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FLEXURAL RESISTANCE OF UNBONDED AND INTERMITTENTLY BONDED FRP-REINFORCED MASONRY PANELS

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ABSTRACT

Unobtrusive FRP rehabilitation techniques were developed to enhance the out-of-plane flexural resistance of masonry wall panels by increasing their ability to absorb energy. In these techniques, unbonded and intermittently bonded FRP reinforcement were used to produce higher rotations and, consequently, large displacements. Experimental and analytical investigations were conducted to evaluate the effectiveness of the developed FRP rehabilitation techniques. Without a design approach available for practitioners, the full potential of these techniques would be hard to realize.

In this paper, a simplified design methodology for calculating the capacity of this type of FRP-reinforced masonry wall panel under out-of-plane pressure is described. The proposed method is based on an analytical model that was developed to predict the post-cracking lateral pressure-displacement response under biaxial bending. The conservation of energy principle is applied to determine the capacity whereas displacement is calculated from rotations employing rigid body mechanics. The resulting wall sub-panels, subsequent to a fully developed crack pattern, are considered to behave as rigid segments that rotate around crack lines. The applicability of the design method is demonstrated by a numerical example.

KEYWORDS: wall panels, out-of-plane resistance, fibre reinforced polymers, unbonded reinforcement, intermittently bonded reinforcement, rigid-body mechanics

INTRODUCTION

The ability of a wall to absorb energy plays a significant role in its lateral load resistance to seismic loads. Due to the linear elastic response of FRP reinforcement until failure, the load-displacement of fully bonded FRP-reinforced members lacks the near constant moment plateau associated with yield of steel. This results in limited deformation and low energy absorption (defined as the area under the load-displacement curve). Higher energy absorption can be obtained using unbonded FRP reinforcement and a ductile-like response can be achieved using intermittently bonded FRP reinforcement when the length of the bonded regions, other than end anchors, is designed purposely for the bond to break prior to rupture of FRP reinforcement.

Deformation incompatibility between epoxy mortars, commonly used to adhere near surface mounted FRP reinforcement to masonry walls, and the original cement/lime mortars is another reason for choosing unbonded reinforcement over fully bonded reinforcement.

An extensive research program was carried out at McMaster University [2] to develop unobtrusive FRP rehabilitation techniques that improve the out-of-plane flexural resistance of unreinforced masonry wall panels by increasing their energy absorption capacities. In these techniques, FRP reinforcement mounted near the surface in epoxy-filled grooves in the bed and head joints, was either unbonded or intermittently bonded to the masonry wall [3]. These techniques meet the stringent requirements of restoration of historical buildings [6] and are cost-effective alternatives applicable to other existing masonry structures.

In order to analyze performance, extend the range of the investigated parameters, and define limitations an analytical model was developed [4]. The model predicts the post-cracking lateral load-displacement response of unbonded and intermittently bonded FRP-reinforced masonry panels under biaxial bending. The response of this type of walls cannot be modelled by conventional approaches assuming local strain compatibility and linear strain gradient. The analytical model is based on balancing internal and external work using rigid body mechanics to satisfy conservation of energy. The experimental observations showed that, after cracking, the wall sub-panels behaved as rigid segments that rotated around crack lines. Through an analytical investigation, a relatively wide range of parameters was covered [5].

Using the principles of this analytical model, the method reported herein was developed to derive design formulae for this type of FRP-reinforced masonry panel for the most common cases of aspect ratios and support condition. The main aspects of this method are the relationship between the vertical and horizontal internal moments of resistance across crack lines and the relationship between rotations in the vertical and horizontal directions and the out-of-plane displacement.

DESIGN ASSUMPTIONS

In developing this design methodology, the following assumptions have been made:

1. Walls are lightly reinforced and failure takes place by rupture of FRP reinforcement.
2. FRP Reinforcement in both the vertical and horizontal directions is either unbonded (end anchored) or intermittently bonded so as to permit intermittent bond to break but FRP reinforcement remains end anchored.
3. Relative rotations between sub-panels of the wall are proportional to the out-of-plane displacement (rigid body rotation).
4. Masonry maximum compressive strain at contact zones between sub-panels is 0.003.
5. The effect of self-weight on enhancing lateral resistance is negligible.

ONE-WAY FLEXURE WALLS

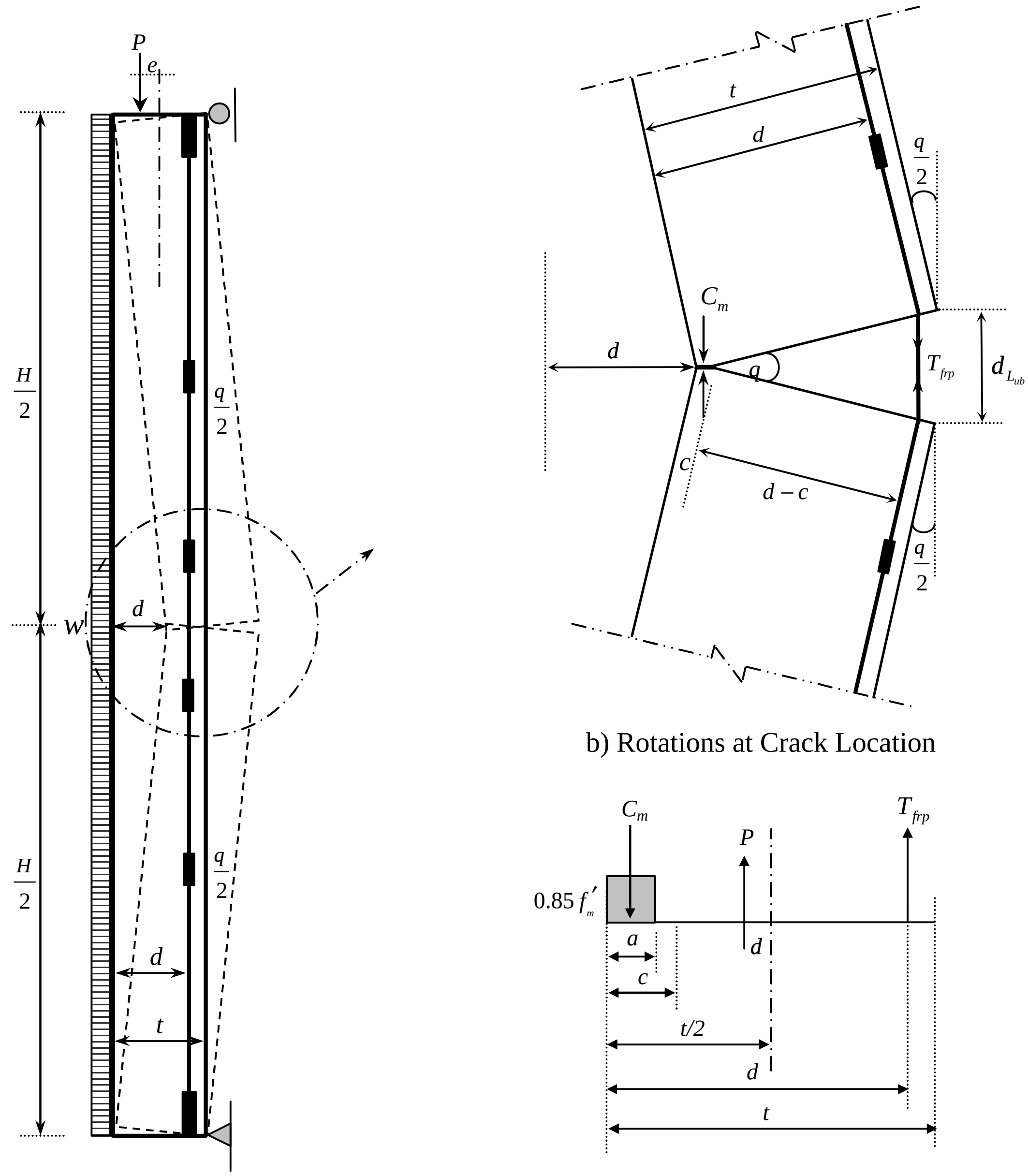
Figure 1 shows a masonry wall simply supported at the top and the bottom and free at its sides. The wall is lightly reinforced with near surface mounted intermittently bonded FRP reinforcement. The rigid body displacement, rotations at the mid-height crack, and forces acting at crack location are shown in Figures 1(a), 1(b) and 1(c), respectively. The ultimate tensile force in FRP reinforcement per unit length of the wall is given by

$$T_{frp_u} = j_{frp} A_{frp} E_{frp} e_{frp_u} \qquad \text{Equation 1}$$

where, j_{frp} = FRP material resistance factor

A_{frp} = area of FRP reinforcement per unit length

E_{frp} = FRP modulus of elasticity, and e_{frp_u} = FRP ultimate strain



a) Rigid Body Displacement

b) Rotations at Crack Location

c) Forces at Crack Location

Figure 1 – One-way Flexure of a Wall Reinforced with Intermittently Bonded FRP

Due to the lack of a better approach and the relative insensitivity of the flexural capacity of unbonded reinforced masonry to the distribution of the compressive stress over the contact zone, masonry is assigned the ultimate strain of 0.003 at the extreme compression fibre. The masonry compressive force per unit length of the wall is then calculated from

$$C_m = j_m c\, 0.85 f'_m a$$ Equation 2

where, j_m = masonry material resistance factor

c = factor depends on the direction of compressive strength relative to bed joints

f'_m = masonry maximum compressive strength, and a = depth of rectangle stress block

From equilibrium of forces, the depth of the compression stress block (a) is determined from

$$C_m - T_{frp} = P$$ Equation 3

Referring to Figure 1(c), the resisting moment is then given by

$$M_r = T_{frp}\left(d - \frac{t}{2}\right) + C_m\left(\frac{t-a}{2}\right) - P\left(d + e\right)$$ Equation 4

where, d = depth of FRP reinforcement

t = thickness of masonry wall

P = axial compressive load

d = wall out-of-plane displacement

e = eccentricity of axial load or minimum eccentricity

Referring to Figure 1(b), the crack angle (q) can be calculated from the elongation (d_{Lub}) in the unbonded FRP reinforcement where the unbonded length of FRP reinforcement (L_{ub}) is approximately equal to the wall height (H) after bond failure of intermittent bond regions.

$$q = \frac{d_{Lub}}{(d-c)} = \frac{L_{ub} e_{frp}}{(d-c)} = \frac{H e_{frp}}{(d-c)}$$ Equation 5

From geometry, the displacement (d) is related to the rotation, q/2, by $d = Hq/4$. Substituting for q from Equation 5 and for $e_{frp} = e_{frp_u}$ at ultimate limit state, the displacement becomes

$$d = \frac{Hq}{4} = \frac{H^2 e_{frp_u}}{4(d-c)}$$ Equation 6

The maximum lateral pressure (w) that the wall can resist is calculated using small deflection theory from

$$w = \frac{8M_r}{H^2}$$ Equation 7

TWO-WAY FLEXURE WALLS

For the wall panel supported on 4-sides shown in Figure 2, if the moments of resistance across crack lines in the vertical and horizontal directions are known, the conservation of energy principle can be applied to calculate the lateral pressure capacity by equating the internal work done by resisting moments to the external work done by the applied pressure. FRP reinforcement in the main direction reaches its ultimate capacity and ruptures causing failure. However, FRP reinforcement in the secondary direction does not necessarily reach its ultimate strain. FRP strain in the secondary direction, say horizontal, is related to that of the main direction, say vertical, by the ratio between the horizontal and vertical rotations (q_h and q_v).

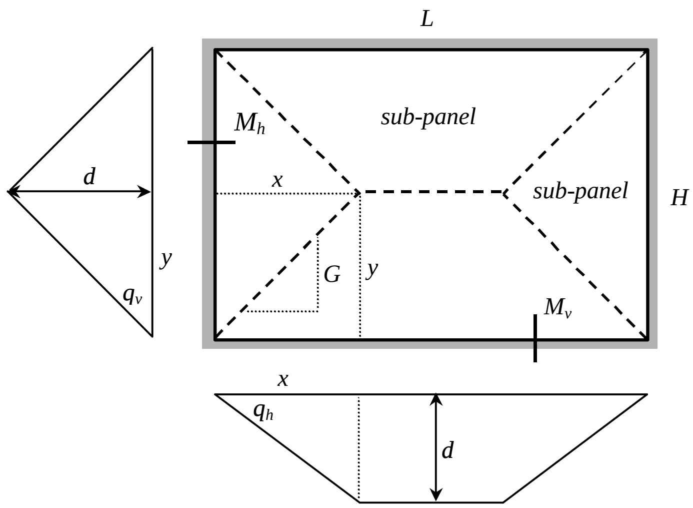


Figure 2 – Rigid Body Displacement and Rotations of Unbonded FRP-Reinforced Panel

From Figure 2 and for small angles so that $\tan q \cong \sin q \cong q$,

$$q_v = d / y \qquad \text{and} \qquad q_h = d / x$$

Therefore, $q_h / q_v = y / x = G$, where G is the slope of diagonal cracks

$$\therefore\ q_h = G\ q_v \qquad \text{Equation 8}$$

From Equation 5 it follows that

$$q_v = \frac{He_{frp_v}}{2(d_v - c_v)} \quad \text{and} \quad q_h = \frac{Le_{frp_h}}{2(d_h - c_h)} \qquad \text{Equation 9}$$

H and L in the above equation represent approximate values of the unbonded lengths of FRP reinforcement in the vertical and horizontal direction, respectively. Considering for lightly reinforced sections that $(d_v - c_v) \approx (d_h - c_h) \cong d$ and substituting the rotations from Equation 9 into Equation 8, the strain in the horizontal FRP reinforcement is related to that in the vertical FRP reinforcement by

$$e_{frp_h} L = G e_{frp_v} H$$

Following the assumption that the main resisting direction is the vertical direction, the strain in the vertical FRP reinforcement (e_{frp_v}) at ultimate state is equal to e_{frp_u} after failure of the inter-mittent bond locations which leaves an unbonded length (*H*) between anchorage points and thus

$$e_{frp_h} = e_{frp_u} GH / L \qquad \text{Equation 10}$$

By a similar analysis, it can be shown that when the situation is reversed and the main resisting direction is the horizontal direction, FRP strain in the vertical direction is given by

$$e_{frp_v} = e_{frp_u} L / GH \qquad \text{Equation 11}$$

Knowing the level of strain in FRP reinforcement in both the vertical and horizontal directions, the moments of resistance, M_v and M_h, in the respective directions across crack lines can be calculated similar to the case of one-way flexure.

Considering the wall sub-panels shown in Figure 2 to be in equilibrium, no additional work is done if compatible displacements are introduced. The pressure capacity of the FRP-reinforced flexure wall panel needs to be determined. The first determinant will be an idealized crack pattern which divides the wall into sub-panels. A point along this crack pattern is chosen and is given a displacement, *d*, in the load direction. The total external work done is the sum of the external work for each wall sub-panel. The total internal work done is the sum of the internal work done along each crack line.

External Work = $\sum W d_c = w \sum a d_c$
Internal Work = $\sum mLq = \sum \left(M_h H q_h + M_v L q_v \right)$

To satisfy conservation of energy, the total external work should equal the total internal work.

$$\therefore w = \frac{\sum \left(M_h H q_h + M_v L q_v \right)}{\sum a d_c} \qquad \text{Equation 12}$$

where W = total load on a wall sub-panel; d_c = displacement of the centroid of that sub-panel
w = pressure per unit area resisted by the wall; $a =$ area of sub-panel
m = bending moment per unit length of crack line
L = length of crack line, and q = angle change at crack line

Figure 3(a) shows idealized crack patterns for solid walls supported along 4-sides. Crack patterns for solid walls supported on 3-sides are shown in Figures 3(b) and 3(c) for walls free at the top and the side, respectively. The lateral pressure capacities for the cases that the designer is likely to encounter were derived using Equation 12 and are summarized in Table 1. The application of the method to other cases is straightforward.

DIAGONAL CRACK SLOPE (G)

The expressions presented in Table 1 require that the slope of diagonal cracks, *G*, be known. The experimental observation [2] showed that the crack patterns of the FRP-reinforced walls were quite similar to those of the corresponding unreinforced walls. This suggests that, for usual cases, the idealized crack patterns for walls lightly reinforced with FRP can be reasonably represented by the crack patterns of the counterpart unreinforced walls. The use of unbonded reinforcement reduces the likelihood that tensile stresses would build-up sufficiently to form a new crack.

Idealized crack patterns are determined by applying the principle of minimum load capacity to Equation 12. The equation is written in terms of the unknown dimensions, which define the crack locations for a complete failure mechanism. The values of these unknown dimensions are those that give the minimum wall capacity. Such values can be found by solving simultaneously the partial differentiation of Equation 12 with respect to each of the unknown dimensions. The values of *G* were calculated for Case (a) of Figure 3 and are given in Table 2.

Table 1 – Capacities of FRP-Reinforced Masonry Panels (Cases of Figure 3)

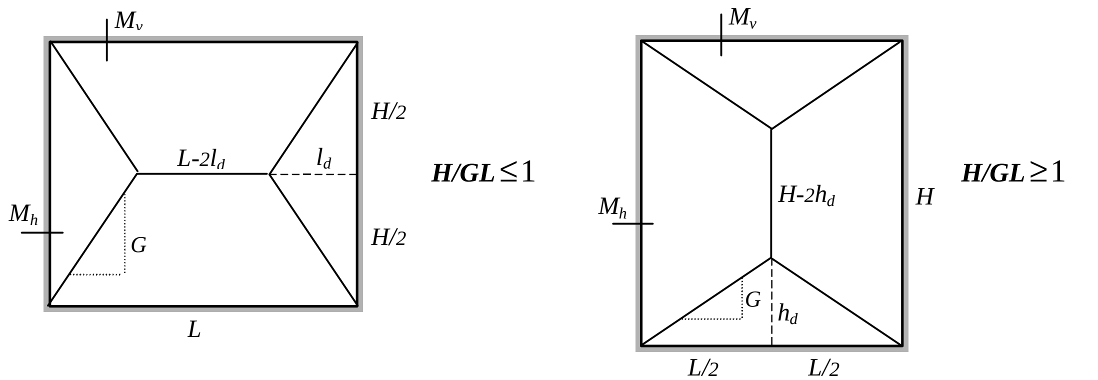
Case	*H/GL*	Lateral Pressure Capacity (*w*)
(a)	≤ 1	$\left[4M_h(G)+4M_v\left(\frac{L}{H}\right)\right] \Big/ \left[\left(\frac{LH}{2}\right)-\left(\frac{H^2}{6G}\right)\right]$
	≥ 1	$\left[4M_h\left(\frac{H}{L}\right)+4M_v\left(\frac{1}{G}\right)\right] \Big/ \left[\left(\frac{LH}{2}\right)-\left(\frac{GL^2}{6}\right)\right]$
(b)	$\leq 1/2$	$\left[2M_h(G)+2M_v\left(\frac{1}{G}\right)\right] \Big/ \left[\left(\frac{HL}{2}\right)-\left(\frac{H^2}{3G}\right)\right]$
	$\geq 1/2$	$\left[4M_h\left(\frac{H}{L}\right)+2M_v\left(\frac{1}{G}\right)\right] \Big/ \left[\left(\frac{LH}{2}\right)-\left(\frac{GL^2}{12}\right)\right]$
(c)	≤ 2	$\left[2M_h(G)+4M_v\left(\frac{L}{H}\right)\right] \Big/ \left[\left(\frac{HL}{2}\right)-\left(\frac{H^2}{12G}\right)\right]$
	≥ 2	$\left[2M_h(G)+2M_v\left(\frac{1}{G}\right)\right] \Big/ \left[\left(\frac{HL}{2}\right)-\left(\frac{GL^2}{3}\right)\right]$

Table 2 – Diagonal Crack Slope, *G*, for Case (a) Support Condition of Figure 3

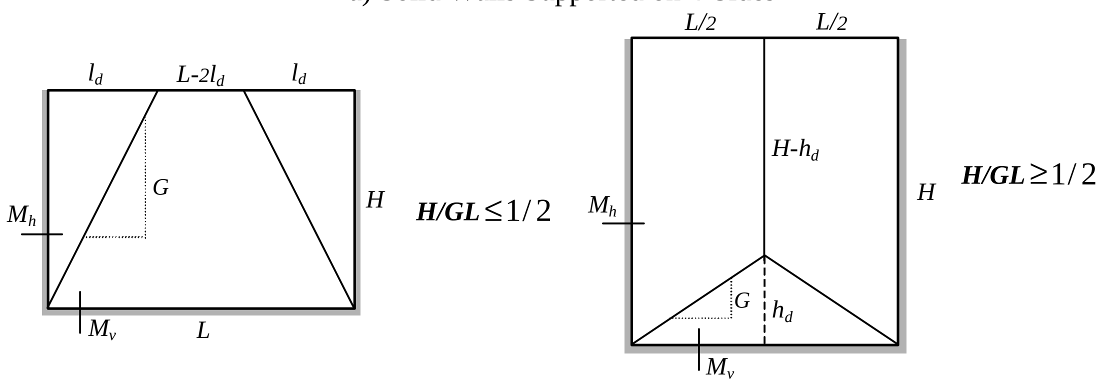
m	***H/L***										
	0.30	**0.40**	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.50**	**3.00**
1.0	0.69	0.71	0.75	0.87	1.00	1.10	1.17	1.23	1.30	1.33	1.40
0.9	0.64	0.71	0.75	0.87	1.00	1.10	1.13	1.20	1.23	1.27	1.33
0.8	0.64	0.67	0.71	0.83	0.93	1.03	1.10	1.13	1.20	1.23	1.27
0.7	0.60	0.63	0.68	0.80	0.90	0.97	1.03	1.10	1.13	1.17	1.20
0.6	0.56	0.60	0.65	0.75	0.87	0.93	1.00	1.03	1.07	1.10	1.13
0.5	0.53	0.57	0.60	0.75	0.83	0.90	0.93	0.97	1.00	1.03	1.07
0.4	0.47	0.52	0.58	0.70	0.77	0.80	0.83	0.87	0.90	0.93	0.97
0.3	0.43	0.48	0.52	0.63	0.70	0.73	0.77	0.80	0.83	0.87	0.90

$\mu = f_{tn}/f_{tp}$ (orthogonal strength ratio)

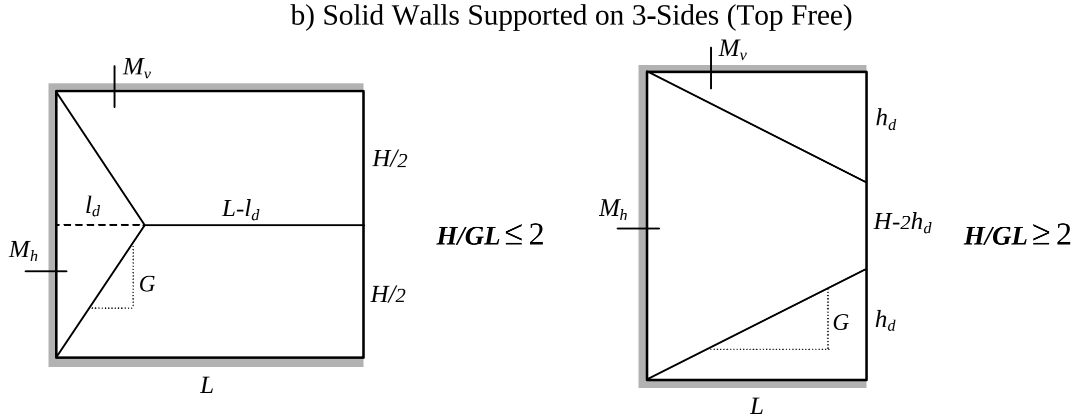
f_{tn} and f_{tp} are masonry flexural tensile strengths normal and parallel to bed joints, respectively



a) Solid Walls Supported on 4-Sides



b) Solid Walls Supported on 3-Sides (Top Free)



c) Solid Walls Supported on 3-Sides (Side Free)

Figure 3 – Idealized Crack Patterns of Masonry Wall Panels

DESIGN OF INTERMITTENT BOND REGIONS

Intermittently bonded reinforcement has the advantage of increasing the energy absorption. It may also be used to reduce lateral displacement if found to be large using unbonded reinforcement. Knowing the limiting length of a bonded region for FRP reinforcement to develop its full tensile capacity enables specifying the length of intermittent bond regions to control whether or not bond fails in these regions. If enabling higher energy absorption is the objective, the length of the intermittent bond regions should be less than the limiting length such that bond failure takes place before rupture of FRP reinforcement. If controlling deflection is the objective, the length of the intermittent bond regions must be greater than this limiting value to prevent bond failure. Obviously, the length of end anchorage must be larger than the limiting length to prevent bond failure in the anchor region. If bond stress is assumed to be constant along the length of bonded regions, the maximum tensile force in FRP reinforcement, F_{max}, before bond failure is the product of maximum bond stress between FRP and masonry, t_{max}, times the surface area of the bonded length of FRP reinforcement.

$$F_{max} = p\, d_{frp}\, l_{b_{lim}}\, t_{max} \qquad \text{Equation 13}$$

where, d_{frp} = diameter of FRP reinforcement

$l_{b_{lim}}$ = limiting bonded length of FRP reinforcement

For FRP reinforcement, the limiting bond length is found by equating the ultimate tensile capacity of the reinforcement, T_{frp_u}, to the maximum tensile force developed in the reinforcement due to bond, F_{max}. The following design values for the bonded length are recommended,

$$l_b < 75\%\ l_{b_{lim}} \rightarrow \text{ to produce bond failure}$$
$$l_b > 150\%\ l_{b_{lim}} \rightarrow \text{ to prevent bond failure} \qquad \text{Equation 14}$$

Numerical Example

A double wythe 3.7 x 2.8 x 0.19 m ($L * H * t$) unreinforced brick masonry wall is simply supported along its 4-sides. Using the material properties given below, determine the amount of unbonded near surface mounted FRP reinforcement required to resist a lateral pressure of 12 kPa.

Masonry		**FRP**
Normal to bed joints	Parallel to bed joints	
f'_{mn} = 27.1 MPa	f'_{mp} = 16.9 MPa	f_{frpu} =2,337 MPa
f_{tn} = 0.3 MPa	f_{tp} = 0.6 MPa	
E_{mn} = 11,600 MPa	E_{mp} = 8,020 MPa	E_{frp} = 147,000 MPa
e_m = 0.003	e_m = 0.003	e_{frp} = 0.0159
j_m = 0.6	j_m = 0.6	j_{frp} = 0.75

H/L = 2.8/3.7 = 0.756 and $m = f_{tn} / f_{tp}$ = 0.3/0.6 = 0.5

From Table 2, G = 0.75 $\therefore$ H/GL = 1.0 (X-Shape crack pattern)

Assuming equal moments of resistance in both directions and substituting for w = 12 kPa in Case (a) of Table 1, $M_v = M_h = 5.0$ kN-m/m

Since the wall height is the short direction, the vertical FRP reinforcement will reach its ultimate strain. Using an approximate value of the moment arm of 0.9d_v where d_v = 175 mm,

$$T_{frp} = M_v / 0.9 d_v = 5.0(10)^6 / 0.9(175) = 32.0 \text{ kN/m}$$

From $T_{frp} = j_{frp} A_{frpv} E_{frp} e_{frpu}$ $\qquad A_{frpv} = 18.0 \text{ mm}^2\text{/m}$

Try 6 - 5 mm FRP rods $\qquad A_{frpv} = 6(10.1)/3.7 = 16.38 \text{ mm}^2\text{/m}$

$$T_{frp} = j_{frp} A_{frpv} E_{frp} e_{frpu} = 0.75\ (16.38)\ 147{,}000\ (0.0159) = 28.7 \text{ kN/m}$$

$C_m = j_m 0.85 f'_{mn}(1000)a = (0.6)\ 0.85\ (27.1)(1000)a = 13.82(a)$ kN/m

From $C_m = T_{frp}$ (no axial load), $a = 2.1$ mm

$M_v = T_{frp}(d - a/2) = 28.7\ (175 - 2.1/2) = 5.0$ kN-m/m

Strain in the horizontal FRP reinforcement is determined from

$e_{frp_h} = e_{frp_u} GH / L = 0.0159\ (0.75)2.8/3.7 = 0.009$

Try 8 - 5 mm FRP rods $\quad A_{frph} = 8(10.1)/2.8 = 29.0\ \text{mm}^2\text{/m}$

$T_{frp} = j_{frp} A_{frph} E_{frp} e_{frph} = 0.75\ (29)\ 147{,}000\ (0.009) = 28.8$ kN/m

$C_m = j_m 0.85 f'_{mp}(1000)a = (0.6)\ 0.85(16.9)\ (1000)a = 8.619(a)$ kN/m

From $C_m = T_{frp}$, $a = 3.34$ mm

$M_h = T_{frp}(d_h - a/2) = 28.8\ (180 - 3.34/2) = 5.14$ kN-m/m

Substituting for M_v and M_h in either of the expressions for Case (a) in Table 1

$w_{\text{Reinf}} = 12.27$ kPa > required = 12 kPa OK

Therefore, use 8 horizontal and 6 vertical 5 mm FRP rods *Ans.*

CLOSURE

The above described design approach is intended to allow designers to make effective use of FRP reinforcement in retrofit applications of unreinforced masonry wall panels where energy absorption enhances resistance to out-of-plane loads. The observed partial and total collapse of masonry façade walls in a seismic event can be avoided by such reinforcement techniques.

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