It is well known that the unit flexural bond strength (defined here as the flexural strength of the brick to lower mortar bed joint associated with any given masonry unit (brick)) varies considerably between units, and that this spatial variability might significantly affect the structural performance and reliability of URM walls in flexure. The paper develops a computational method to predict the strength for URM walls subject to one-way vertical bending considering unit-to-unit spatial variability of flexural bond strength. We characterise the probability distributions of wall strength and examine how spatial variability in unit flexural bond strength affects the variability of first cracking load, second cracking load and peak load and behaviour of clay brick URM walls. This is done using 3-D nonlinear Finite Element Analyses and stochastic analysis in the form of Monte Carlo simulations. Varying COVs (0.1, 0.3 and 0.5) of unit bond strength are considered. The mean and variance of wall strength are estimated to show the effect of spatial variability of bond strength on wall strength.

KEYWORDS: stochastic, masonry, bond strength, spatial variability, structural reliability

INTRODUCTION

The Australian masonry design code (AS3700-2011) [1] has been in a limit states format since 1988. Although it is commonly believed that current design models are conservative, the actual level of safety of masonry structures is not known. It is unclear how to compare the structures designed according to the masonry design codes with the structures designed using other materials in terms of reliability (or safety) and whether different masonry walls and other structural elements have similar levels of reliability. The problem is compounded by the fact that the strength properties of masonry are highly variable, particularly for the unit-to-unit spatial variability of flexural bond strength, due to variations in the quality of workmanship, the weather during construction, and the materials from location to location, all within one structure. However, many existing stochastic analyses of structures assume uniform bond strength in the wall, rather than considering the unit-to-unit spatial variability of bond strength, the latter being a more realistic approach in examining material variability. In fact, the existence and importance of spatial variability in the masonry wall have been observed in past studies. For instance, [2] discussed the effects of random variation in the flexural strength of brick work as early as 1976. Then [3] and [4] stressed the importance of considering this factor and used Monte Carlo techniques to model its effects in analysis. [5] suggested that assuming statistical independence of individual unit strengths provides wall capacities consistent with experimental results for vertical one-way bending in 1991. [6] measured the statistical parameters of bond strengths by the bond wrench test on 19 building sites in Melbourne. These genuine data provided strong evidence for the high spatial variability in flexural bond strength. The unit-to-unit spatial variability in flexural bond strength was also considered in the masonry reliability analysis by [7]. [8] estimated the characteristic masonry strength by taking into account the unit compressive strength when doing the masonry analysis calculations, but only in the form of the measured mean and standard deviation of unit strength rather than unit-to-unit spatial variability. In recent studies, [9, 10] examined experimentally the extent of spatial correlation between unit flexural bond strengths within clay brick walls and recommended that each unit has a flexural bond strength that is statistically independent of its neighbours. In addition, random field analyses were used to model the performance of geotechnical and structural systems where such systems are subject to spatially varying parameters (e.g., [11]) and [12] successfully used random field analysis to model the spatial variability of corrosion damage to concrete structures. This, undoubtedly, increases the importance, feasibility and rationality of considering the spatial variability of bond strength in masonry analyses.

While there have been a number of studies of the effects of variability and workmanship on the strength of structural masonry [2, 13-16], very few studies have considered computational methods to calculate the structural reliability of masonry structures. However, [7] and [17] developed preliminary 'proof-of-concept' techniques to estimate the structural reliability of masonry walls for vertical one-way bending and compression loading. [7] developed a structural reliability model to calculate the probability of failure for masonry walls in flexure, considering the unit-to-unit variability of bond strength, and showed the important effect that the unit-to-unit spatial variability could have on strength prediction and structural reliability.

The current paper presents a computational method, using 3-D nonlinear Finite Element Analyses and stochastic analysis in the form of Monte Carlo simulations, to calculate the strength prediction and structural reliability for URM walls subjected to one-way vertical bending. It provides statistical evidence to illustrate the significant importance of considering the unit-to-unit spatial variability of flexural bond strength by comparing the probability distributions obtained from non-spatial and spatial analysis, for the first cracking load (the load at which cracking first occurs in the wall), the second cracking load (the load at which tensile cracking appears in the midheight region of the wall) and the peak load.

PROBABILISTIC MODEL

A probabilistic model is generated before the establishment of non-spatial and spatial analysis models. In this section, a 3-D non-linear FEA model of a full sized, single leaf clay brick URM wall of dimensions 2.5 m × 2.5 m is generated. This sized wall was chosen because it is realistic for both the height and length of the wall, and also, based on the context of assuming the load redistribution system hypothesis of failure [7], a 2.5 m long wall may have potentially a greater chance of a weak joint at which cracking could initiate compared to a wall with shorter length, as there are more joints across the length of the wall. The same principle holds true in the height direction.

The structural configuration considered here is a single skin infill panel simply supported at the top and bonded at the bottom (one-way vertical bending). The boundary conditions for the full sized wall model are the same as in many practical situations. That is, the bottom course of the wall is bonded by a layer of mortar to an underlying floor or footing, and at the top, the wall is restricted from lateral (out-of-plane) displacement but can move in the vertical direction. A uniform lateral pressure load is applied over the full face of the wall.

The interface material model used herein is the combined cracking-shearing-crushing model [18], also known as the composite interface model. This composite interface model is appropriate to simulate fracture, frictional slip as well as crushing along material interfaces [18]. The brick units are modelled as linear elastic, while the mortar joints are modelled with interface elements, which obey the nonlinear behaviour described by this combined cracking-shearing-crushing model [19, 20].

The detail of material parameters to be used in the 3-D FEA analysis of the full wall is listed in Table 1. The element types and mesh density are summarized in Table 2.

Table 1: Summary of Material Parameters to be used in the 3-D FEA Model

Horizontal and vertical mortar joint interface elements	linear normal stiffness modulus	353 N/mm^3
	linear tangential stiffness modulus	146 N/mm^3
	tensile bond strength	variable
	tensile fracture energy	variable
	cohesion	0.65 N/mm^3
	tangent friction angle	0.75
	tangent dilatancy angle	0.6
	tangent residual friction angle	0.75
	confining normal stress	-1.2 N/mm^2
	exponential degradation coefficient	5
	capped critical compressive strength	20 N/mm^2
	shear traction control factor	9
	compressive fracture energy	15 N/mm
	equivalent plastic relative displacement	0.12
	shear fracture energy factor	0.15
Expanded brick elements	brick young's modulus	20000 N/mm^2
	brick's Poisson's ratio	0.15
	brick density	1800 kg/m^3
Potential brick cracks (all values are artificially high to force cracking in mortar joints and not brick joints)	linear normal stiffness modulus	1000 N/mm^3
	tangential normal stiff modulus	1000 N/mm^3
	tensile strength	2 N/mm^2
	fracture energy	0.5 N/mm

In this probabilistic model, the fracture energy is related to the tensile strength by the following expression [21].

$$G_{I}^{f} = 0.01571 \times f_t + 0.0004882 \quad (1)$$

Table 2: Summary of 3-D FEA Element Type and Mesh Selection for the Full Sized Wall

Brick/Mortar Bodies \ Element/Mesh Selection	Element Types	Mesh Density
Bricks	HE20 CHX60	2×4×1
Mortar Joints	IS88 CQ48I	2×4×1
Mid-length Brick Interface Element	IS88 CQ48I	1×4×1

For a large data set, Normal, Lognormal and Weibull distributions can be used to represent brickwork properties [22]. [21] found that the distribution of unit flexural bond strengths for full sized clay brick URM walls is best represented by the truncated Normal probability distribution and this was adopted for the current study. Based upon an extensive literature review, a mean flexural bond strength of 0.4 MPa was selected. The choices of COV being 0.1, 0.3 and 0.5 encompass those reported in [6] for 19 building sites in Melbourne, and are consistent with the observations of [21].

NON-SPATIAL ANALYSIS MODEL

The non-spatial analysis model is generated making use of the 3-D non-linear FEA full wall model and Monte-Carlo computer simulation techniques. Non-spatial analysis is the scenario considering a stochastic analysis with the full sized wall having non-spatially varying unit flexural bond strength for each realisation. It means the bond strength is identical for each unit in the wall. Many existing stochastic analyses of structures assume this scenario.

The main procedure for non-spatial analysis model generation is as follows. ① Establish the expressions describing the relationships between the masonry unit flexural bond strength and the wall cracking loads, and between the masonry unit flexural bond strength and the wall peak loads, respectively; ② Generate a random variable as the wall unit flexural bond strength, following a truncated Normal distribution, and find the corresponding cracking loads and peak load of the wall using the best fit curve interpolation of the non-spatial runs; ③ Repeat step ② for 100,000 runs for COV = 0.1, 0.3 and 0.5, respectively; ④ Produce the histograms and distributions of wall cracking loads and peak loads for each COV of bond strengths. These are presented in Figures 2 and 3.

SPATIAL ANALYSIS MODEL

Spatial analysis is the scenario considering a stochastic analysis with the full sized wall having spatially varying unit flexural bond strengths, assuming that there is no spatial correlation in bond strength existing between each unit in the wall.

150 FEA simulations for each COV are deemed as the reasonable number based on the convergence check of peak load and COV (see Figure 1). The model considering spatial variability of unit flexural bond strength (μ_{ft} = 0.4 MPa, COV = 0.1, 0.3 and 0.5) is obtained in the following way. ① Generate a set of random numbers, as the tensile bond strengths of the masonry, following the truncated Normal probability distribution; ② Assign the strengths, and the associated fracture energy values according to Equation (1), to horizontal (bed) and vertical

(perpend) mortar joints in the FEA model. In this process a unique strength is assigned to the mortar bed joint along the complete length of each masonry unit (brick) and a unique strength is assigned to the mortar perpend joint over the complete height of each unit; ③ Run the FEA model and collect its failure progress (i.e., the first cracking load, second cracking load, peak load, and load versus deflection curve); ④ Repeat steps ①, ② and ③ for 150 runs. The resulting histograms, distributions and load versus deflection curves are shown in Figures 2, 3 and 5.

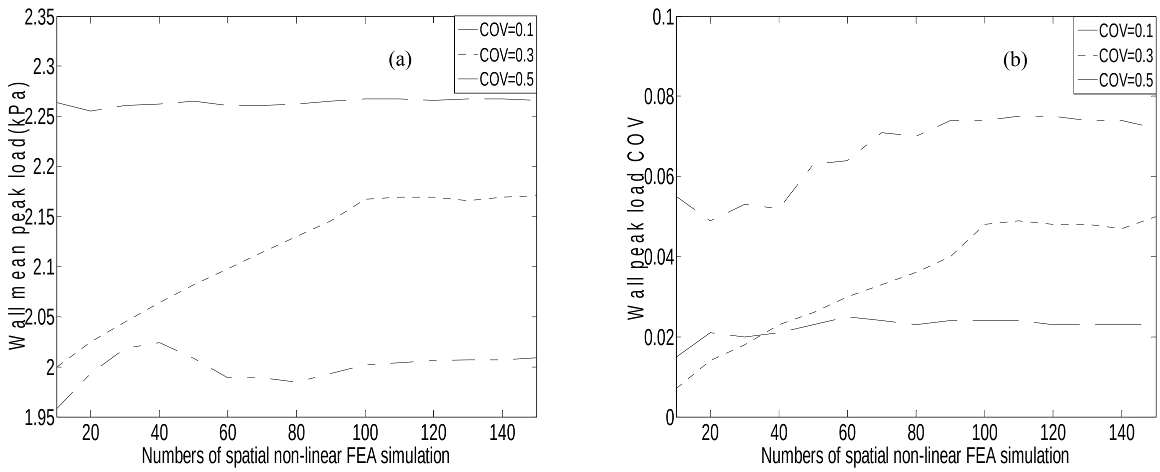

Figure 1: Convergence: a) Convergence of mean peak load with increased number of simulations; b) Convergence of peak load COV with increased number of simulations

COMPARISON OF NON-SPATIAL AND SPATIAL ANALYSES

The results, including the first cracking, second cracking and peak loads obtained from non-spatial and spatial analysis models are compared for the case of mean of 0.4 MPa and COV of 0.3 in Figure 2. Table 3 shows that the mean values and standard deviations without considering spatial variability of unit bond strength (Non-spatial) are 59%, 70%, 4.4% higher, and 50%, 60%, 77% higher than those obtained from a spatial analysis for first cracking, second cracking and peak loads, respectively.

Table 3: Summary of First Cracking, Second Cracking and Peak Loads for Non-Spatial and Spatial Analyses μ_{ft} = 0.4 MPa, COV = 0.3

μ_{ft} = 0.4 MPa COV = 0.3	Non-Spatial Analysis			Spatial Analysis		
	Mean (kPa)	σ (kPa)	COV	Mean (kPa)	σ (kPa)	COV
First Cracking Load	1.24	0.34	0.27	0.51	0.17	0.33
Second Cracking Load	1.97	0.55	0.28	0.59	0.22	0.38
Peak Load	2.27	0.48	0.21	2.17	0.11	0.05

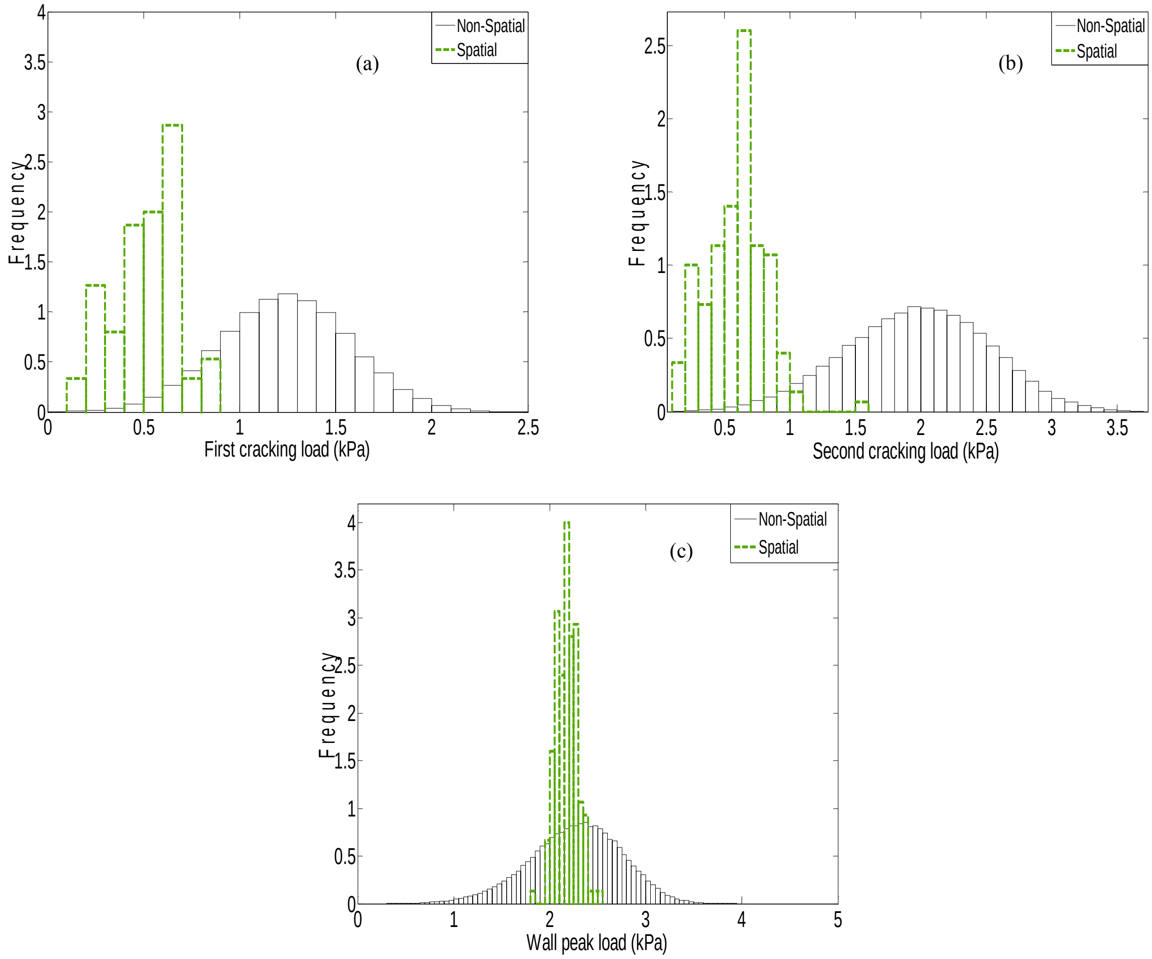

Figure 2: Histograms: a) First Cracking Loads; b) Second Cracking Loads; c) Peak Loads for Non-Spatial and Spatial Wall Simulations μ_{ft} = 0.4 MPa, COV = 0.3

This observation is intuitive for the first cracking load. The first cracking load is the load at which cracking first occurs anywhere in the wall, so this will occur at a lower wall pressure load for the spatial case due to the presence of units of lower than average strength than in the non-spatial case for which all units have equal strength. When a pressure load is applied to the wall, the maximum moment in the wall will develop at the base of the wall initially. For the wall with a uniform (Non-spatial) unit tensile strength, the first cracking would always appear in the bottom course of masonry. While for the wall with unit-to-unit spatial variability, both the moment and the random presence of lower than average unit tensile strengths will determine the location that the first cracking occurs (weakest link theory [7]). According to the statistics from all the spatial analysis simulations, only 38.7% of first cracking occurs in the bottom course, the remaining 61.3% of first cracking takes place elsewhere before the cracking appears in the bottom course due to the presence of "weak" joints in the wall, resulting in lower first cracking loads, on average, than for the non-spatial simulations. As the first cracking loads in the spatial analysis are influenced by the weaker bonds initiating the first cracking, the standard deviation in the spatial analysis is also lower than in the non-spatial. The same principle holds true for the second cracking and peak loads.

It can be seen in Figures 2/c, 3 and Table 4 that ignoring the spatial variability of the unit bond strength (Non-spatial analysis) overestimates the mean strength (peak load) of the wall while at the same time underestimates the wall strength in the lower tail of the histograms in all three cases of COV = 0.1, 0.3 and 0.5. The degree of overestimation of mean and underestimation in the lower tail becomes larger when the COV of unit tensile strength increases. As it is the lower tail of the histograms which are of most interest during masonry structural design and structural reliability analysis, the non-spatial scenario will over design the structure, particularly for large values of COV.

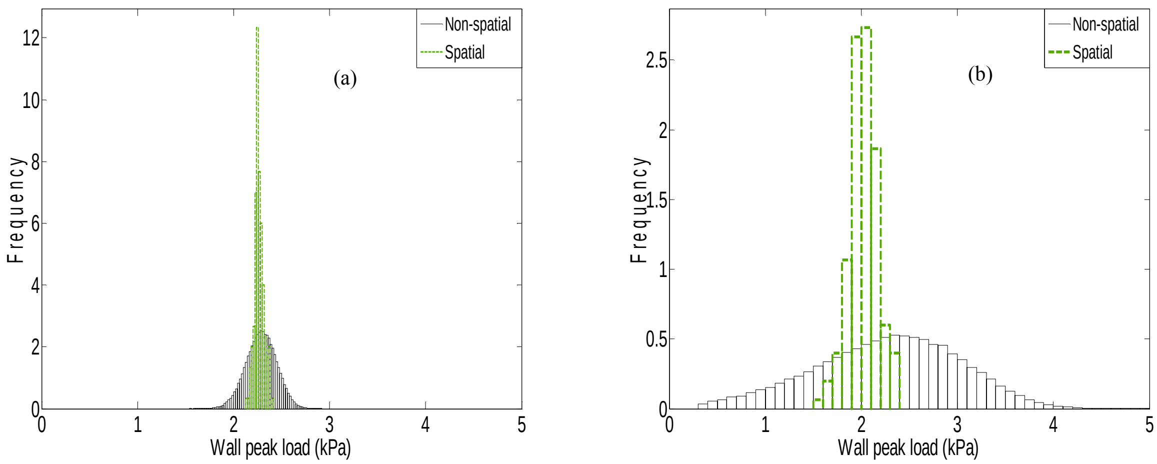

Figure 3: Histograms of Peak Loads for Non-Spatial and Spatial Wall Simulations μ_{ft} = 0.4 MPa: a) COV = 0.1; b) COV = 0.5

Table 4: Summary of Peak Loads for Non-Spatial and Spatial Analyses

Bond Strength		Non-Spatial Wall Peak Loads Analysis					Spatial Wall Peak Loads Analysis				
Mean (MPa)	COV	Mean (kPa)	Min (kPa)	Max (kPa)	σ (kPa)	COV	Mean (kPa)	Min (kPa)	Max (kPa)	σ (kPa)	COV
0.4	0.1	2.29	1.55	2.95	0.16	0.07	2.27	2.13	2.37	0.05	0.02
	0.3	2.27	0.34	3.95	0.48	0.21	2.17	1.81	2.51	0.11	0.05
	0.5	2.27	0.34	4.76	0.75	0.33	2.01	1.52	2.32	0.14	0.07

FAILURE MODE COMPARISON

The failure behaviours of the wall for non-spatial and spatial analysis also differ greatly. Figure 4 shows two examples of failure behaviours classified by the crack opening width, ignoring widths below 0.003 mm. Examining a propped cantilever in linear beam theory, the bending moment at the fixed end (the bottom in current case) is greater than the bending moment in the middle area. Therefore, when a pressure load is applied to the wall the maximum moment in the wall will develop at the base of the wall. For non-spatial (uniform) tensile strength, it is expected that the course at the base of the wall will crack first. For the non-linear finite element analyses used in the current study, the post peak strength associated with the mortar joints allows the joint to continue resisting moment as it softens during which the moment is redistributed to the midheight region of the wall and the total load on the wall is able to further increase. When the moment in the wall in the midheight region reaches the moment capacity of the wall cross section, cracking occurs at midheight (second cracking) typically over multiple courses (Figure 4a). For the majority of simulations conducted during the current study second cracking occurred prior to peak load but for some of the non-spatial analysis with high unit flexural bond strength, the peak load occurred prior to second cracking with second cracking occurring during the post peak branch of the load versus wall displacement response. For the chosen example with unit-to-unit spatial variability, the first cracking occurs in the "weakest link" where the ratio of moment to moment capacity (or tensile stress to tensile strength) is greatest at a unit located in the 11th course from the bottom of the wall. However, the crack does not develop as the adjacent units are strong enough to bear the redistributed load. Then the weaker units in the bottom course (where the moment is greater) begin to crack. Before the cracking extends completely across the bottom course, cracking in the 16th course from the bottom of the wall and in adjacent courses appear before failure.

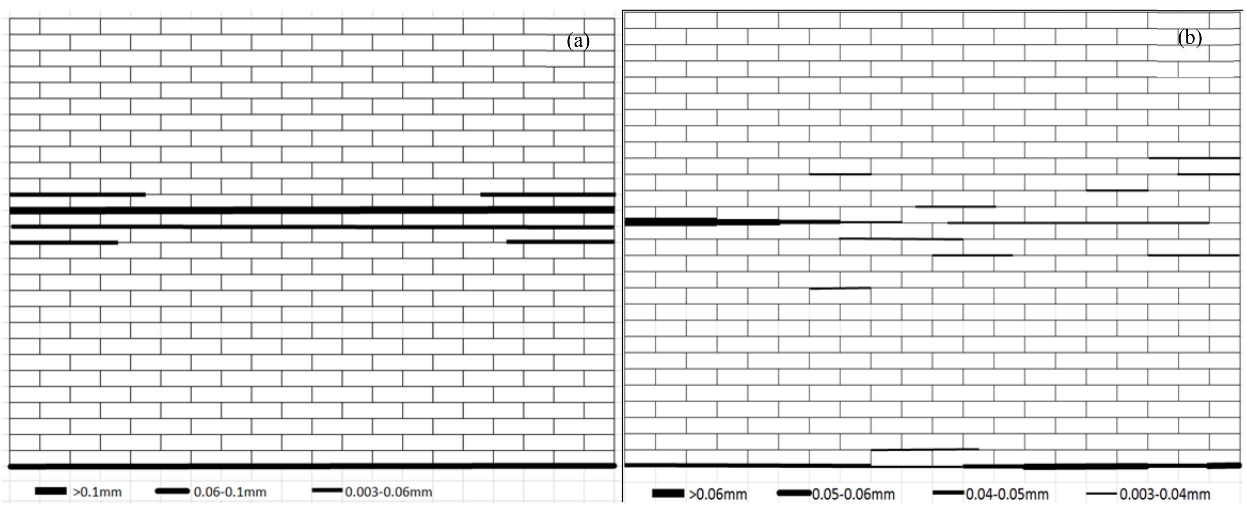

Figure 4: Failure Mode for Non-Spatial and Spatial Bond Strength: a) Non-Spatial Bond Strength f_t = 0.4 MPa; b) an Example with Spatial Bond Strength (μ_{ft} = 0.4 MPa COV = 0.3)

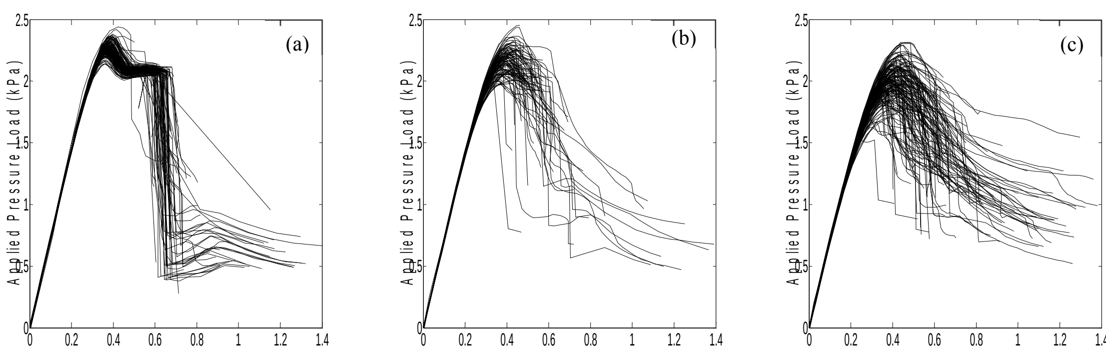

Figure 5: Load Versus Deflections for Spatial Wall Simulations μ_{ft} = 0.4 MPa: a) COV = 0.1; b) COV = 0.3; c) COV = 0.5

The load versus deflection curves also indicate the progression of wall failures. Figure 5 shows the all load versus deflection curves for the spatial analyses for μ_{ft} = 0.4 MPa and COV = 0.1, 0.3 and 0.5. None of the curves are the same and they overlap much considering the unit-to-unit spatial variability.

CONCLUSION

A computational method and probabilistic model have been developed to calculate the strength prediction for URM walls subject to one-way vertical bending. The probabilistic information in this model is unit-to-unit spatial variability of flexural bond strength. A comparison of the first cracking loads, second cracking loads and peak loads between spatial and non-spatial analysis has been made, showing the importance of considering spatial variability in structural analysis for URM walls. It has been found that a non-spatial analysis overestimates the first cracking and second cracking loads of the wall, and underestimates the peak load of the wall in the lower tail of the distribution of wall strengths compared with a spatial analysis. Also, the failure modes of the wall are more realistic for the spatial analysis.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support of the Australian Research Council via Linkage Project LP0669538 as well as technical assistance provided by Doctor Robert Peterson.

REFERENCES

1. AS3700. (2011) Australian Standard, in Masonry Structures, Sydney.
2. Baker, L.R. and Franken, G.L. (1976) “Variability Aspects of the Flexural Strength of Brickwork” 4th International Brick Masonry Conference, Brussels.
3. Baker, L.R. (1981) “The flexural action of masonry structures under lateral load”, Deakin University.
4. Lawrence, S.J. (1983) “Behaviour of brick masonry walls under lateral loading” University of New South Wales.
5. Lawrence, S.J. (1991) “Stochastic Analysis of Masonry Structures” Computer Methods in Structural Masonry.
6. McNeilly, T., Scrivener, J., Lawrence, S.J. and Zsembery, S. (1996) “A Site Survey of Masonry Bond Strength” Australian Civil/Structural Engineering Transactions, IEAust.
7. Stewart, M.G. and Lawrence, S.J. (2002) “Structural reliability of masonry walls in flexure” Masonry International, 5(2):48-52.
8. Stewart, M.G. and Lawrence, S.J. (2007) “Model Error, Structure Reliability and Partial Safety Factors for Structural Masonry in Compression” Masonry International, 20(3):107-116.
9. Heffler, L.M., Stewart, M.G., Masia, M.J. and Corrêa, M.R.S. (2008) “Statistical analysis and spatial correlation of flexural bond strength for masonry walls” Masonry International, 21(2):59-70.
10. Corrêa, M.R.S., Masia, M.J., Stewart, M.G. and Heffler, L.M. (2012) “An experimental and statistical analysis of the flexural bond strength for masonry walls”. Australian Journal of Structural Engineering, 13(2):139-148.

11. Der Kiureghian, A. and Ke, B.-J. (1988) “The Stochastic Finite Element Method in Structural Reliability” Probabilistic Engineering Mechanics.
12. Vu, K.A.T. and Stewart, M.G. (2005) “Predicting the Likelihood and Extent of RC Corrosion-Induced Cracking” Journal of Structural Engineering, ASCE, 131(11)1681-1689.
13. Lawrence, S.J. and Lu, J.P. (1991) “Cracking of Brickwork Walls with Lateral Loading” Proceedings of the Asia-Pacific Masonry Conference, Singapore.
14. Lawrence, S.J. and Cao, .T. (1988) “Cracking of Non-loadbearing Masonry Walls Under Lateral Forces” 8th International Brick Masonry Conference, Dublin.
15. Lawrence, S.J. (1978) “The Flexural Behaviour of Brickwork” North American Masonry Conference, Colorado.
16. Fyfe, A.G., Middleton, J. and Pande, G.N. (2000) “Numerical Evaluation of the Influence of Some Workmanship Defects on the Partial Factor of Safety for Masonry” Masonry International, 13(2):48-53.
17. Drake, B., Stewart, M.G. and Lawrence, S.J. (2004) “Variability and Structural Reliability of Structural Masonry in Compression” 7th Australasian Masonry Conference, Newcastle.
18. Hendriks, M. and Wolters, B. (2007) “Diana Finite Element User’s Manual” Release 9.2. TNO DIANA BV, P.O. Box 49, 2600 AA Delft, The Netherlands.
19. Lourenco, P.B. and Rots, J.G. (1997) “A multi-surface interface model for the analysis of masonry structures” Journal of Structural Engineering, ASCE, 123(7):660-668.
20. Van Zijl, G.P.A.G. (2000) “Computational Modelling of Masonry Creep and Shrinkage” Delft University of Technology.
21. Heffler, L.M. (2009) “Variability of Unit Flexural Bond Strength and its Effect on Strength in Clay Brick Unreinforced Masonry Walls Subject to Vertical Bending” Faculty of Engineering and Built Environment, University of Newcastle, NSW, Australia.
22. Lawrence, S.J. (1985) “Random Variations in Brickwork Properties” 7th International Brick Masonry Conference, Melbourne.