



COMPARISON OF IN-PLANE MASONRY SHEAR MODELS

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ABSTRACT

Shear behaviour in masonry assemblages can be quite complex and difficult to model. This is demonstrated by the fact that provisions for in-plane masonry shear design from country to country are vastly different. The objective of this paper is to analyze current shear models in order to determine their reliability. The models considered include: CSA S304.1-04, ACI 530-08, AS 3700-2001, NZS 4230-2004, Eurocode 6, BS 5628-2:2005, Anderson and Priestley (1992), and Voon and Ingham (2007). Numerous test data were gathered from the literature for comparison with these models with the intention of establishing the most effective model.

KEYWORDS: masonry, in-plane shear

INTRODUCTION

There is a general consensus amongst researchers that masonry has three main in-plane failure modes, including: sliding, diagonal, and flexural. Sliding failure occurs as two courses separate horizontally along the bed joints. Diagonal failure exhibits very brittle behaviour characterized by rapid strength degradation soon after the maximum strength is realized. Diagonal failure can include failure at the head joints, bed joints, and through the units. Flexural failure occurs by tensile yielding of the vertical reinforcement at the heel of the wall or crushing of the masonry at the toe of the wall. It is generally the preferred mode of failure because tensile failure of the steel is more ductile and effective in dissipating energy. These failure modes are depicted in Figure 1 below.

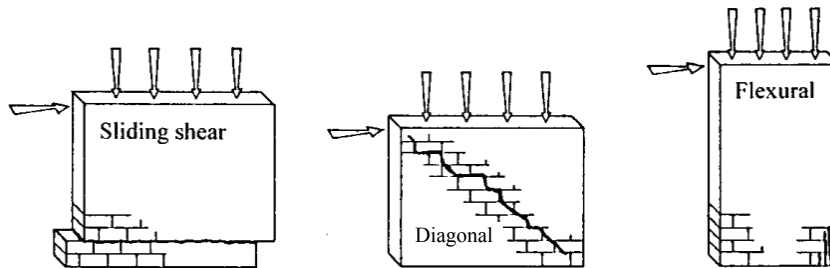


Figure 1: Masonry In-Plane Shear Failure Modes [1]

While in-plane shear failure modes may be agreed upon, a consensus on the analytical prediction of shear wall failure has yet to be reached. International masonry standards have very different methods for predicting shear failure, and new shear models are frequently being proposed based on more current research. This paper reviews several international standards and proposed models for comparison to each other as well as to experimental test data. All strength and material reduction factors are included in the analysis in order to compare predicted design values. Some variables have been modified for consistency throughout this paper.

CANADIAN STANDARD – CSA S304.1-04 [2]

In the Canadian masonry standard, provisions for calculating in-plane masonry shear capacity are provided based on the diagonal and sliding shear modes of failure. Although not explicitly stated, there is an expectation that walls also be designed for in-plane flexural shear resistance. This is determined by an analysis similar to flexure for beams accounting for axial load. In this paper it is assumed that the moment acting on the wall comes from only the applied shear force acting at the top of the wall.

The diagonal shear resistance is determined by accounting for masonry shear strength, applied axial load, and horizontal reinforcement as shown in Equations 1 and 2. In Equation 1, $M_f/V_f d_v$ must not be less than 0.25 nor more than 1.0. In this paper, where the only moment acting on the wall is that due to the shear force at the top of the wall, and the effective depth of the wall is taken as 0.8L, this portion of the equation reduces to $H/0.8L$.

$$v_m = 0.16 \left(2 - \frac{M_f}{V_f d_v} \right) \sqrt{f'_m} \quad (1)$$

$$V_n = \phi_m (v_m t d_v + 0.25P) \gamma_g + \left(0.60 \phi_s A_h f_y h \frac{d_v}{s} \right) \leq 0.4 \phi_m \sqrt{f'_m} t d_v \gamma_g \quad (2)$$

The third mode of failure, sliding shear, is calculated based on the applied axial load, the vertical reinforcement, and the friction between the sliding surfaces, as shown in Equations 3 and 4. To be conservative, in the calculations presented here, only the vertical reinforcement in the middle third of the wall was accounted for as recommended by [3]. In addition, the sliding surface was assumed to be masonry to smoothed concrete or steel to yield a smaller coefficient of friction.

$$P_2 = 0.9P + \phi_s A_t f_{yt} \quad (3)$$

$$V_n = \phi_m \mu P_2 \quad (4)$$

US STANDARD – TMS 402-08/ACI 530-08/ASCE 5-08 [4]

The US masonry standard is similar to the Canadian standard with a few exceptions. The design shear capacity is based only on diagonal shear failure and flexural shear failure. Sliding shear failure provisions are included only for autoclaved aerated concrete masonry. Furthermore, US design standards rely on strength reduction factors rather than material reduction factors.

The US masonry standard recommends the nominal strength for combined flexure and axial load be based on applicable conditions of equilibrium. Therefore, the flexural resistance was found in a similar manner as above. The diagonal shear resistance is again based on a summation of the resistance provided by the masonry, applied axial load, and reinforcement as shown in Equations 5 through 8 below. In these equations, $M_f/V_f d_v$ must not be greater than 1.0 and was again taken as $H/0.8L$ in this paper.

$$V_{nm} = \left[4.0 - 1.75 \left(\frac{M_f}{V_f d_v} \right) \right] A_n \sqrt{f'_m} + 0.25P \quad (5)$$

$$V_{ns} = 0.5 \left(\frac{A_h}{S} \right) f_{yh} d_v \quad (6)$$

$$V_n = \phi_v (V_{nm} + V_{ns}) \leq \phi_v V_{nmax} \quad (7)$$

$$\begin{aligned} V_{nmax} &= 6A_n \sqrt{f'_m} \text{ for } \frac{M_f}{V_f d_v} \leq 0.25 \\ &= 4A_n \sqrt{f'_m} \text{ for } \frac{M_f}{V_f d_v} \geq 1.0 \\ &= \text{linearly interpolated for } 0.25 \leq \frac{M_f}{V_f d_v} \leq 1.0 \end{aligned} \quad (8)$$

AUSTRALIAN STANDARD – AS 3700-2001 [5]

The Australian standard also bases its design on flexural and diagonal shear. The capacity reduction factor, ϕ , is 0.75 for reinforced masonry under all loading action effects.

The diagonal shear resistance in this standard accounts for the masonry and reinforcement shear resistance, but does not attribute any resistance to an applied axial load. The diagonal shear resistance is found through Equations 9 and 10 below.

$$V_n \leq \phi (f_{vr} t d_v + 0.8 f_{yt} A_s) \quad (9)$$

$$f_{vr} = \left(1.50 - 0.5 \frac{H}{L} \right) MPa \quad (10)$$

The flexural shear resistance is found by checking the wall in both bending and overturning. The bending resistance is found through Equations 11 and 12, again assuming for the purposes of this paper that the moment comes from the shear force acting at the top of the wall. The overturning resistance is similar to the flexural shear capacity considered by the Canadians and the Americans and is based on the applied axial force and the vertical reinforcement and calculated from Equations 13 and 14.

$$M_d \leq \phi f_{yt} A_{sd} d_v \left[1 - \frac{0.6 f_{yt} A_{sd}}{(1.3 f'_m) t d_v} \right] \quad (11)$$

$$A_{sd} = \min \left\{ \frac{(0.29)1.3f'_m t d_v}{f_{yt}} \text{ or } A_t \right\} \quad (12)$$

$$V_n = \phi \left[\frac{k_{sw} P \frac{L}{2} + f_{yt} A_t (L - 2l')}{H} \right] \quad (13)$$

$$k_{sw} = 1 - \frac{P}{t d_v f'_m} \quad (14)$$

NEW ZEALAND STANDARD – NZS 4230:2004 [6]

In this masonry standard, design must be based on the observation type of the masonry. Type C allows the masonry to be built without construction observation, Type B must be inspected during construction to ensure the design is being interpreted correctly and generally carried out as specified, and Type A has the same requirements as Type B with the additional requirement of construction supervision at all critical stages to ensure the standards of workmanship and materials are of consistently high quality. For the purposes of this paper, Type B observation was assumed as the commentary stated this was the default method for most structural masonry. The strength reduction factors used were 0.75 for shear and 0.85 for flexure. The design shear value is based on all three modes of failure.

Once again, the flexural shear resistance is found based on satisfying applicable conditions of equilibrium and compatibility of strain. The diagonal shear resistance is calculated as a sum of the contributions of the masonry, axial stress, and shear reinforcement through Equations 15 through 21 below. In these equations, for Type B observation, v_g and v_{bm} are 1.50 MPa and 0.70 MPa, respectively, and C_3 is 0.8 for walls.

$$V_n = \phi_v v_n b_w d_v \quad (15)$$

$$v_n = v_m + v_p + v_s \leq v_g \quad (16)$$

$$v_m = (C_1 + C_2) v_{bm} \quad (17)$$

$$C_1 = 33 \rho_w \frac{f_{yh}}{300} \quad (18)$$

$$\begin{aligned} C_2 &= 1.5 \text{ for } \frac{H}{L} < 0.25 \\ &= 0.42 \left[4 - 1.75 \left(\frac{H}{L} \right) \right] \text{ for } 0.25 \leq \frac{H}{L} \leq 1.0 \\ &= 1.0 \text{ for } \frac{H}{L} > 1.0 \end{aligned} \quad (19)$$

$$v_p = 0.9 \frac{P}{b_w d_v} \tan \alpha \quad (20)$$

$$v_s = C_s \frac{A_s f_{yk}}{b_w s} \quad (21)$$

The sliding shear resistance is found through Equation 22 below, conservatively using a coefficient of friction of 0.7 assuming masonry against smoothed concrete or steel here.

$$V_n = \phi_{vf} (A_{vf} f_{yf} + P) \quad (22)$$

EUROPEAN STANDARD – EUROCODE 6 [7]

In this standard, material strengths are divided by partial safety factors to get design values rather than multiplied by reduction factors. The partial safety factors, γ_{M_s} , are 2.2 and 1.15 for masonry and steel, respectively. The design resistance in this standard is based on diagonal shear and flexural shear.

The diagonal shear resistance is calculated by Equations 23 through 26 below. For aggregate concrete masonry and general purpose mortar, the initial masonry shear strength is 0.20 MPa.

$$V_n = V_{Rd1} + V_{Rd2} \quad (23)$$

$$V_{Rd1} = f_{vd} t L \quad (24)$$

$$f_{vd} = f_{vko} + 0.4 \sigma_d \leq 0.065 f'_m \quad (25)$$

$$V_{Rd2} = 0.9 A_{sw} f_{yd} \quad (26)$$

The flexural shear resistance is calculated by Equations 27 and 28. The design shear force was again found here by assuming it acts at the top of the wall.

$$M_n = A_t f_{yd} z \leq 0.4 f_d t d_v^2 \quad (27)$$

$$z = d_v \left(1 - 0.5 \frac{A_t f_{yd}}{t d_v f_d} \right) \leq 0.95 d_v \quad (28)$$

BRITISH STANDARD – BS 5628-2:2005 [8]

The British standard is similar to the Eurocode, as seen in Equations 29 through 33. One exception was that the initial shear strength of the masonry is always 0.35 MPa as seen in Equation 30. In addition, the partial safety factor for masonry is 2.3 in flexure and compression and 2.0 in shear.

$$v = \frac{f_v}{\gamma_{mv}} + \frac{A_s}{L t} \left(\frac{f_{yk}}{\gamma_{ms}} \right) \quad (29)$$

$$f_v = 0.35 \text{MPa} + 0.6 \sigma_d \leq 1.75 \text{MPa} \quad (30)$$

$$V_n = v t L \quad (31)$$

$$z = d_v \left(1 - \frac{0.5 A_t f_{yt} \gamma_{mm}}{t d_v f'_m \gamma_{ms}} \right) \quad (32)$$

$$M_n = \frac{A_t f_{yt} z}{\gamma_{ms}} \leq 0.4 \frac{f'_m t d_v^2}{\gamma_{mm}} \quad (33)$$

ANDERSON AND PRIESTLEY [9]

Anderson and Priestley developed a shear model after analyzing data from tests carried out through the University of Berkley [10, 11], the University of Colorado [12], and the Building Research Institute in Japan [13]. Given that this model did not include reduction factors or partial safety factors, the material reduction factors from the Canadian standard are used to get a design value for shear resistance in this paper. This model accounts for all three modes of in-plane shear failure.

The equation for the diagonal shear was initially based on the contributions of the masonry, the vertical reinforcement in the middle third of the wall, the axial stress, and the horizontal reinforcement. After data fitting, Anderson and Priestley found that the vertical steel in the middle third of the wall had no effect on the diagonal shear resistance, resulting in Equations 34 through 37 below.

$$V_n = V_{nm} + V_{np} + V_{ns} \quad (34)$$

$$V_{nm} = k_1 k_2 b_1 \sqrt{f'_m} L t \quad (35)$$

$$V_{np} = b_2 P \quad (36)$$

$$V_{ns} = b_4 A_h f_{yh} \frac{L}{S} \quad (37)$$

The sliding shear resistance is determined through Equations 38 and 39 below. It was assumed for this paper that the applied moments had reached M_n , found below in Equation 41, so the compression steel force balanced the tension steel force. Therefore only the vertical steel in the middle third of the wall was accounted for in C. Finally, the flexural shear resistance is found through Equations 40 and 41. Once again, the moment was assumed to derive from the shear force acting at the top of the wall.

$$C = A_v f_{yv} + P \quad (38)$$

$$V_n = \mu C \quad (39)$$

$$a = \frac{A_v f_{yv} + P}{0.85 f'_m f} \quad (40)$$

$$M_n = \frac{1}{2} [(A_v f_{yv} + P)(L - \alpha) + A_t f_{yt}(L - 2d')] \quad (41)$$

VOON AND INGHAM [14]

The shear model proposed in the 2004 New Zealand standard was largely based on research completed by Voon and Ingham. The proposed model was modified for simplicity for adoption in the national standard. The model presented here is the original proposed masonry shear equation. This equation is for diagonal shear only, and is shown in Equations 42 through 49. As this model was developed for use in the New Zealand standard, for comparison in this paper, the methods for determining the sliding and flexural shear resistance were taken from NZS 4230:2004 and the strength reduction factors of 0.75 for shear and 0.85 for flexure were used.

$$V_n = V_{nm} + V_{np} + V_{ns} \leq 0.33A_n \sqrt{f'_m} \quad (42)$$

$$V_{nm} = v_m t d_v \quad (43)$$

$$v_m = k(C_a + C_b) \sqrt{f'_m} \quad (44)$$

$$C_a = 0.022 \rho_v f_{yv} \quad (45)$$

$$C_b = 0.083 \left[4 - 1.75 \left(\frac{M_f}{V_f L} \right) \right] \text{ where } 0.25 \leq \frac{M_f}{V_f L} \leq 1.0 \quad (46)$$

$$V_{np} = 0.9P \tan \alpha \quad (47)$$

$$V_{ns} = A_n f_{yt} \frac{D_{eff}}{s} \quad (48)$$

$$D_{eff} = L - 2d' - l_{dh} \quad (49)$$

COMPARISON TO EXPERIMENTAL TEST RESULTS

The experimental data used for comparison basis throughout this paper was taken from sources [9] and [14] for a total of 75 walls. Figure 2 shows the factors of safety for each of the 75 test walls. The factors of safety were calculated as a ratio of the predicted design shear value from each model to the experimental shear force. Table 1 shows the mean and standard deviation of the factor of safety for each shear model compared to these test walls. It also shows the percentage of walls in which the models predicted the correct mode of failure. In the test data, 51 of the walls failed in diagonal shear, 15 failed in flexural shear, 2 failed in sliding shear, and 7 failed in a combination of two or more of the failure modes. For the combined failures, if the model predicted the failure in any one mode in that combination, it was assumed to predict the correct failure mode.

Table 1: Model Comparison with Test Data

Model	F.S.	Standard Deviation	Correct Failure Mode
CSA S304.1-04	3.14	1.22	58.7%
ACI 530-08	2.01	0.55	73.3%
AS 3700-2001	3.75	1.78	56.0%
NZS 4230:2004	2.10	0.54	72.0%
Eurocode 6	5.29	2.61	41.3%
BS 5628-1:2005	4.91	2.22	32.0%
Anderson and Priestley	2.41	0.92	25.3%
Voon and Ingham	1.75	0.50	68.0%

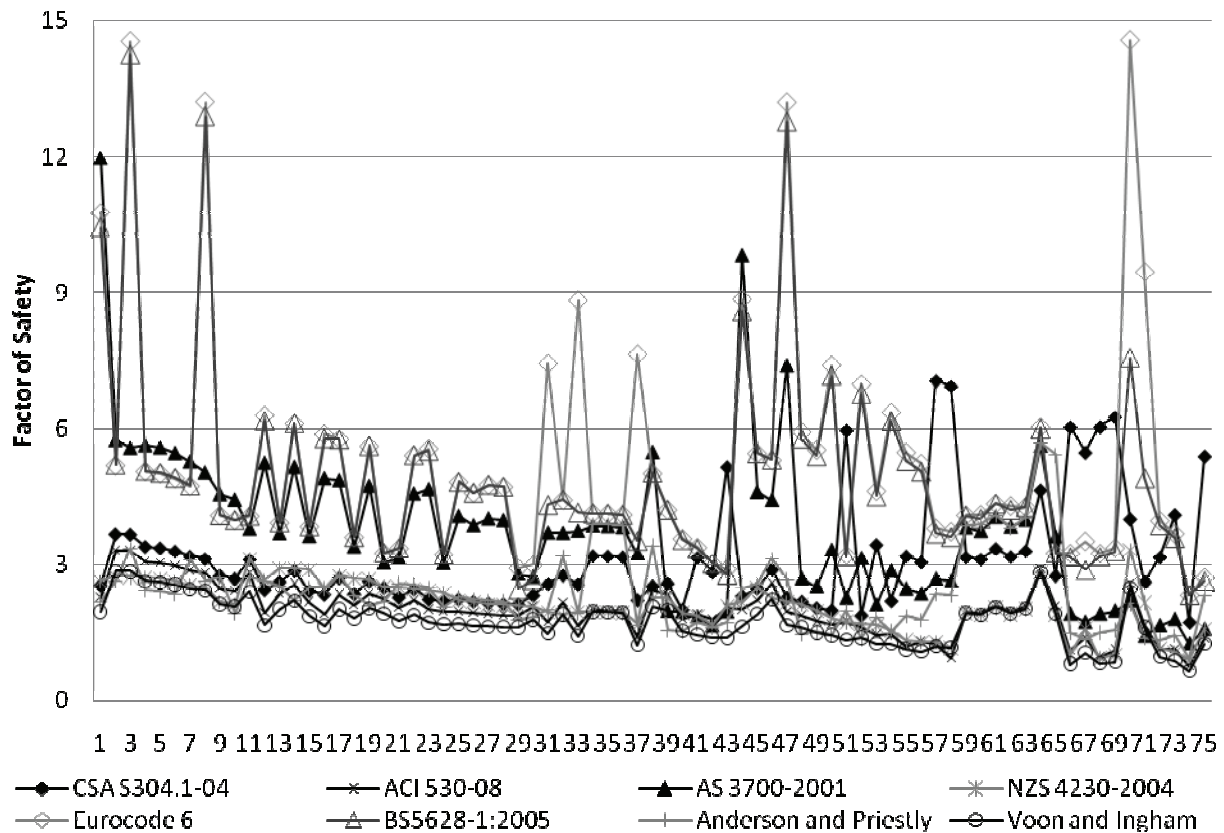


Figure 2: Factors of Safety for Test Walls

DISCUSSION AND CONCLUSIONS

The test data were broken down into subsets by various factors such as axial load, amount of reinforcing, and masonry compressive strength. For each subset of data, a new mean factor of safety was computed for each analytical model and compared to the means from the overall data set. In the data set, ten walls which had zero applied axial load. For these walls, the mean factor of safety for the Canadian model is 5.55, an increase of approximately 77 percent. In the rest of the models, the mean factor of safety decreased anywhere from 3 to 45 percent. Furthermore,

the Canadian model predicted that each of these walls would fail in sliding shear, when they actually failed in either diagonal or flexural. Therefore, the sliding shear resistance for the Canadian standard appears to be very conservative in cases where there is no applied axial load.

The amount of horizontal reinforcement appeared to have little effect on the mean factors of safety for most of the models. In the Anderson and Priestley model, however, walls with little to no horizontal reinforcement showed a decrease in the mean factor of safety of almost 30 percent. In contrast, walls with a large amount of horizontal reinforcement had an increase in the mean factor of safety of 33 percent.

For walls with a large amount of vertical reinforcement in the middle third of the wall, each of the codes appeared to be much less conservative with the mean factors of safety reducing anywhere from 22 to 51 percent. The Canadian model, however, was more conservative with the mean factor of safety increasing by 35 percent. Conversely, for walls with very little reinforcement in the middle third of the wall, most of the models were more conservative, again with the exception of the Canadian model. The Australian model had the greatest increase in the mean factor of safety at 66 percent. In the European and British models, having a small amount of vertical reinforcement in the outer third of the wall greatly increases the conservativeness of the model. For the rest of the models, there was little change in the factors of safety whether there was a small or large amount of vertical reinforcement in the outer third of the wall.

For the purposes of comparison, walls with masonry strength less than 15 MPa were considered to have low masonry strength. For these walls, the American, Australian, European, British, and Voon and Ingham models were considerably more conservative with increases in the mean factors of safety of anywhere from 22 to 38 percent. Walls with masonry strength more than 25 MPa were considered to have high masonry strength, and generally there was no significant effect on the mean factor of safety.

For walls with an aspect ratio of 1.0 or greater, there was little effect on the mean factor of safety for each of the models. There was only one wall with an aspect ratio less than 1.0. This wall had an increased factor of safety of 72 percent for the Canadian model. For the rest of the models, this wall had a decrease in the factor of safety up to 58 percent.

The diagonal shear model proposed by Voon and Ingham, combined with the models for sliding and flexure from the New Zealand standard, produced the lowest mean factor of safety, as well as the lowest standard deviation for the factor of safety. Furthermore, it was one of the best at predicting the correct failure mode. The simplified version of this model used in the New Zealand standard also had a relatively low mean factor of safety with a small standard deviation. It was also slightly more accurate in predicting the failure mode. The US standard had factors of safety and standard deviation similar to that of the New Zealand standard. The fact that it does not have a method for determining sliding shear resistance was irrelevant since only two walls actually failed in sliding. The Canadian and Australian standards had higher mean factors of safety and standard deviations, and were less accurate in predicting the failure mode. Finally, the British and European standards proved to be the least accurate shear models. These two models frequently predicted the walls would fail in flexure, indicating that their flexural shear provisions are very conservative.

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LIST OF SYMBOLS

a	depth of the compression block (mm)	P_2	axial dead load and factored tensile resistance at yield of vertical reinforcement (kN)
A_h	area of horizontal reinforcement (mm^2)	p_w	reinforcement ratio
A_n	net cross-sectional area (mm^2)	s	spacing of horizontal reinforcement (mm)
A_s	cross-sectional area of reinforcement (mm^2)	t	thickness of wall (mm)
A_{sw}	area of shear reinforcement (mm^2)	v	shear stress (MPa)
A_t	area of vertical reinforcement in outer third of the wall (mm^2)	v_{bm}	basic type-dependent shear strength of masonry (MPa)
A_v	area of vertical reinforcement in the middle third of the wall (mm^2)	V_{exp}	applied experimental shear force (kN)
A_{vf}	area of shear-friction reinforcement (mm^2)	V_f	factored applied shear force (kN)
b_w	effective web width (mm)	v_g	maximum permitted type-dependent total shear stress (MPa)
C	maximum compressive force in masonry (kN)	v_m	shear strength attributed to the masonry (MPa)
$C_{1,2,3}$	shear strength coefficients	V_n	nominal shear strength (kN)
$C_{a,b}$	shear strength coefficients	v_n	total shear stress (MPa)
d'	distance from vertical reinforcement to edge of the wall (mm)	V_{nm}	nominal masonry shear strength (kN)
D_{eff}	effective depth of section (mm)	V_{nmax}	maximum nominal shear strength (kN)
d_v	effective depth (mm)	V_{np}	nominal axial stress shear strength (kN)
f_d	design compressive strength of masonry (MPa)	V_{ns}	nominal shear strength provided by horizontal reinforcement (kN)
f_{yh}	yield strength of horizontal steel (MPa)	v_p	shear strength attributed to axial stress (MPa)
f'_m	masonry compressive strength (MPa)	V_{Rd1}	design value of shear resistance attributed to masonry (kN)
f_v	characteristic shear strength of masonry (MPa)	V_{Rd2}	design value of shear resistance attributed to reinforcement (kN)
f_{vd}	design shear strength of masonry (MPa)	v_s	shear strength attributed to reinforcement (MPa)
f_{vko}	initial shear strength of masonry (MPa)	z	lever arm (mm)
f_{yd}	design strength of reinforcing steel (MPa)	α	angle formed between lines of axial load action and resulting reaction on a component ($^\circ$)
f_{yf}	yield strength of shear-friction steel (MPa)	γ_g	factor to account for partially grouted walls
f_{vr}	shear stress value (MPa)	γ_M	partial safety factor for materials
f_{yt}	yield strength of vertical reinforcement in outer third of the wall (MPa)	γ_{mm}	partial safety factor for strength of masonry
f_{yv}	yield strength of vertical reinforcement in middle third of the wall (MPa)	γ_{ms}	partial safety factor for strength of steel
H	height of wall (mm)	γ_{mv}	partial safety factor for shear strength of masonry
k	ductility reduction factor	μ	coefficient of friction
L	length of wall (mm)	ϕ_m	resistance factor for masonry
l_{dh}	shear reinforcement development length (mm)	ϕ_s	resistance factor for steel
M_n	nominal flexural resistance (kN-m)	ϕ_v	strength reduction factor for shear
M_f	factored applied moment (kN-m)	ρ_v	ratio of vertical reinforcing steel
M_{Rd}	design moment resistance (kN-m)	σ_d	design compressive stress (MPa)
P	applied axial load (kN)		