IDENTIFICATION OF SHEAR MECHANICAL PARAMETERS OF MASONRY PIERS FROM DIAGONAL COMPRESSION TEST

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ABSTRACT

In the seismic design and assessment of ordinary masonry buildings the prediction of the strength of masonry piers subject to in-plane lateral forces plays a crucial role. Different simplified models are present in literature and codes to describe the failure modes of piers (*Rocking*/*Crushing*, *Bed Joint Sliding*, *Diagonal Cracking*) and to predict their load bearing capacity. In general, they are based on simple idealizations of the limit strength domain of masonry through few mechanical parameters. Referring in particular to the *Diagonal Cracking* failure mode, two models are usually adopted: that of Turnšek and Čačovič (1970) and that of Mann and Müller (1980). These models are dependent on two main mechanical parameters: the cohesion of mortar joints, usually obtained through the triplet test, and the tensile strength of masonry, usually derived by the diagonal compression test. Aim of this paper is to synthetically analyse the physical meaning, the experimental evaluation and the proper use of these parameters. Moreover, a method to relate the result of the diagonal compression test to a "mean" value of the cohesion is proposed. This latter test offers two main advantages: a versatile application to different types of masonry (also irregular ones); the capability of providing "mean" mechanical parameters, representative of the whole masonry.

KEYWORDS: masonry, pier strength, in-plane shear behaviour, diagonal compression test.

INTRODUCTION

The prediction of the strength of masonry piers subject to in-plane lateral forces plays a crucial role in the seismic design and assessment of ordinary masonry buildings.

A "direct" approach to the estimation of the strength of masonry piers consists in performing experimental tests able to simulate reality as closely as possible, in terms of boundary conditions and acting forces. Through these tests, the limit strength domain of piers of given slenderness and given masonry type may be obtained in the space of the applied forces. Although quite accurate and reliable, such approach is costly and time-consuming since it requires a large number of tests to be performed. Moreover, in most cases, it is technically inapplicable to existing buildings due to its highly destructive nature. For these reasons, a "indirect" approach, based on simplified theoretical models, is usually adopted.

Simplified models present in literature and codes are oriented to describe specific failure modes which may occur in piers (*Rocking*/*Crushing*, *Bed Joint Sliding*, *Diagonal Cracking*). They are generally based on: the approximate evaluation of the local/mean stress state produced by the applied forces on predefined points/sections of the pier; the assessment of its admissibility with reference to the limit strength domain of the constituent material, usually idealized through simple schematizations based on few mechanical parameters. This approach, whose theoretical reliability has been already assessed by the authors in [1], requires thus the experimental evaluation of the limit strength domain of the material (the masonry) instead of that of the structural element (the pier). This is an evident technical advantage. However, the experimental determination of the mechanical parameters adopted to define the limit strength domain of masonry still poses many problems, mainly related to test settings and mechanical interpretations of the results.

Aim of this paper is to synthetically analyse the physical meaning, the experimental evaluation and the proper use of the mechanical parameters on which the most diffused simplified models for the prediction of the load-bearing capacity of piers are based. Particular attention will be paid to the *Diagonal Cracking* failure mode and to the two models which are usually adopted to describe it [2,3]. Such models describe different masonry types (Figure 1) and may provide very different predictions of the strength [1]. In Turnšek and Čačovič's model [2], the limit strength domain is defined through a single "global" parameter of the material: the tensile strength of masonry, usually determined by the diagonal compression test. In Mann and Müller's model [3], the domain is defined on the basis of "local" parameters related to the single constituents of the material: the cohesion and friction coefficients of joints, usually evaluated through the triplet test, and the tensile strength of blocks. The experimental evaluation of these latter "local" parameters, and in particular of the cohesion and the friction coefficients, may poses some problems in those masonry where the mortar joints are not regularly arranged (Figure 1b).

a. b.

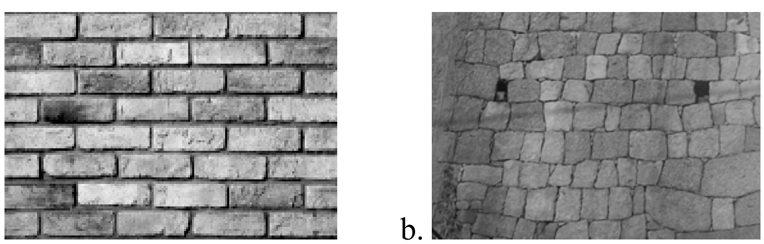


Figure 1: Different types of masonry pattern

On one hand, the triplet tests is hardly performable on not perfectly regular assemblages; on the other hand, the results obtained would not be representative of the entire set of joints of the masonry, due to the large scatter which may characterize them. For these reasons, the paper investigates the possibility of using the diagonal compression test to obtain a "mean" evaluation of the cohesion and the friction coefficient of mortar joints of a given masonry. Such test offers two main advantages: a versatile application to different typologies of masonry; the capability of providing "mean" mechanical parameters, representative of the whole masonry.

CRITERIA FOR THE SHEAR STRENGTH PREDICTION OF PIERS

Observation of seismic damage to complex masonry walls, as well as laboratory experimental tests, showed that masonry piers subjected to in-plane loading may have two typical types of behaviour, to which different failure modes are associated: flexural behaviour (*Rocking*, *Crushing*); shear behaviour (*Sliding Shear Failure*, *Diagonal Cracking*). The occurrence of different failure modes depends on several parameters: the geometry of the pier; the boundary conditions; the acting axial load; the mechanical characteristics of the masonry constituents (mortar, blocks and interfaces); the masonry geometrical characteristics (block aspect ratio, in-plane and cross-section masonry pattern).

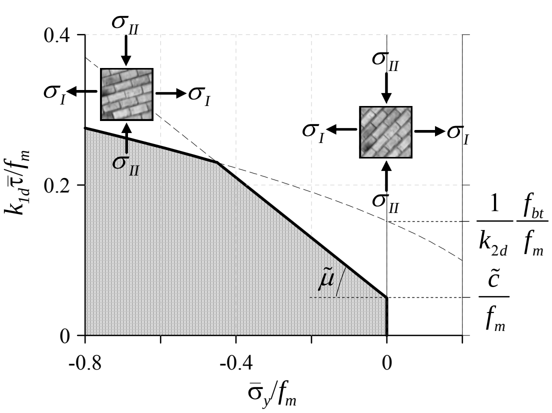
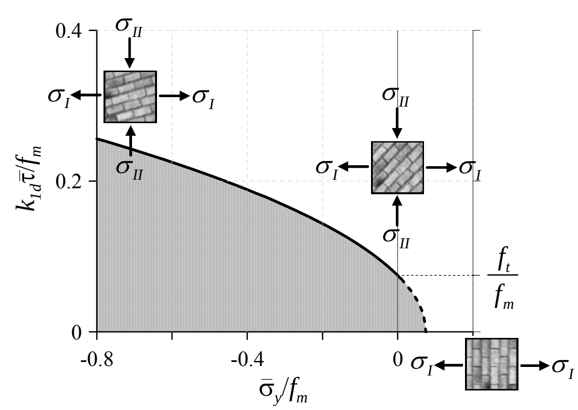
The most common simplified models present in the literature for the prediction of the strength of masonry piers are based on the choice of a "reference" stress σ_c (either shear, normal or principal stress) and of a "reference" point or section on which it should be calculated. Its admissibility is assessed by comparison with a proper limit stress domain of the material. Focusing the attention on the *Diagonal Cracking*, it is possible to recognize two main types of models: (a) models describing masonry as an equivalent isotropic material such as proposed by Turnšek and Čačovič [2], considering indistinctly the development of cracks along principal stress directions; (b) models describing masonry as a composite material such as proposed by Mann and Müller [3], considering separately the development of cracks along its constituting components (joints and blocks). These models are summarized in Table 1, where: $\bar{\tau} = V/A$ and $\bar{\sigma}_y = N/A$ are the mean shear and normal stresses acting on the cross section of the pier (V and N being the overall shear strength of the pier and the applied axial load, respectively, and A being the transversal area of the panel); k_{1d} is the ratio between the shear stress at the centre of the pier and the mean shear stress $\bar{\tau}$; k_{2d} is the ratio between the shear stress applied on a block and the local shear stress at its centre (usually k_{2d} =2.3); σ_I and σ_{II} are the calculated principal stresses; c and μ are the "local" cohesion and friction coefficients of mortar joints, respectively; φ is a parameter describing the interlocking of masonry pattern ($\varphi = 2h/b$, h and b being the height and width of blocks); f_{bt} is the tensile strength of blocks and f_t is the tensile strength of masonry.

The formulation proposed by Turnšek and Čačovič is based on the assumption of masonry as an isotropic material. It considers as reference stress σ_c the maximum principal stress acting at the centre of the pier σ_I; it must not exceed the tensile strength of masonry f_t. This latter parameter is assumed as constant in any loading direction (isotropic limit stress domain).

The formulation proposed by Mann and Müller is based on two main hypotheses: (a) bricks are much stiffer than mortar joints; (b) the mechanical properties of head joints are negligible. Since no shear stresses can be transferred through head joints, blocks are subjected to a torque; equilibrium can be attained only by a vertical force couple, leading to a non-uniform distribution of the compression stresses on bed joints. Concerning joints, the model assumes the shear stress acting at the centre of the pier as reference one. The limit domain is defined through a Mohr-Coulomb type criterion, where $\tilde{c}$ and $\tilde{\mu}$ are "global" properties of masonry taking into account the geometrical characteristics of the pattern. Concerning block, the model adopts the maximum principal stress acting on the block at the centre of the pier as reference one; it must not exceed the tensile strength f_{bt}.

The two models may provide very different predictions of the strength. Their reliability depends on the degree of anisotropy of the type of masonry examined [1]. Coherently with the hypotheses adopted, Turnšek and Čačovič's criterion is more suitable to describe those masonries which tend to behave as homogeneous and isotropic materials, whereas Mann and Müller's theory is more appropriate for masonry which behaves as anisotropic material.

Table 1: Models aimed to interpreter the *Diagonal Cracking* failure mode

	Mann and Müller [1980]		Turnšek and Čačovič [1970]
	Mortar joints	*Blocks*	
Reference stress σ_c	$k_{1d}\bar{\tau}$ Shear stress at the centre of the pier	$\sigma_I = \frac{\bar{\sigma}_y}{2} + \sqrt{(k_{1d}k_{2d}\bar{\tau})^2 + \left(\frac{\bar{\sigma}_y}{2}\right)^2}$ Maximum principal stress in the block at the centre of the pier	$\sigma_I = \frac{\bar{\sigma}_y}{2} + \sqrt{(k_{1d}\bar{\tau})^2 + \left(\frac{\bar{\sigma}_y}{2}\right)^2}$ Maximum principal stress at the centre of the pier
Limit domain	$\sigma_c = \tilde{c} - \tilde{\mu}\bar{\sigma}_y$ (1)	$\sigma_c = f_{bt}$ (2)	$\sigma_c = f_t$ (3)
Mechanical Parameters	$\tilde{c} = c\frac{1}{1+\mu\varphi}$ $\tilde{\mu} = \mu\frac{1}{1+\mu\varphi}$	f_{bt}	f_t
σ_y-τ			

In Turnšek and Čačovič's model, the limit stress domain of masonry is defined in a *direct* way, by evaluating experimentally the tensile strength f_t. Two different types of tests may be adopted: the diagonal compression and the racking test. In both cases, f_t is calculated by assuming that the failure of masonry sample derives from the attainment of the maximum principal stress at its centre. In the first case, the strength is obtained for a ratio $\sigma_I/\sigma_{II} = -0.3$ and a loading direction angle 45° (see the following paragraph). In the second case, the test is interpreted according to Turnšek and Čačovič's model, the strength being inversely calculated from Equation (3) on the basis of the axial load applied and maximum shear load attained. It is worth noting that, for $\bar{\sigma}_y$=0, f_t corresponds to the strength of masonry in a pure shear condition ($\sigma_I/\sigma_{II} = -1$ and a loading direction angle 45°). For this reason, in engineering practice and codes the parameter f_t is often associated with the cohesion of mortar joints c, obtained through the triplet test. It is worth noting that, if the limit stress domain of the material would actually be isotropic, the values of f_t obtained from these different types of test should theoretically be equal. In practice, this is not true. The limit stress domain of masonry is far to be perfectly isotropic. Thus, different values of f_t may be obtained depending on the test type, and associated stress field (Figure 2). For the same reason, in racking test, different values of the tensile strength may be obtained depending on the axial load applied, due the variation of both the loading direction angle and the ratio σ_I/σ_{II}.

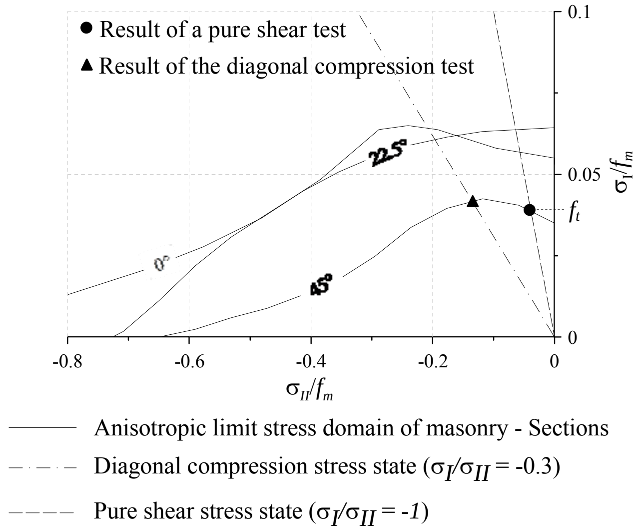


Figure 2: Sensitivity of f_t on the test type and on the stress field associated

In Mann and Müller's model, the limit stress domain of masonry is defined in a *indirect* way, on the basis of the experimental evaluation of micromechanical parameters of its constituents (c, μ and f_{bt}). The types of experimental tests most widely used in common practice are: the triplet test, suitable to define local mechanical properties of mortar joints such as c and μ, and the splitting or bending tests suitable to define the tensile strength of blocks. For industrialized masonries, this approach presents the great advantage of determining the parameters through easy and inexpensive tests on small specimens. Nevertheless, it may pose some problems in the case of historic hand-made masonries. On the one hand, for certain type of masonries (e.g. irregular stone masonries), tests may be performed only with great difficulty due to the irregularity of the pattern. On the other hand, the determination of "punctual" parameters in a material which may present many dishomogeneities may determine a high dispersion of the results. For these types of masonries, thus, an alternative way to determine the parameters should be found.

The attention is focused on the diagonal compression test. Such test offers two main advantages: a versatile application to different typologies of masonry; the capability of providing "mean" mechanical parameters, representative of the whole masonry; the on-site performability.

MECHANICAL INTERPRETATION OF THE DIAGONAL COMPRESSION TEST

In diagonal compression test, a square masonry panel is subjected to a compressive force P applied on its diagonals. The collapse of the panel is usually associated with the development of a crack starting from its centre.

In the standard interpretation of the test, the diagonal tensile strength of masonry f_{dt} is obtained by assuming that the panel collapses when the principal tensile stress σ_I at its centre attains its maximum value. Brignola et al. [4] has recently demonstrated that this interpretation of the test is reliable and that, in non-linear range, the stress redistribution occurring in the panel does not affect the value of σ_I computed with the elastic solution.

As demonstrated theoretically by Frocht [5] and as can be easily shown by a finite element analysis [4,6], the elastic solution provides the following stress state at the centre of the panel: $\sigma_x = \sigma_y = -0.56\,P/A$, $\tau = 1.05\,P/A$, corresponding to a ratio $\sigma_I/\sigma_{II} = 0.3$ (the loading direction angle being always 45°). The actual value of f_{dt} should be then computed as: $f_{dt} = \sigma_I = 0.5\,P/A$.
It is worth noting that, in most codes and standards [7,8], the tensile stress σ_I is calculated by assuming a uniform shear stress distribution within the panel, which leads to the following stress state at its centre: $\sigma_y=\sigma_x=0$, $\tau = \left(1/\sqrt{2}\right)P/A$ (*A* being the transversal area of the panel). Under these hypotheses, the diagonal tensile strength of masonry f_{dt} is calculated, in practice, as if the panel would be in a pure shear stress state (σ_I/σ_{II} =-1, loading direction angle 45°). The value of f_{dt} is indeed computed as: $f_{dt} = \sigma_I = 0.7\,P/A$.
Figure 3 shows the comparison between these two different interpretation of the diagonal compression test by the Mohr's circle. In the following, reference will be made to the theoretical evaluation of the stress state of the panel, leading to $f_{dt} = 0.5\,P/A$.

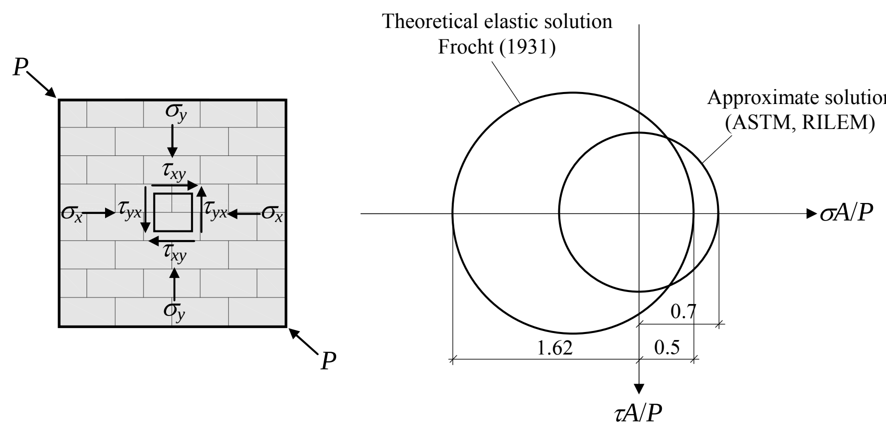


Figure 3: Interpretations of the diagonal compression test by the Mohr's circle.

In the case of Turnšek and Čačovič's model, the tensile strength of masonry may be directly assumed as $f_t = f_{dt}$.
In the case of Mann and Müller's model, the problem of relating the strength f_{dt} to the local parameters of mortar joints *c* and μ should be considered (the parameter φ is valuable on the basis of the masonry pattern). In principle, the results of two experimental tests would be necessary. They could be obtained by performing two diagonal compression tests, as an example applying in the first one only the force *P* and adding in the second one a compressive load on two opposite sides of the panel. However, since in most of the cases the result of a single test is available (usually in absence of additional compressive loads), in the paper a method to correlate f_{dt} to the cohesion *c*, by assuming the friction coefficient μ on the basis of an expert judgement, is proposed.

THE USE OF THE DIAGONAL COMPRESSION TEST TO DETERMINE THE COHESION OF MORTAR JOINTS

As already stressed, Mann and Müller's model should be employed for those masonries showing a clearly anisotropic behaviour. A crucial point in the experimental evaluation of the strength of an anisotropic material is the assessment of the actual stress state: in fact, different results may be obtained depending on the ratio σ_I/σ_{II} and on the loading direction angle.

A simple idea to evaluate the cohesion of mortar joints through the diagonal compression tests could be based on inversely calculating c from the Mann and Müller's limit stress domain. However, in diagonal compression test the stress state occurring at the centre of the panel is characterized by an unit ratio between σ_x and σ_y components: a reliable interpretation of the test result cannot disregard the σ_x component, which is nevertheless neglected in Mann and Müller's model. The problem of considering the σ_x component could be faced by adopting a "modified" version of Mann and Müller's model developed by Dialer [9], in which also the resistance contribution of head joints is taken into account. However, this model seems to adopt too much simplified hypotheses. In particular, it assumes that head joints are always compressed, whereas it can be demonstrated that they are compressed or tense depending on the ratio between σ_x and τ components. Moreover, it is worth noting that, in both Mann and Müller's and Dialer's models, also bed joints are assumed to be always entirely compressed: actually, depending on the ratio between σ_y and τ components, this could be not verified. In order to guarantee the equilibrium of blocks for any stress state, it is necessary to assume that part of bed joints may be tense. For these reasons, a more complex domain formulated by Calderini and Lagomarsino [1,10] is here adopted. It is built on a simplified micromechanical basis, adopting hypotheses partially coincident with those of Mann and Müller's model (neglecting of the mechanical properties of head joints; assumption of a Mohr-Coulomb's type law for compressed bed joints). It considers the complete stress state acting on the material (σ_x, σ_y and τ). Moreover, it takes into account also those situations in which head joint are open and bed joints are not completely compressed: thus, equilibrium is guaranteed for any stress state. In particular, the failure of mortar joints is described by a discrete set of equations depending on the sign of the normal stresses acting on the bed joints and on the opening/closing state of head joints. Table 2 summarizes these equations. It can be observed that, when bed joints are partially compressed and partially tense (State B), the domain depends, besides on friction coefficient μ, cohesion c and interlocking coefficient φ, on the tensile strength of bed joints.

Table 2: Set of equations defining the limit domain of joints proposed in [1,10].

State of bed joints	Head joint condition	Equation	
A. Entirely compressed	Open	$\tau = \frac{1}{1+\mu\varphi}\left(c - \varphi\sigma_x - \mu\sigma_y\right)$	(4)
	Closed	$\tau = c - \mu\sigma_y$	(5)
B. Partially compressed, partially tense	Open	$\tau = -\frac{\varphi\left(\sigma_x + \chi\sigma_y\right) \pm \sqrt{\varphi^2\left(\sigma_x + \chi\sigma_y\right)^2 - \left(1+\chi\varphi^2\right)\left(\varphi^2\sigma_x^2 + \chi\sigma_y^2 - c^2\right)}}{1+\chi\varphi^2}$ Note[1]	(6)
	Closed	The following system of equations has be to numerically solved in $\overline{\tau}$ and α: $\begin{cases} \left[\eta^2\left(2-\mu\varphi\right)^2 + \chi\varphi^2\right]\tau^2 + 2\left[\eta^2\left(2-\mu\varphi\right)\mu + \chi\varphi\right]\sigma_y\tau + \left[\eta^2\mu^2 + \chi\right]\sigma_y^2 - c^2 = 0 \\ \eta^2\left[\left(2-\mu\varphi\right)\tau + \mu\sigma_y\right]^2 - \alpha^3 c^2 = 0 \end{cases}$ Note[2]	(7)

[1] χ is the square of the ratio between the cohesion c and the tensile strength of mortar joints.
[2] $\eta = \alpha/\left(1+\alpha\right)$, α being a damage variable ($0<\alpha<1$).

Figure 4 shows different sections of the domain in the plane σ_y-τ, for different values of σ_x (the domain is adimensionalized with respect to the compressive strength of masonry f_m). It can be observed that the domain is divided in two regions, corresponding to State *A* and *B* of Table 2. The threshold between State *A* and *B* depends only on the interlocking φ of the considered type of masonry. The field of the domain described by Equation (4) (bed joints entirely compressed and head joints open), coincides for σ_x =0 with Equation (1) of Mann and Müller's model; Equation (4) is nevertheless replaced by Equation (6) for low values of compression.

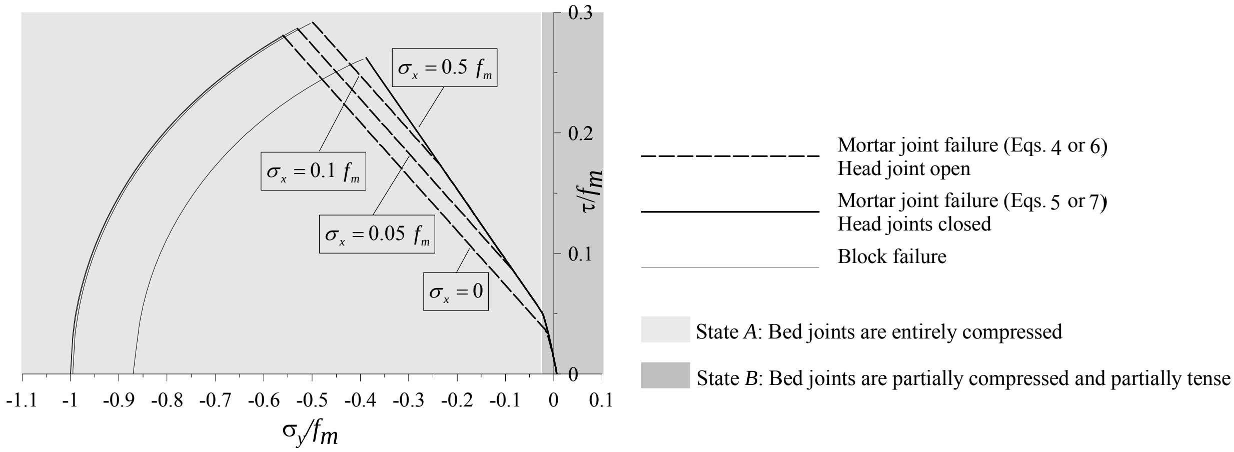


Figure 4: Limit domain of the constitutive law adopted [1,10].

The relationship between f_{dt} and the cohesion *c* can be obtained by finding the point of the limit domain corresponding to the stress state occurring at the centre of the panel in the diagonal compression test ($\sigma_x = \sigma_y = -0.56\,P/A$, $\tau = 1.05\,P/A$). It can be demonstrated that, for the particular stress state assumed, the threshold between State *A* to State *B* corresponds to $\tan(\varphi)=\sigma_y/\tau$, that is b/h =3.75. This means that, for masonry with ratio b/h<3.75, the standard diagonal compression test produces tensile stresses on bed joints and the relationship between f_{dt} and the cohesion *c* depends on their tensile strength. Figure 5a shows as the dependence on this latter mechanical parameter is not negligible for values of b/h lower than 3.75. However it is worth noting that the dependence on the tensile strength of mortar joint may be overcome by altering the stress state in order to move the threshold between State *A* and State *B* to lower values of b/h. In particular, the stress state may be modified by applying a lateral compression $q=Q/A$ on two sides of the panel. Figure 5b shows, for masonries characterized by different ratios b/h, the minimum value of *q* to be applied in order to guarantee that the stress state produced by the diagonal compression test at the centre of the panel is of type *A*.

Table 3 shows the equations to determine *c* as a function of the load *P* and *Q* applied, as far as the stress state *A* is concerned. In particular, *k*=1.875 is the ratio between the shear and normal stress σ_x components of the stress state at the centre of the panel.

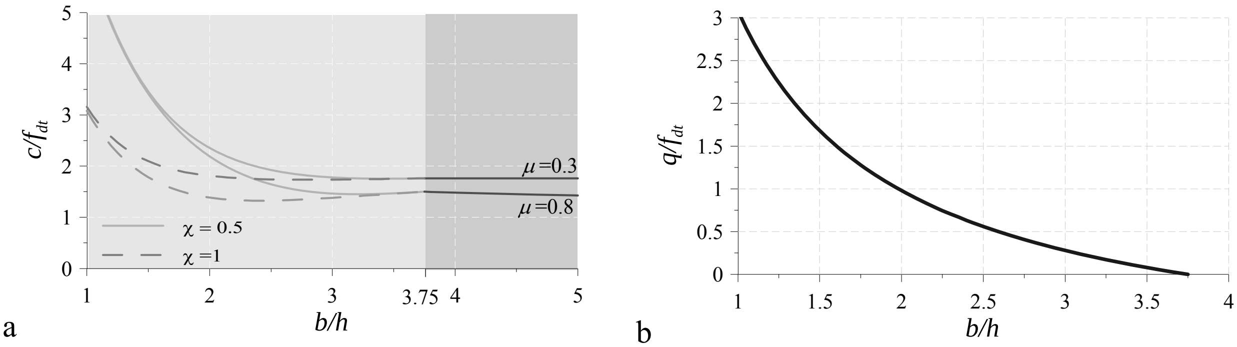


Figure 5: Sensitivity of the ratio c/f_{dt} on the interlocking (φ=2h/b) and on χ; b) load q to be applied as a function of b/h.

Table 3: Expressions of the parameter β

Head joint condition	Cohesion of mortar joints	
Open μ>1/k	$c = -0.56\left[\varphi + \mu - k\left(1 + \mu\varphi\right)\right]\frac{P}{A} + \left[\varphi - k\left(1 + \mu\varphi\right)\right]\frac{Q}{A}$	(8)
Closed μ≤1/k	$c = -0.56\left(\mu - k\right)\frac{P}{A} - k\frac{Q}{A}$	(9)

FINAL REMARKS

In this paper, a critical review of the physical meaning, the experimental evaluation and the proper use of the mechanical parameters, which the most widely used simplified models for the prediction of the load-bearing capacity of piers are based on, has been proposed. Particular attention to the mechanical parameters on which the two main models proposed in literature and codes to interpreter the *Diagonal Cracking* failure mode are founded, has been paid. In particular they are: the tensile strength of masonry f_t in the case of model proposed by Turnšek and Čačovič, usually obtained by diagonal compression test; the cohesion c and friction μ coefficients of mortar joints in case of model proposed by Mann and Müller, traditionally obtained by performing the triplet test. It has been stressed as the use of the diagonal compression test seems particularly attractive and effective. In fact it offers two main advantages with respect to the use of the triplet test: a versatile application to different typologies of masonry (triplet tests is hardly performable on not perfectly regular assemblages); the capability of providing “mean” mechanical parameters, representative of the whole masonry (the results obtained by triplet test would not be representative of the entire set of joints of the masonry, due to the large scatter which may characterize them). A crucial point for the mechanical interpretation of the results obtained by the diagonal compression test is represented by the correct evaluation of the stress field which occurs in the panel. Regarding this, it has been highlighted as the interpretation provided by most of codes and standards [7,8] is based on an approximate assumption of a uniform shear stress distribution, which actually doesn’t correspond to the theoretical one provided by the elastic solution. This incorrect assumption leads to results which are on the unsafe side.

Finally, in the case of Mann and Müller’s model, the problem of relating the strength f_{dt} to the local parameters of mortar joints c and μ has been attempted (the parameter φ is valuable on the basis of the masonry pattern). In principle, the results of two experimental tests would be necessary. However, since in most of the cases the result of a single test is available, in the paper a method to correlate f_{dt} to the cohesion c, by assuming the friction coefficient μ on the basis of an expert judgement, is proposed.

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