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STRUCTURAL RELIABILITY OF REINFORCED CONCRETE BLOCK MASONRY WALLS IN CONCENTRIC COMPRESSION

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ABSTRACT

The safety and reliability of reinforced grouted concrete block masonry is not accurately known. The present paper develops a probabilistic model to calculate the structural reliability of typical reinforced grouted concrete block masonry walls in compression designed to Chinese standards, loaded concentrically in compression. The effect of probability distribution of model error, material strengths, live load type, structural safety class, live-to-dead ratio, reinforcement ratio, discretization of wall thickness and load effect combination were considered when calculating structural reliabilities. When using the recommended distribution of model error for typical structures the existing (design) safety levels were found to be close to the target reliability for concentric compression for second class safety grade structures which comprise the majority of building stock in China. However, the reliability-based code calibration showed that design loads could be increased and decreased by 13.6% and 16.7% for first and third class safety grade structures, respectively.

KEYWORDS: structural reliability, probability, reinforced grouted masonry, concrete block, compression, masonry.

INTRODUCTION

There is increasing interest in China in the use of reinforced grouted concrete block masonry structures instead of the more traditional unreinforced clay brick masonry for it has many advantages such as improved bearing capacity and earthquake-resistance performance, energy-efficiency and smaller foundations. The Chinese code GB 50003 [1] is used for the design of reinforced grouted concrete masonry structures. Like many modern codes of practice for structural engineering design, the Chinese code GB 50003 [1] is presented in a 'limit state' format. Limit state specifications for structural steel and reinforced concrete (RC) design have been successfully developed over the past 25 years from reliability-based methods. However, although there are many reinforced grouted block masonry structures in China, their actual level of safety and reliability is not accurately known. Clearly, there is a need for a reliability analysis to assess the safety of new and existing reinforced grouted block masonry structures which may be used to recommend any changes to GB 50003 [1], as well as to codes internationally.

Only a few studies have focused on the reliability analysis of masonry structures (e.g. [2-5]). Recent reliability research [2] has given very preliminary statistics and reliability analysis of reinforced grouted concrete block walls in concentric compression, and the results indicate that reliability indices are greater than a target reliability index of 3.7. However, Yang and Shi [2] have not fully addressed the derivation of model errors, the effect of probability distribution of model error, the weighting assigned to live-to-dead load ratios, discretization of unit thickness, load effect combination and structural safety class on structural reliability. In addition, the collected experimental data for concentric compression was relatively small. This is of little surprise as reinforced grouted concrete block masonry is a challenging structural component to model probabilistically due to many components including concrete hollow block, mortar, reinforcement and grout in the holes and horizontal grooves, and the strength properties of reinforced grouted concrete block masonry are highly variable. So to date, no comprehensive procedure for the calculation of structural reliabilities of reinforced grouted concrete block masonry structures in compression has been developed.

This paper presents new and improved probabilistic models to calculate the structural reliability of single storey height reinforced grouted concrete block masonry walls in concentric compression. This paper follows the methodology of Stewart and Lawrence [3] who developed a probabilistic model to calculate the structural reliability of typical unreinforced brick masonry walls designed in Australia loaded in compression. Stewart and Lawrence [3] found that the existing safety levels of masonry were much higher than expected and recommended that the design capacity for Australian masonry for walls loaded concentrically in compression can be increased by up to 66%.

The different factors influencing the reliability of reinforced grouted concrete block masonry in compression are studied, including model error, grouted block masonry strength, load effect combinations, reinforcement ratio, live load type, structural safety class, live-to-dead load ratio and discretization of wall thickness. A key challenge is the prediction of model error, which was obtained from a statistical analysis of 109 axially loaded wall test results. Finally, the reliabilities of reinforced grouted concrete block masonry wall subjected to concentric compression are compared with the target reliability index recommended from Chinese Standard GB 50068 [6]. While the probability and reliability methods described herein are applied to a specific national code and structural configuration, these methods are also applicable to reliability-based assessments of other masonry codes or limit states.

RELIABILITY AND TARGET RELIABILITY INDEX FOR REINFORCED GROUTED CONCRETE BLOCK MASONRY

Failure of a structural element occurs when the load effect (S) exceeds the resistance (R). Structural reliability may be expressed as a probability of failure (p_f) or 'reliability index' (β):

$$p_f = P_r(R - S \leq 0) = P_r(G(R,S) \leq 0) = \int_0^\infty F_R(x)\, f_S(x)dx \qquad (1)$$

$$\beta = -\Phi^{-1}(p_f) \tag{2}$$

where Φ^{-1} is the inverse standard normal distribution function, G( ) is the limit state function where G( )≤0 denotes failure of the structure, $f_S(x)$ is the probability density function of load effect and $F_R(x)$ is the cumulative probability density function of the resistance. For more detail see Stewart and Melchers [7].

ISO 2394 [8] recommends β_T =3.8 for ultimate strength limit state design where consequences of failure are great and relative cost of safety measures are moderate. A target reliability index of β_T =3.8 was used by Stewart and Lawrence [3] for unreinforced masonry walls in compression. According to Chinese Standard [6] the target reliability is decided by the structural failure mode for a certain safety class (see Table 1). Although, the mechanical properties of reinforced grouted concrete block masonry loaded in compression and flexure show some ductile characteristics similar to RC structures, brittle properties can still be evidently found (see [9] for detailed description). Reinforced grouted concrete block masonry structures are a relatively new structural system in China, so there have been little opportunity to observe failure modes in practice. Hence, it is assumed, perhaps conservatively, that the failure mode is brittle and it follows from Table 1 that the target reliability index of β_T =3.7 for second safety class structures is recommended in this paper.

Table 1: Target Reliability Indices β_T (adapted from [6]).

Type of Failure	Safety Grade		
	First Class	Second Class	Third Class
Ductile Fracture	3.7	3.2	2.7
Brittle Fracture	4.2	3.7	3.2

MODEL ERROR

A model error (ME) characterizes the accuracy and variability of a strength or capacity prediction model:

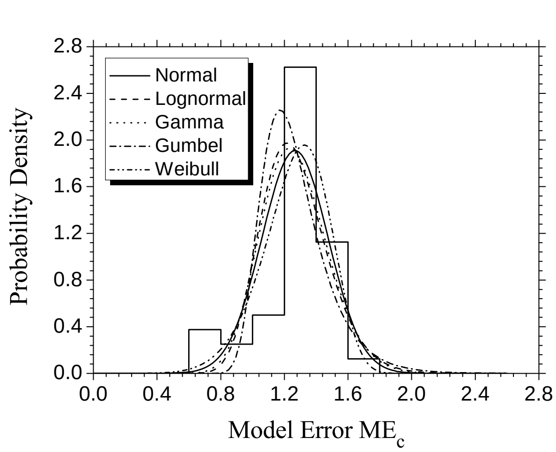
$$\mathrm{ME} = \frac{\text{Wall test capacity}}{\text{Capacity calculated from predictive model}} \tag{3}$$

A database of 109 axially loaded wall test results carried out in China from published literature is used to characterize model error. All walls have wall thickness of t=190 mm, height to thickness ratio H/t varying from approximately 3-15, block hole ratio varying 0.40-0.50 and reinforcement ratio varying from 0-0.54% (see [9] for full description of all test data). The predicted wall capacity is calculated from a predictive model based on Chinese code GB 50003 [1]. The mean predicted wall capacity ($\overline{R}$) in concentric compression can be calculated as:

$$\overline{R} = \varphi_{og}(f_{gm}A + 0.8 f'_{ym}A'_s) \tag{4}$$

where f_{gm} is the actual strength of grouted concrete block masonry; f'_{ym} is the mean compression yield strength for reinforcement; A is cross-sectional area of wall; A'_s is cross-sectional area of vertical reinforcement; φ_{og} is the wall stability factor depending on slenderness in concentric compression. For full details of the steps needed to obtain a predicted capacity based on wall test data, see [9].

A total of 109 wall test results were sourced to calculate the mean and COV (coefficient of variation) for model error ME_c. The histogram and various fitted probability distributions for concentric compression are shown in Figure 1. The Kolmogorov-Smirnov test found that no probability models were rejected at the 5% significance level. Figure 2 is used to help select the best fit probability distribution of model error. When the CDF^{-1} of a particular probabilistic mode sits on the 1:1 line then this indicates that the probabilistic model is a perfect fit to the data. Figure 2 shows that all distributions of model error under-estimate the lower tail of the histogram and so the probability of failure will be under-estimated because structural reliabilities are most sensitive to the lower tail of the probability distribution of resistance. In this case, the Weibull distribution is the most conservative as it produces the lowest estimates of structural reliability and provides the closest fit to the 1:1 line and so it is recommended for concentric compression.



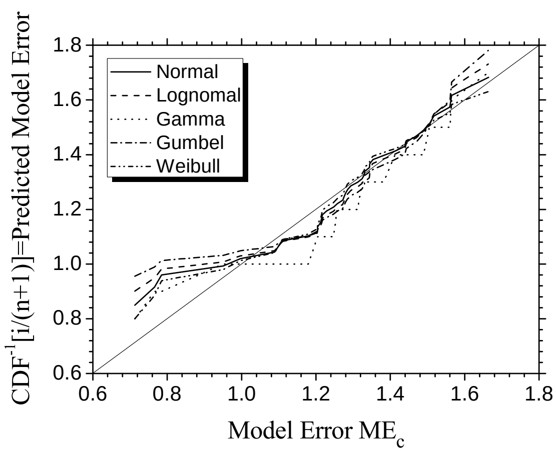


Figure 1: Probability Distributions of Model Error **Figure 2: Inverse CDF Plots**

The load-carrying capacity measured from wall tests are subjected to sources of uncertainty including variability of test procedures and specimen variability. So, it is necessary to exclude these variabilities in the estimation of ME variabilities. The coefficient of variation of model error V_{ME} is:

$$V_{ME} = \sqrt{V_m^2 - V_{test}^2 - V_{spec}^2} \tag{5}$$

where V_m is the COV obtained directly from a comparison of the measured and predicted strength (see Table 2); V_{test} represents the coefficient of variation in the measured loads due to the accuracy of test measurements and definitions of failure; and V_{spec} represents uncertainties due to differences between strength of the test specimen and control specimens, variation in actual specimen dimensions from those measured, etc. Ellingwood, et al. [10] suggest that at least for testing RC beams and columns V_{test} varies from 0.02 to 0.04, and V_{spec} is about 0.04. The statistics of model error assuming that V_{test} =0.02 and V_{spec} =0.04 are shown in Table 2.

Table 2: Statistics for Model Error

Eccentricity	Sample Sizes	Mean	V_m	V_{ME}	Distribution
e/t=0	109	1.267	0.167	0.161	Weibull

DESIGN LOADS AND LOAD MODEL

The current Chinese code GB 50003 [1] applies two load effect combinations. The maximum load effect for a single storey height wall is

$$S_{\max} = \max[\gamma_0(1.2G_k + 1.4Q_k),\ \ \gamma_0(1.35G_k + 0.98Q_k)] \tag{6}$$

where γ_0 is the importance coefficient of the structure. γ_0=1.1 if structural safety grade is first class; γ_0=1.0 for second class; γ_0=0.9 for third class. G_k, Q_k is the characteristic value for the dead and live load, respectively.

Most masonry buildings in China conform to a second class structural safety grade (i.e. γ_0=1.0). The design live-to-dead ratio will vary depending on type and size of building. Since most reinforced grouted concrete block masonry walls in China support RC slabs (and not timber floors). The weightings (w_i) assigned to live-to-dead load ratio proposed by Ellingwood [10] are adjusted to allow for an increased likelihood of lower live-to-dead load ratios [9], see Table 3.

Table 3: Weightings (w_i) for Dead and Live Load for Reinforced Grouted Concrete Block Masonry Structures

Q_k/G_k	0.25	0.50	1.0
Weighting (w_i)	45%	45%	10%

STRUCTURAL RELIABILITY ANALYSIS IN CONCENTRIC COMPRESSION

The ultimate strength limit state equation for reinforced grouted black masonry walls under concentric compression is

$$G(x) = ME_c \times \varphi_{og} \times t \times b(f_{gm} + 0.8 f'_{ym}\rho_s) - (D + L_p) \tag{7}$$

where

ME_c is the model error;

f_{gm} is the actual value of compressive strength for grouted concrete block masonry;

f'_{ym} is actual value of vertical compressive yield strength of reinforcement;

ρ_s is reinforcement ratio; b is actual wall width;

φ_{og} is stability factor of wall depending on slenderness, $\varphi_{og} = \dfrac{1}{1+0.001(H/t)^2}$, H is wall height and t is the wall thickness;

D is deal load; L_p is the peak live load in 50 years.

The design thickness t_{nom} of a wall is

$$t_{nom} = \frac{S_{\max}}{\varphi_{og,nom} b(f_g + 0.8 f'_y \rho_s)} \tag{8}$$

where

f'_y is the design value of compressive yield strength for vertical reinforcement, for HRB335 grade f'_y =300 MPa;

$\varphi_{og,nom}$ is the design value of the wall stability factor, $\varphi_{og,nom} = \dfrac{1}{1+0.001(H/t_{nom})^2}$;

f_g is the design value of compressive strength for grouted concrete block masonry, $f_g = f + 0.6\alpha f_c$;

where f is the design value of compressive strength for block masonry which is obtained directly from GB 50003 [1] and influenced by strength grades for block and mortar. The design value of prism grout (i.e. concrete) compressive strength f_c is obtained directly from Chinese Code for Design of Concrete Structure, see [9] for more details.

If t_{nom} is substituted into Equation (7), then the design size is based on the configuration with the highest reliability; hence the limit state generalizes to

$$G(x) = ME \times \lambda_{fA} \times \lambda_{\varphi_{og}} \times S_{\max} - (D + L_p) \tag{9}$$

where λ_{fA} represents the effect of actual strength differing from design strength:

$$\lambda_{fA} = \frac{f_{gm} + 0.8 f'_{ym} \rho_s}{f_g + 0.8 f'_y \rho_s} \tag{10}$$

and $\lambda_{\varphi_{og}}$ represents the effect of actual value of φ_{og} differing from design value of $\varphi_{og,nom}$:

$$\lambda_{\varphi_{og}} = \frac{\varphi_{og}}{\varphi_{og,nom}} = \frac{1+0.001(H/t_{nom})^2}{1+0.001(H/t)^2} \tag{11}$$

The strength of grouted concrete block masonry (f_{gm}) is composed of block masonry (f_m) and grout strength ($f_{c,m}$). Statistical parameters for grouted concrete block masonry strength (f_{gm}) should be based on the variability of measured strengths by standard unit grouted concrete block specimens. However, such statistics (f_{gm}) are not available and so to characterize the statistics for f_{gm} statistical parameters for f_m and $f_{c,m}$ are required, where $f_{gm} = f_m + 0.94\alpha f_{c,m}$, α is equal to the block hole ratio, and so

$$\lambda_{fA} = \frac{(f_m + 0.94\alpha f_{c,m}) + 0.8 f'_{ym}\rho_s}{f_g + 0.8 f'_y \rho_s} \tag{12}$$

The statistical parameters for f_m, $f_{c,m}$ and f'_{ym} used in Equation (12) are (i) statistics of grout strength $f_{c,m}$ are shown in Table 4; (ii) probabilistic distribution for block masonry compression strength f_m is normally distributed with mean=1.0f_k (f_k is the characteristic value of the compression strength) and COV=0.17 and (iii) compressive yield strength of reinforcement f'_{ym} is normally distributed with mean=1.14f'_{yk} and COV=0.07 for general reinforcement of HRB 335 grade (f'_{yk} is the characteristic value of the compression yield strength where f'_{yk}=335 MPa). The statistics for λ_{fA} will be influenced by grout strength and ratio of reinforcement.

If t_{nom}=190 mm and COV=0.02 [9], then a Monte-Carlo simulation analysis of Equation (11) shows that mean($\lambda_{\varphi_{og}}$)=1.00 and COV($\lambda_{\varphi_{og}}$)=0.01.

Table 4: Summary of Statistical Parameters

Variable		Mean	COV	Distribution	Reference
Concentric	ME_c	1.267	0.161	Weibull	
	$\lambda_{\varphi_{og}}$	1.0	0.01	Normal	
f_m : MU10, Mb7.5		4.01 MPa	0.17	Normal	[9]
MU15, Mb7.5		5.78 MPa			
MU15, Mb15		7.38 MPa			
MU20, Mb10		7.92 MPa			
MU20, Mb20		10.07 MPa			
$f_{c,m}$: Cb20		22.24 MPa	0.23	Normal	[9]
Cb30		28.34 MPa	0.19		
Cb40		36.18 MPa	0.16		
f'_{ym}		381.9 MPa	0.07	Normal	[9]
D		1.06G_k	0.070	Normal	[9]
L_p (office)		0.524Q_k	0.288	Gumbel	[9]
L_p (residence)		0.644Q_k	0.233	Gumbel	[9]

MU: the strength grade of block (in MPa); Mb: the strength grade of mortar (in MPa); Cb: the strength grade of grout (in MPa).

DISCRETIZATION OF THICKNESS

It is recognized that blocks are manufactured in discrete sizes and so designers will normally adjust support conditions, span lengths and structural systems to optimize unit thickness selection. However, it is most likely that unit thickness will still need to be 'rounded-up' (i.e., discrete size greater then design thickness). This 'rounding-up' may be incorporated into a reliability analysis by increasing $\lambda_{\varphi_{og}}$ and t. The effect of this 'rounded-up' on structural reliability can be significant [4]. For example, if the average oversizing is estimated to be 10% (i.e. t=1.1t_{nom}) then mean resistance increases by 16%. A similar phenomena also occurs for RC and structural steel sections, but the mean oversizing is estimated to be approximately only 5% [11]. Although reliability-based code-calibration studies ignore this influence (e.g. [2,10]) this effect is potentially higher for masonry than other materials, and so its effect on structural reliability is considered in the present study.

RESULTS

A general-purpose structural reliability analysis program CALREL is used as the computational software incorporating First Order Reliability Method (FORM) to calculate structural reliability [12]. The live load is Q_k=2.0 kPa for residential and office buildings, second safety class importance coefficient of γ_0=1.0, single storey wall height H=2800 mm~3000 mm and block hole ratio of 0.46 are typical for reinforced grouted concrete block masonry in China. The probability models for dead (D) and peak 50-year live loads (L_p) are given in Table 4. Material strength grade combinations, which satisfy practical designs, vary from MU10, Mb7.5, C20 to MU20, Mb20, C40. The reinforcement ratio can vary from ρ_s=0.1% to ρ_s=1.0%.

Table 5 shows that the reliabilities (β) of wall in concentric compression for Q_k/G_k=0.5 for residential and office floor loading and for all probability distributions of model error. The reliability indices for residential occupancy are slightly lower than office loading due to the high mean live load for residences. The reliability indices are not obviously influenced by reinforcement ratio. However, reliabilities are very sensitive to the probability distribution of model error and slightly influenced by compressive grouted block masonry strength. As expected, the recommended distribution for model error (Weibull) produces the lowest reliability indices. Hence, to be conservative reliability indices to be used for code calibration will be derived from a Weibull distribution of model error. This is also convenient as ISO 2394 [8] recommends a lognormal or Weibull distribution for resistance when using their recommended target reliability indices.

Table 5: Structural Reliabilities (β) for Concentric Compression, for Q_k/G_k=0.5

Reinforcement Ratio	Model Error Distribution	MU10, Mb7.5, Cb20 (f_g=5.15MPa)		MU15, Mb7.5, Cb30 (f_g=7.56MPa)		MU15, Mb15, Cb30 (f_g=8.56MPa)		MU20, Mb10, Cb40 (f_g=10.22MPa)		MU20, Mb20, Cb40 (f_g=11.57MPa)	
		residence	office	residence	office	residence	office	residence	office	residence	office
ρ_s=0.1%	Normal	4.49	4.56	4.53	4.60	4.47	4.55	4.52	4.60	4.47	4.54
	Lognormal	4.52	4.59	5.13	5.22	5.19	5.30	5.64	5.76	5.61	5.74
	Gamma	4.66	4.71	5.43	5.51	5.57	5.65	6.19	6.28	6.25	6.34
	Gumbel	4.54	4.60	5.21	5.30	5.31	5.41	5.85	5.96	5.86	5.98
	Weibull	4.04	4.11	3.95	4.03	3.90	3.98	3.94	4.01	3.88	3.96
ρ_s=1.0%	Normal	4.40	4.48	4.38	4.46	4.35	4.43	4.40	4.48	4.36	4.44
	Lognormal	4.93	5.03	5.37	5.50	5.40	5.53	5.74	5.88	5.69	5.83
	Gamma	5.26	5.34	5.96	6.06	6.07	6.17	6.65	6.76	6.67	6.78
	Gumbel	5.02	5.11	5.59	5.71	5.66	5.78	6.13	6.26	6.11	6.25
	Weibull	3.84	3.92	3.80	3.88	3.77	3.85	3.82	3.90	3.78	3.86

An average reliability index (β_{av}) considering the range of live-to-dead load ratios is:

$$\beta_{av} = -\Phi^{-1}\left(\sum_{i=1}^{3} w_i p_{fi}\right) \tag{13}$$

where w_i and p_{fi} are the weighting and the probability of failure for the i^{th} live-to-dead load ratio, respectively. The values of w_i are shown in Table 3.

Figures 3 and 4 show that the average reliability index β_{av} is slightly higher than the target reliability index of β_T =3.7 as a function of reinforcement ratio and design compressive strength of grouted block masonry for various live-to-dead load ratios.

Although, there are very few reinforced grouted concrete block masonry walls constructed for first and third safety classes in China, this paper also calculates reliability indices as a function of importance coefficient (γ_0), see Figure 5. Figure 5 shows that the reliability indices are less than the target reliability of β_T =4.2 for the first safety class but greater than the target reliability of β_T =3.2 for the third safety class. To ensure that the target reliability is met for first and third safety classes Figure 5 shows that (i) γ_0 should increase from 1.1 to 1.25 for the first safety class; and (ii) γ_0 should decrease from 0.9 to 0.75 for the third safety class. These are significant changes in the importance coefficient, which if implemented would (i) increase design loads by 13.6% for first safety class and (ii) decrease design loads by 16.7% for third safety class. A reliability analysis for other limit states (such as seismic, shear) may be needed to assess if changes to γ_0 are warranted for these limit states.

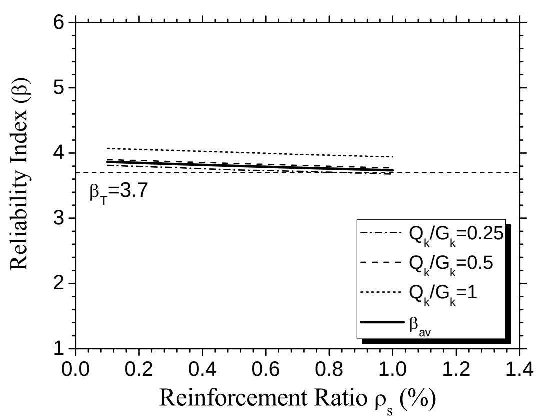


Figure 3: Reliability Indices for Reinforcement Ratio and Load Ratio (Weibull, MU15, Mb15, Cb30)

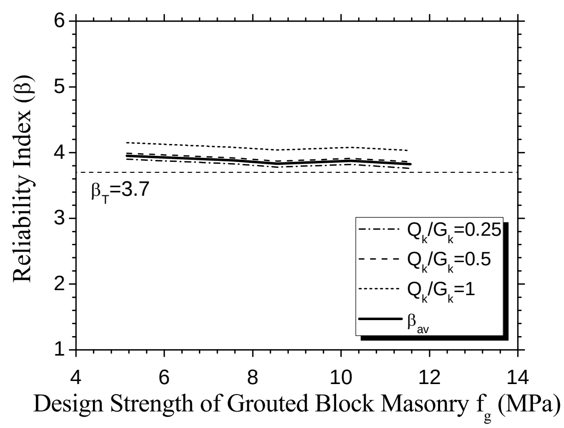


Figure 4: Reliability Indices for Design Strength and Load Ratio (Weibull, ρ_s =0.3%)

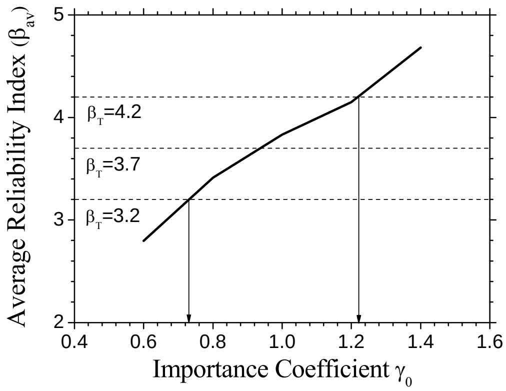


Figure 5: Average Reliability Indices (β_{av}) for Importance Coefficient γ_0 (Weibull, ρ_s=0.3%, MU15, Mb15, Cb30)

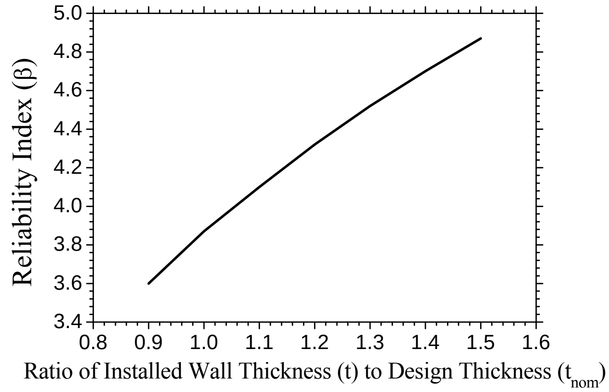


Figure 6: Reliability Indices for Discretization of Wall Thickness (Weibull, ρ_s=0.3%, MU15, Mb15, Cb30)

While discretization of wall thickness is not considered for reliability-bases code calibration, its effect on structural reliability is of interest. Figure 6 shows that reliability indices for reinforced grouted concrete block masonry walls are obviously influenced by discretization of wall thickness. For example, Figure 6 shows that if wall thickness is rounded up by only 10% ($t=1.1t_{nom}$) then the reliability index will increase from 3.87 to 4.10.

CONCLUSIONS

Model error and structural reliabilities have been calculated for reinforced grouted concrete block masonry single storey walls loaded concentrically in compression. The effect of distribution of model error, material strengths, live load type, structural safety class, live-to-dead load ratio, reinforcement ratio, wall thickness discretization were considered. Reliabilities are very sensitive to the probability distribution of model error. The Weibull distribution provides the lowest reliability indices. Average reliabilities were compared to a target reliability index, and it is found that when using the recommended model error distribution the average reliability indices meet the target reliability index for concentric compression for second class safety grade structures which comprise the majority of building stock in China. However, the reliability-based code calibration showed that design loads could be increased and decreased by 13.6% and 16.7% for first and third class safety grade structures, respectively. Also, there is a need for collection and statistical analysis of more test data for reinforced grouted block walls loaded concentrically in compression.

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