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SIZE EFFECT IN VERTICALLY SPANNING UNREINFORCED MASONRY WALLS

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ABSTRACT

The flexural design of sections of masonry spanning vertically with simple supports at top and bottom is usually carried out using simple bending theory and the flexural tensile strength of the material. Simple masonry walls behave in a brittle manner and the flexural strength is known to be highly variable. Simple stochastic analysis can be used to provide an improved behaviour model. The principles of this analysis are outlined. A database of wall test results gathered from around the world is used to examine the 'model error' in the simple bending model and shows that there is clear evidence of a size effect, resulting in longer and higher walls having lower strength. It is shown that adjustment of predicted wall strength by the simple stochastic analysis can greatly reduce this bias, producing more consistent estimates of wall strength across a range of lengths and heights.

KEYWORDS: unreinforced masonry; lateral loading; walls; stochastic analysis; design; size effects

INTRODUCTION

Unreinforced masonry subjected to flexure behaves in a brittle way, such that the material resistance is lost locally when a crack occurs. Some structural arrangements will have the capacity to redistribute stresses and display a pseudo-plastic behaviour, for example laterally loaded walls subjected to two-way bending, where considerable resistance can be present after first cracking [1]. However, not all structures have this capability, and this paper deals with sections of masonry that are subjected to bending with simple supports at the top and bottom edges only (referred to as vertical bending). These structures typically display very brittle behaviour, with failure under sustained load occurring immediately after the formation of a crack in a bed joint.

It is well known that masonry properties are highly variable and statistical parameters for some properties have been discussed previously [2]. It has been shown [3] that the bond strengths of individual masonry units are not spatially correlated in a wall, and bond strength is therefore treated in this paper as a random variable throughout the wall. This variability can mean that conventional deterministic analyses are not accurate and often over-estimate the strength of the structure. The variability can be taken into account in some instances by a simple stochastic analysis [4, 5]. This paper builds on previous work by applying a stochastic analysis to a larger database of wall test results, drawn from around the world. It is part of a broader investigation of the flexural behaviour of masonry [6, 7, 8, 9, 10] and reliability of masonry design [11, 12].

Design of simple vertically-spanning sections of masonry is usually carried out using the simple bending model and the flexural tensile strength of the material. For example, the Australian Masonry Structures code [13] calculates a moment of resistance in vertical bending as the product of a flexural tensile strength (f'_{mt}) and section modulus (Z_d), plus an allowance for any vertical stress from self-weight and applied vertical load (within certain limits). The implicit assumption is that the flexural tensile strength is the same everywhere in the wall, which is clearly not the case. It will be shown that random variation in material strength reduces the wall strength significantly and that this should be taken into account in design.

Work of this nature has been hampered in the past by the lack of a consistent measure of the flexural tensile strength of masonry, free of self-weight effects. The bond wrench test, developed in Australia, is becoming increasingly commonly used and provides such a measure. While the exact configuration of the test varies in different parts of the world, such differences are relatively small compared with the effect of ignoring strength variability when analysing a wall. The use of a proxy flexural test such as the wallette developed in the United Kingdom and used in Eurocode 6 [14], which is influenced by self-weight and masks the effect of strength variability, makes precise analysis difficult and therefore hinders the development of improved design methods that are based on rational understanding rather than being empirical.

SIMPLE STOCHASTIC ANALYSIS

In carrying out stochastic analysis of masonry in flexure, each masonry unit is considered to be a single element, bonded to the surrounding masonry through its bed joint. Each unit is placed independently by the bricklayer and, for the purposes of analysis, is considered to have an individual value of bond strength. No consideration is given to any variation of strength within the bed area of the unit as it would have no bearing on the results. In other words, the population considered is that of the flexural tensile strength of individual units, as measured by the bond wrench.

In considering the behaviour of a structural element such as a vertically spanning wall, two concepts can be applied – a series type of behaviour, or weakest-link approach, and a parallel type of behaviour, or bundle-of-threads approach. In the first, failure of a unit results in failure of the structural element; in the second, failure of a unit allows the possibility of stresses to be distributed to surrounding elements, with the possibility of further incremental failures until the resistance falls sufficiently that the structural element as a whole will fail. For reasons of space and simplicity, the second concept is not applied in this paper; the analysis uses only the first concept.

To analyze a simple vertically-spanning section of masonry wall, it is considered as comprising a number of courses, with each course having a random strength sampled from a population of course strengths. The first step in the analysis is to derive the statistical parameters for the population of course strengths. For the present purposes, this is done by using a weakest-link approach, that is, by assuming that the strength of a course is the same as the strength of the weakest individual masonry unit in it. This is achieved by the use of order statistics, for which tables are available [15] giving the weakest of a sample taken from a Normally distributed population. For this analysis, the effect of any variation of the form of distribution from the Normal is expected to be small and the Normal distribution is therefore used.

By order statistics, the strength of the weakest unit f_{mt}^{*} is predicted to be given by Equation 1:

$$f_{mt}^{*} = f_{mt} - K_l \sigma_j \quad (1)$$

Where f_{mt} is the mean strength of individual units, σ_j is the standard deviation of individual unit strengths and K_l is a factor from order statistics. As an example, for a course length of ten units, K_l = 1.54 [15].

To predict wall strength from the parameters of the distribution of course strengths, a probabilistic failure analysis is used, as described previously [5]. This analysis takes account of the number of courses in the wall, the type of loading (i.e. shape of the bending moment diagram) and the self-weight. It calculates a distribution of wall strengths from the distribution of course strengths, allowing for failure of the course with the most adverse combination of bending moment and strength. The use of probabilistic failure analysis is given by Equation 2:

$$f_{mt}^{**} = K_h \, f_{mt}^{*} \quad (2)$$

Where f_{mt}^{**} is the mean wall strength, f_{mt}^{*} is the mean course strength from Equation 1, and K_h is a factor derived from the probabilistic failure analysis.

The two types of analysis – calculating modified statistics for the strength of courses from the parameters of the individual unit strengths and calculating statistics for the strength of walls from the parameters for course strengths – are independent and can be applied sequentially. However, when both are applied, it is necessary to apply the analysis to determine course strength before the analysis to allow for the number of courses. Both analyses are applied in this paper, individually and jointly, to a large database of wall test results.

DATABASE OF WALL TEST RESULTS

The relationship between the strengths of vertically spanning walls and the flexural strength of the masonry has been examined by compiling a database of test results drawn from around the world. For test results to be considered relevant, they were required to be from tests on unreinforced masonry, have sufficient details of support and loading configurations as well as dimensions of units and the wall, and to have no secondary influences such as arching between supports or render applied to the wall. Furthermore, it was essential that the wall strength be accompanied by reporting of a simple measure of flexural strength, free of self-weight effects, either from individual unit bond strengths measured by the bond wrench, or from the strengths of simple stack-bonded beams that could be used to derive the parameters of the bond strength distribution for individual units. Unfortunately, there are many reports of tests that do not provide this essential information and are therefore of no use for this exercise.

A total of 114 wall test results for clay and concrete masonry were sourced from 13 research reports [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. For each of the test results, the wall strength was predicted by the simple bending model and expressed as a dimensionless model error (ME) by dividing the measured strength by the predicted strength. The data come from tests carried out in Australia, USA and Canada, and an initial assessment of the data (not presented here because of space limitations) showed that there is no significant trend related to the country of origin. All data were therefore considered together, although they were divided into two groups based on material – clay (102 test results) and concrete (12 test results). Plotting of the aggregated model error against length (Figure 1) and against height (Figure 2) shows evidence of ME reducing with increase in both length and height, particularly for the clay masonry walls. This indicates that there are reductions in wall strength for both length and height of the wall, which are not taken into account in the simple bending model.

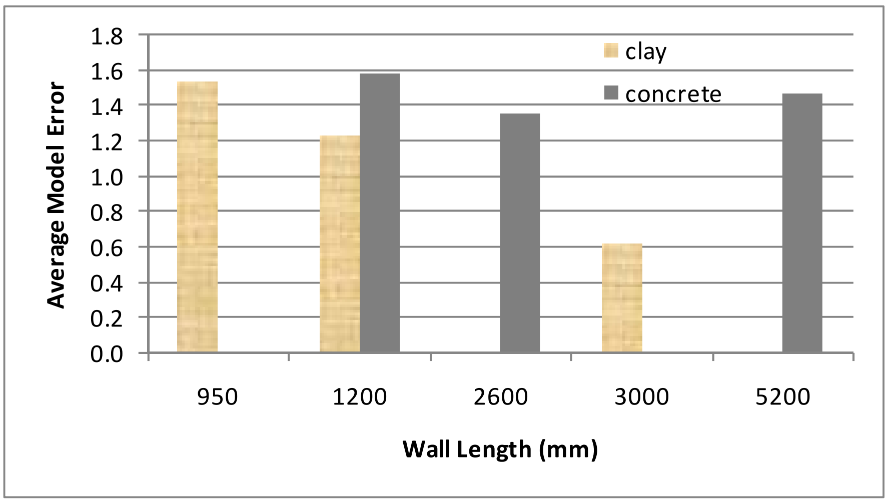

Figure 1: Average Model Error versus Length

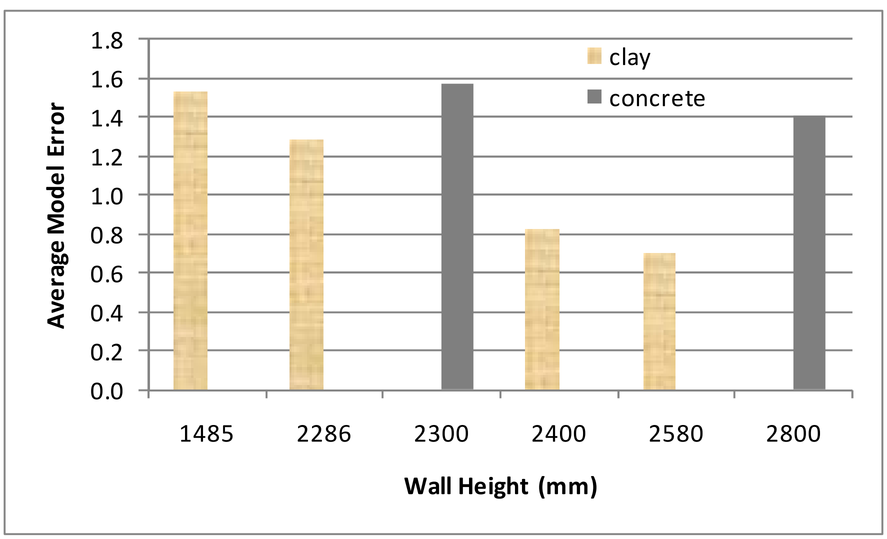

Figure 2: Average Model Error versus Height

The aim of the present analysis is to use a stochastic approach to develop an improved model that reduces this bias in the predicted strengths produced by the simple bending model.

ADJUSTMENT FOR WALL LENGTH

When the predicted wall strength is adjusted for length, using the order statistics factor for the weakest unit in the course (Equation 1), the resulting model errors are as shown in Figure 3. There is an increase in ME overall, but for the model to be an improvement, it should demonstrate a reduction in the downward trend or bias with increasing length. This can be expressed as a variation from the mean; as a way of measuring this, the coefficient of variation (CV) of the average model error for each wall length is used.

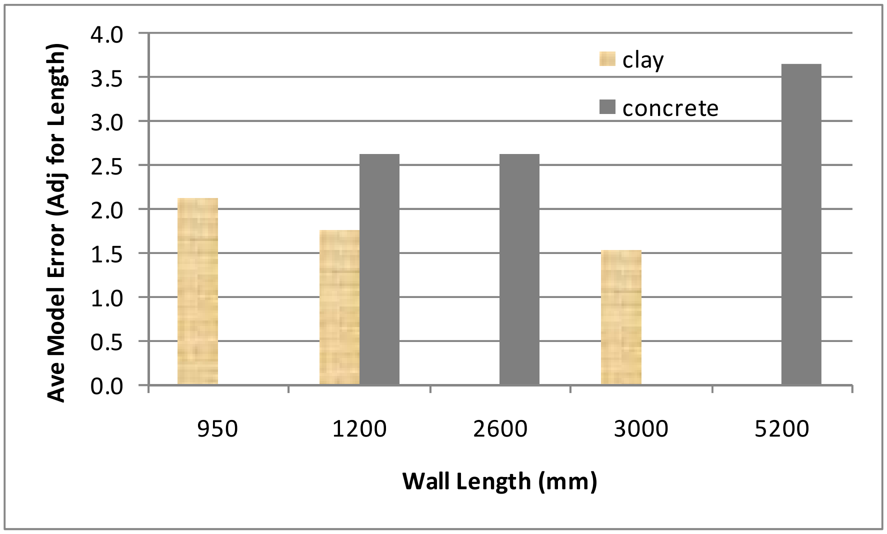

Figure 3: Model Error Adjusted for Length

Table 1 shows, for clay and concrete wall tests, the average ME for each wall length, with the mean, standard deviation (SD) and CV of ME. The base case (without adjustment) is shown and, for comparison, the statistics of ME for the case where the predicted wall strength is adjusted for length (as shown in Figure 3). The variability of ME from the mean, as measured by CV, is markedly reduced for the clay walls (0.17 compared with 0.42) and somewhat increased for the concrete walls (0.20 compared with 0.08). This indicates that the bias in ME related to the length of the wall has been reduced, at least in the case of the clay masonry walls. Note that there are many more test results for clay than for concrete (102 as against 12 walls), so the results for clay must be accorded much greater statistical significance than those for concrete. It is also worth noting that, after adjustment, the CV for concrete walls is approximately the same as that for the clay walls.

ADJUSTMENT FOR WALL HEIGHT

A similar process can be followed to adjust the data for the bias related to the height of the wall. When the predicted wall strength is adjusted for height, using the probabilistic analysis factor for the number of courses in the wall (Equation 2), the resulting model errors are as shown in Figure 4. For comparison, and to avoid distorting the statistics, ME is plotted against length, although each wall test result has been corrected for height.

Table 1: Model Error versus Length

	Base Case		Adjusted for Length Bias	
Length (mm)	**ME Clay**	**ME Concrete**	**ME Clay**	**ME Concrete**
950	1.535		2.136	
1200	1.229	1.579	1.755	2.639
2600		1.360		2.633
3000	0.617		1.547	
5200		1.467		3.651
Mean	1.127	1.469	1.813	2.974
SD	0.467	0.110	0.299	0.586
CV	0.42	0.08	0.17	0.20

Table 2 shows the ME statistics for the height adjustment case. As for the length adjustment, there is an increase in ME overall and a noticeable decrease in variation from the mean, indicating a reduction in the bias of the behaviour model related to height. In this case, the reduction in CV occurs for both clay (0.36 compared with 0.42) and concrete (0.05 compared with 0.08) walls, but is less marked for the clay walls than was the reduction obtained by adjustment for length.

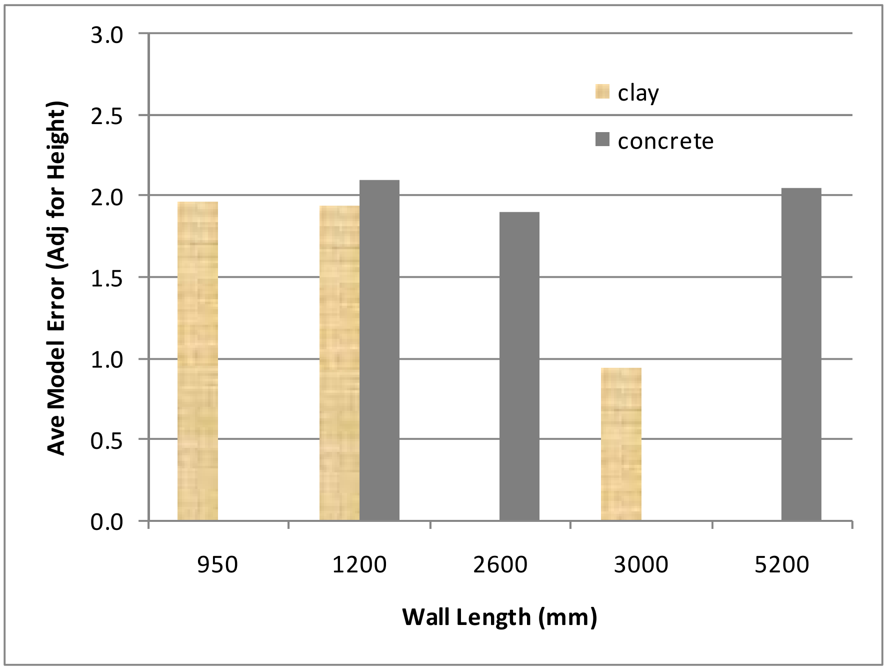

Figure 4: Model Error Adjusted for Height

Table 2: Model Error versus Length (Adjusted for Height Bias)

Length (mm)	ME Clay	ME Concrete
950	1.962	
1200	1.933	2.094
2600		1.903
3000	0.937	
5200		2.053
Mean	1.611	2.017
SD	0.584	0.101
CV	0.36	0.05

ADJUSTMENT FOR BOTH LENGTH AND HEIGHT

When the adjustments for length and height bias are both applied, the resulting model errors are as shown in Figure 5. Table 3 shows the corresponding ME statistics.

The variation from the mean for clay walls (CV = 0.10) is the lowest of any of the cases considered and is markedly lower than the base case (CV = 0.42). This indicates that the bias demonstrated for length and height of the wall has been greatly reduced. While the variation from the mean for the concrete masonry walls (CV = 0.21) is higher than for the base case, this is a much smaller data set than that of clay masonry walls and the results could therefore be expected to be less precise. The increased variation for concrete walls is introduced by the adjustment for length (see above) and seems to be entirely related to the 5200 mm long walls, which comprise only three specimens out of the total data set of 114 walls. It is possible that experimental error in these wall test results has introduced this effect. If these walls were omitted from the database, the variability for the concrete masonry walls would be much lower after adjustment.

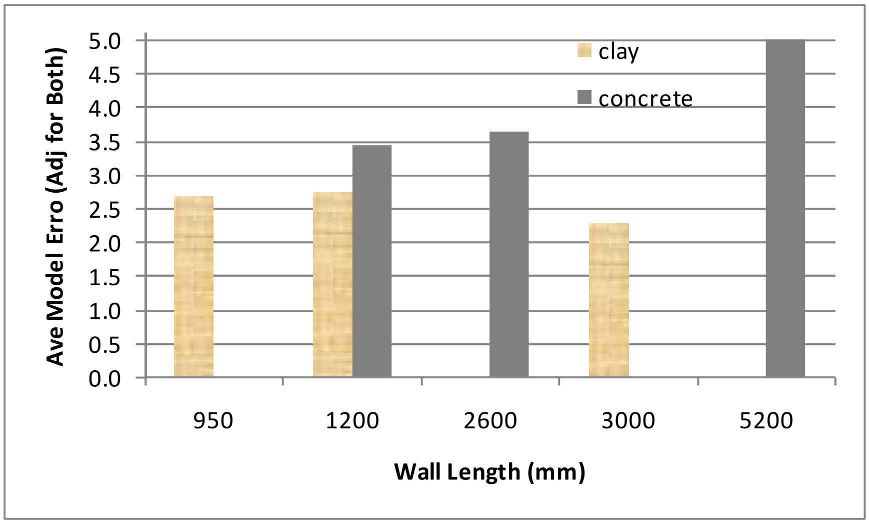

Figure 5: Model Error Adjusted for Both Length and Height

Table 3: Model Error Adjusted for Length and Height Bias

Length (mm)	ME Clay	ME Concrete
950	2.698	
1200	2.740	3.435
2600		3.641
3000	2.279	
5200		5.012
Mean	2.572	4.029
SD	0.255	0.857
CV	0.10	0.21

CONCLUSIONS

What has been presented here is, because of space limitations, a simplified analysis, and no attempt is made to draw definite conclusions or recommend improved design equations. The aim has been to demonstrate that bias exists in the commonly used simple bending model, as a consequence of random variation in bond strength. This bias shows in the form of size effects for both length and height, and can be removed by simple stochastic analysis.

Removal of the bias has the effect of increasing the overall model error, showing that the simple bending model, with adjustment for length and height effects, underestimates the strength of walls. This is likely to be caused by effects occurring between initial cracking and complete failure, such as the partial sharing of strength between adjacent units and progressive development of cracks in the wall (even though failure appears to be instantaneous under sustained load). It might also be the case that end effects and departure from ideal support conditions in the tests has enhanced the measured strength of the walls. Work is continuing [3], in an effort to understand better the behaviour of walls in the phase between first cracking and failure.

Further refinement of the analysis is possible and will be pursued. The overall model error should be taken into account in a reliability analysis, leading to an appropriate partial safety factor (capacity reduction factor) for design. The present analysis demonstrates that using a stochastic approach to the behaviour model can lead to a consistent level of model error across a range of wall length and height. There is an urgent need for more test data to facilitate analysis of this nature and it is essential that any reported wall test results should be supported by adequate information, especially corresponding measurements of flexural strength using the bond wrench method.

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