A PROBABILISTIC APPROACH FOR THE INTERPRETATION OF LONG TERM DAMAGE OF HISTORIC MASONRY

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ABSTRACT

In some cases, the assessment of the safety of historic buildings may require the evaluation of their vulnerability toward the effects of persistent loading. The problem of achieving a reliable lifetime estimate of this kind of structures involves many uncertainties. Therefore a probabilistic approach seems to be needed.

The lifetime assessment of ancient structures is investigated here through a simple stochastic approach, modelling the results of laboratory creep tests on the masonry of ancient buildings.

KEYWORDS: historic masonry, long term behaviour, probabilistic approach, fragility curves

INTRODUCTION

The time dependent behaviour of ancient masonry buildings, often characterized by non-homogeneous load-bearing sections, is considered among the factors affecting their structural safety. Together with other synergistic aspects, this has proven to be an important cause of collapse of some monumental buildings during the last fifteen years.

Exploiting the material coming from different ancient structures, several experimental procedures have been adopted to understand the phenomenon, from creep to pseudo-creep tests. Various rheological models have been applied to interpret the results [1]. Here a study on the material taken from the crypt of the Cathedral of Monza (Italy) will be described, and its interpretation by means of a probabilistic model will be proposed, aimed to the individuation of an aleatory variable as a significant index of vulnerability, and to the solution of the classic problem of reliability in stochastic conditions.

PROVENANCE OF THE TESTED MATERIAL

Following the project for the rearrangement of the Museum of the Cathedral of Monza in 1994, a door was opened on the northern wall of the crypt. The removed material was collected for experimental tests. According to the historical information, the construction of the crypt (concluded in 1577) is nearly contemporary to the construction of the bell tower (1592-1605), that was badly damaged by compression and is now undergoing a repair intervention [2]. As it has become clear after the collapses of the last fifteen years, towers as well as pillars of cathedrals turn out to be particularly vulnerable to the effects of persistent loading. Therefore, obtaining better experimental knowledge of their creep behaviour has become crucial. Since considerable amounts of historical masonry are not normally available, a good opportunity was taken of gaining original XVI century masonry.

EXPERIMENTAL TESTS

Different kinds of mechanical tests were previously carried out on the masonry of the crypt of Monza, including monotonic tests, cyclic tests and a creep test. The results were discussed in [3].

Two series of pseudo-creep tests, a fairly rapid and convenient testing procedure, were carried out on prisms of dimensions 200mm x 200mm x 350mm, applying the load by subsequent steps corresponding to 0.25 MPa kept constant for 10800 sec. (Figures 1 and 2).

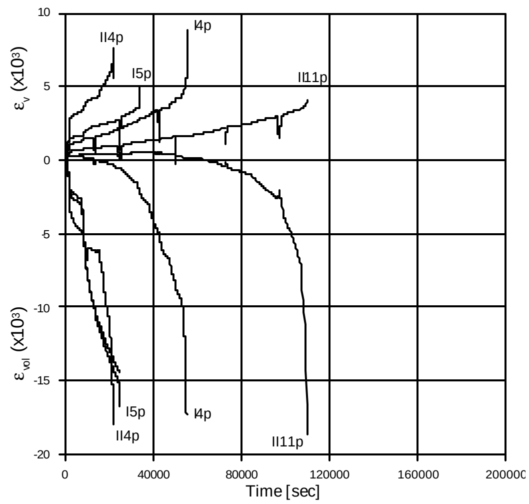


Figure 1 - Pseudo-creep tests: first series

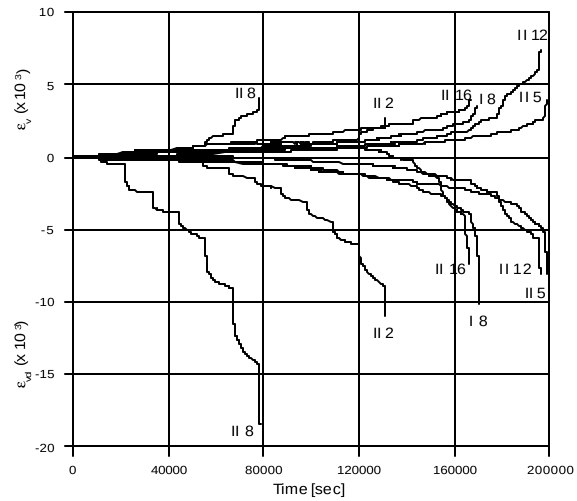


Figure 2 - Pseudo-creep tests: second series

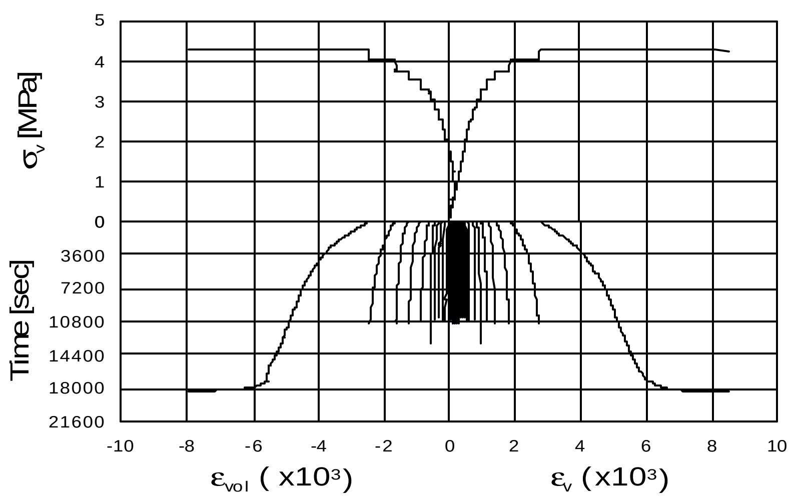


Figure 3 - Pseudo-creep test on prism II 12

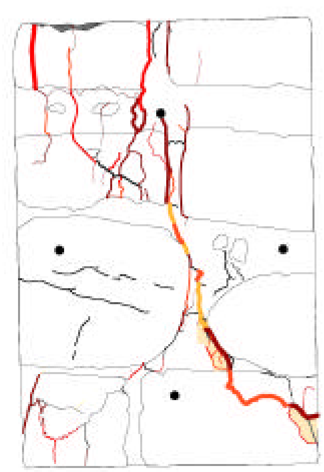


Figure 4 - Face B of prism II 12 at the end of test

During the first series, this load history was preceded by a monotonic phase up to a stress of 2.25 MPa, equal to 65% of the average short-term strength obtained by monotonic tests. The second series followed a more regular procedure. In both cases a clear dilatancy phenomenon can be observed, with considerable volumetric strain developing when approaching failure, a typical feature of brittle materials.

The results of a test on a single specimen of the second series are shown in Figure 3. Considering the trend of the stress-strain plot, it can be noted that the behaviour can be considered linear elastic below a stress value of 2.5 MPa. Correspondingly, the strain-time plot shows that within this interval only primary creep develops. After that level, the stress-strain diagram indicates non-linear behaviour, and correspondingly the strain-time plots exhibit the steady-state (or secondary) and, eventually, the tertiary creep phases. In Figures 4 and 5 the crack pattern across the specimen at the end of the test are shown.

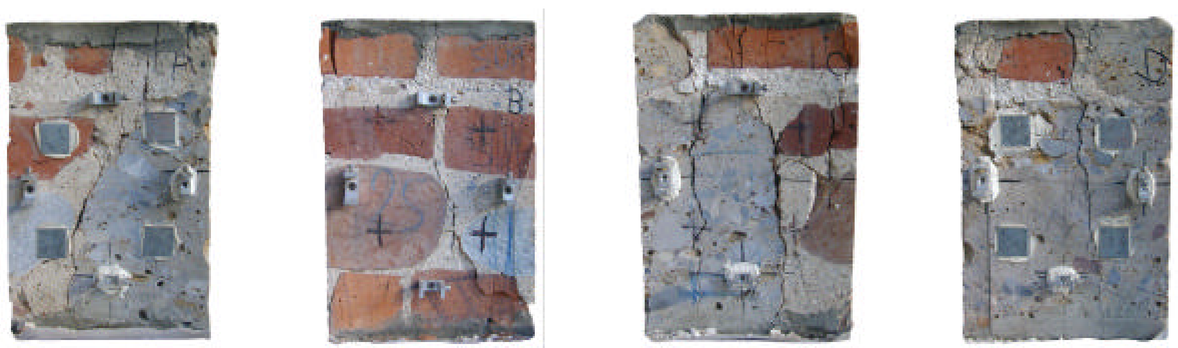


Figure 5 - crack pattern of prism II 12 at the and of test, face A, B, C, D

It is apparent from the drawings that the prism was characterized by the presence of a large portion of stone, occupying most of face D and large parts of faces A and C. The crack pattern has basically developed in a sub-vertical direction, with fissures opening preferably along discontinuities already present at start of the test, whereas the bricks were not cracked.

A comparison between the mechanical properties of the crypt of Monza and those obtained from the ruins of the tower of Pavia, previously tested by a similar procedure [1], is shown in Figures 6 and 7. It can be seen that in the case of Monza, lower values of the maximum stress and higher values of maximum strain were recorded on average, indicating the lower quality of the masonry constituting the crypt. This was also indicated by the presence of a diffused crack pattern on the prisms before testing. This is probably due to the use of a poor mortar, given the non-noble role of this masonry which formed the Cathedral foundations, and also to the coring procedure used for the extraction of the material.

Before carrying out the compression tests, every prism had been non-destructively characterized by transparency sonic tests. The measured peak stress obtained from the pseudo creep tests has been plotted in Figure 8 vs. the sonic velocity and compared to the equivalent values obtained from tests on the ruins of the tower of Pavia. An interesting direct relationship can be clearly observed that applies to the whole data. Again, the results confirm the bad quality of the masonry of Monza, showing lower values of sonic velocity and of compressive strength than those of Pavia.

Considering the last load step for each specimen, the secondary creep rate before collapse has been calculated and then related to the duration of the last load step, which can be regarded as the residual life of the material. In Figure 9 these values have been plotted comparing the masonry of Monza (first and second series) with those of the Tower of Pavia. Though the number of test results is not particularly large, an interesting inverse relationship can be found, which seems to apply to the two materials considered together, as well as to other brittle materials like concrete, subjected to creep and fatigue tests [4]. A strong correlation exists between creep time to failure and secondary creep rate which, accordingly, can be used as a reliable parameter to predict the residual life of a material element subjected to a given sustained stress.

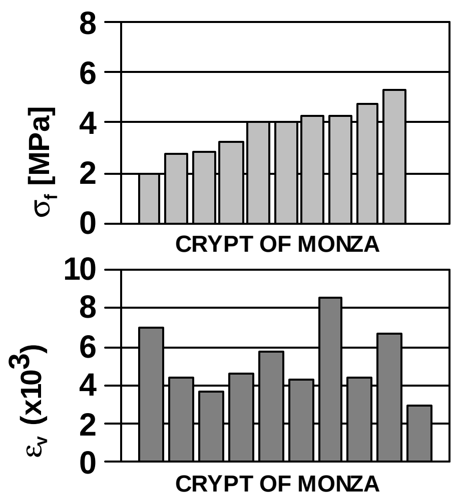


Figure 6 - Ultimate stress and maximum strain from pseudo-creep tests of first and second series on the crypt of Monza

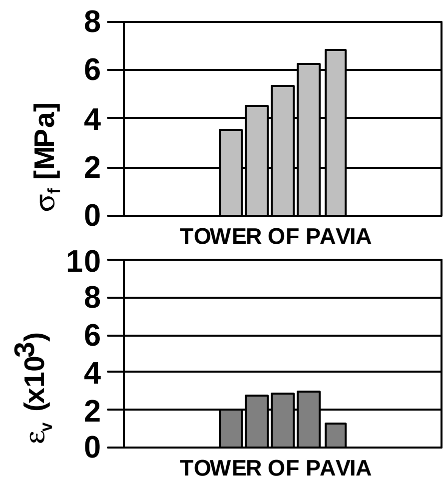


Figure 7 - Ultimate stress and maximum strain from pseudo-creep tests on the ruins of the Tower of Pavia

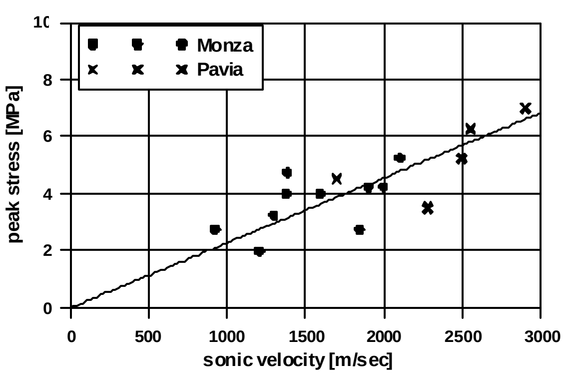


Figure 8 - Peak stress of pseudo-creep tests vs. sonic velocity

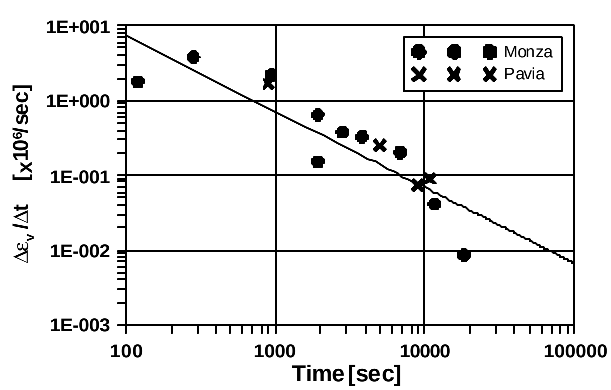


Figure 9 - Secondary creep rate before failure vs. duration of the last load step

STOCHASTIC MODELLING OF STRAIN-RATE BEHAVIOUR

We have shown previously [5, 6] that the strain evolution connected with a given stress history of a viscous material like a historic multiple leaf masonry can be described through the parameter $\dot{e}$. $\dot{e}$ is defined as the secondary creep strain rate, i.e. the slope of the linear part of the strain vs. time diagram shown by the specimen subject to the constant stress level s for a certain time interval. For each s the high randomness connected with the changing of velocity, due to the high non-homogeneity of the masonry studied, led us to consider $\dot{e}$ as a random variable (r.v.) with a certain distribution of the values (Figure 10).

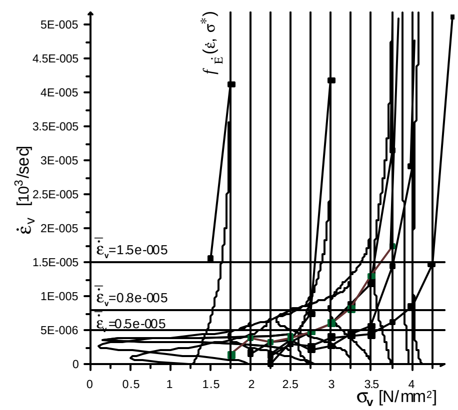


Figure 10 - Interpolation of the strain-rate and the modelling of $f_{\dot{E}}(\dot{e}, s^*)$

Following this way, the deformation process can be interpreted as a stochastic process of the r.v. $\dot{e}$. The strain-rate also depends on the stress level s corresponding to the deformation being recorded. Therefore, for each stress level s the strain-rate $\dot{e}$ (measured in 10^3/sec) can be modelled with a probability density function (p.d.f.) $f_{\dot{E}}(\dot{e}, s)$ that is dependent on both the stress s and on the strain rate $\dot{e}$. Since the experimental measurements are taken at discrete stress values s^* and the p.d.f. $f_{\dot{E}}(\dot{e}, s)$ is dependent on the velocity $\dot{e}$ and on s^* = constant: $f_{\dot{E}}(\dot{e}, s^*)$. Therefore, at each stress value s^* the modelling of the strain-rate behaviour depends only on the r.v. $\dot{e}$.

In order to model $f_{\dot{E}}(\dot{e}, s^*)$, at every stress level s^* a family of theoretical distributions has to be chosen. There is no doubt that the choice of a distribution to model a given phenomenon has to be connected to the physical aspects of the phenomenon itself, and to the characteristics of the distribution function in its tail, where often no experimental data can be collected. This last aspect of the matter can be investigated by analysing the behaviour of the immediate occurrence rate function $f_{\dot{E}}(\dot{e}, s^*)$ connected with the chosen distribution function:

$$f_{\dot{E}}(\dot{e}, s^*)d\dot{E} = \Pr\{\dot{E} < \dot{e} \le \dot{E} + d\dot{E} \mid \dot{e} \ge \dot{E}\} \qquad \text{Equation 1}$$

More details are provided on this subject in [7] and [8].

The recorded experimental data can show dispersion around the average value of $\dot{e}$, especially for large values of the strain-rate. As stated before, this is probably due to the randomness connected with the high non-homogeneity of the masonry studied.

Knowledge of the physical phenomenon has shown a relationship between the secondary creep strain-rate before failure and the residual life of the material (Figure 9) [9, 10]. Therefore a conventional value of $\dot{e}$ may be assumed as a critical value indicating a safety limit. Therefore, at a given stress level s^* the probability to record the critical velocity connected with the secondary creep safety limit increases if the strain-rate $\dot{e}$ increases. Consequently, it seems correct to assume that, at a given stress level s^* the higher is the strain-rate, the higher is the probability to have the secondary creep strain-rate in the interval $\{\dot{\mathrm{E}} < \dot{e} \le \dot{\mathrm{E}} + d\dot{\mathrm{E}}\}$. The assumed hypothesis, as a satisfied (but not unique) physical interpretation of the decay process, leads to model the loss $\dot{e}$ at the time s^* with a Weibull distribution (Figure 10) as follows:

$$f_{\dot{\mathrm{E}}}(\dot{e}, s^*) = a_1 r_1 (r_1 \dot{e})^{a_1 - 1} \exp\left[-(r_1 \dot{e})^{a_1}\right] \quad \text{Equation 2}$$

The estimation of the shape parameters a_1 and r_1 have been made through a computer code involving the maximum likelihood method. This modelling can be obtained also through the function FMIN present in the MATLAB code.

This family of distributions presents an immediate occurrence rate function (Equation 1) increasing as the value of $\dot{\mathrm{E}}$ increases and tending to ∞ as $\dot{\mathrm{E}} \to \infty$; this fact seems to respect the physical interpretation of the strain-rate behaviour as previously described.

It is furthermore interesting to evaluate the probability for the system of reaching or exceeding a given deformation level $\bar{\dot{e}}$ over a stress history. This probability can be seen as the shadowed area above $\bar{\dot{e}}$ as shown in Figure 11 [7, 8].

This area can be calculated by using the survival function $\Im_{\dot{\mathrm{E}}}(\dot{e}, s^*) = 1 - F_{\dot{\mathrm{E}}}(\dot{e}, s^*)$ where $F_{\dot{\mathrm{E}}}(\dot{e}, s^*)$ is the cumulative distribution function of the p.d.f. (Equation 2).

The computation of $\Im_{\dot{\mathrm{E}}}(\dot{e}, s^*)$ is possible with the use of any kind of computer code for numerical integration.

For different strain-rate levels $\bar{\dot{e}}$, the survival function has been evaluated for all stress levels s^*. The calculated values allow one to plot an experimental "*fragility curve*" connected to each chosen strain-rate level (see Figure 12) [7, 8].

Following this approach, the deterioration process can be treated as a reliability problem [8].

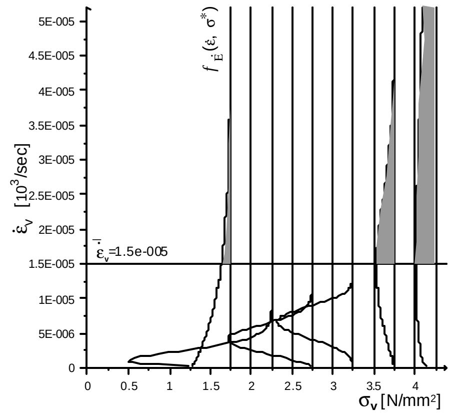


Figure 11 - Exceedance probability to cross the threshold $\bar{\dot{e}}$

Indeed [11], the reliability $R(t)$ concerns the performance of a system over time and is defined as the probability that the system does not fail during the time t. Here, this definition is extended and $\bar{R}(s)$ is assumed as the probability that a system exceeds a given significant strain-rate $\bar{\dot{e}}$ with a stress s . The random variable used to quantify reliability is $\bar{\Sigma}$ which is just the stress to exceed the strain-rate $\bar{\dot{e}}$. Thus, from this point of view, the reliability function is given by [11, 12]:

$$\bar{R}(s) = \Pr(\bar{s} > s) = 1 - F_{\bar{s}}(s) \qquad \text{Equation 3}$$

where $F_{\bar{s}}(s)$ is the distribution function for $\bar{s}$ and represents the theoretical modelling of the experimental fragility curves.

In order to model the experimental fragility curves and to evaluate $F_{\bar{s}}(s)$, a Weibull distribution has been chosen [8, 13] as follows:

$$F_{\bar{s}}(s) = 1 - \exp\left[-(r_2 s)^{a_2}\right] \qquad \text{Equation 4}$$

In fact, this distribution seems to be a good interpretation of the physical phenomenon: the larger is the stress level, the higher is the probability that a critical velocity $\bar{\dot{e}}$, connected with the creep phenomenon, will happen for a σ value included in the next $(\bar{s} + d\bar{s})$ interval. Therefore, distributions with the function $f_{\bar{s}}(s)$ increasing with s and tending to ∞ as $s \rightarrow \infty$ are needed. Also this time, the Weibull distributions satisfy this requirement.

FRAGILITY CURVES MODELLING ON EXPERIMENTAL DATA

As said above, Figure 10 shows the interpolation of the strain-rate vs. stress and the modelling of $f_{\dot{E}}(\dot{e}, s)$; on it, as possible critical strain-rate values, three different $\bar{\dot{e}}_v$ have been identified, well below the values shown in Figure 9 and immediately preceding failure, connected with the initiation of the secondary creep phase. In Figure 12 the experimental and theoretical fragility curves connected with these thresholds are reported. They describe the probability of exceeding the critical thresholds as a function of the stress level reached, σ_V.

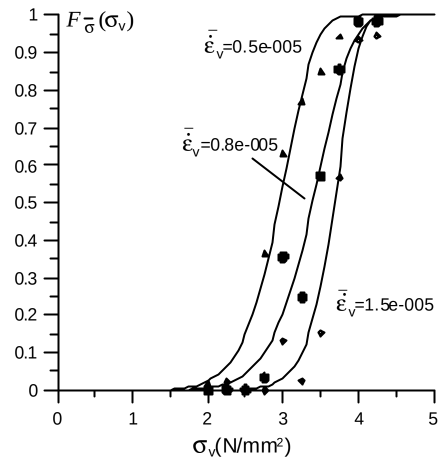


Figure 12 - Vertical strain-rate: experimental (symbols) and theoretical (lines) fragility curves

In Figure 12 it is evident that the high value present in the experimental fragility curves corresponding to the stress value $\bar{s}$ = 3N/mm^2 affects the fitting: the theoretical curves seem to underestimate the probability to reach the stain-rate thresholds $\bar{\dot{e}}_v = 0.08e-005$ and $\bar{\dot{e}}_v = 1.5e-005$ (Table 1).

Table 1 – Experimental and Theoretical $F_{\bar{s}}(s)$ for two different thresholds at $\bar{s}$ = 3N/mm^2

$\bar{s}$ = 3N/mm^2	Experimental $F_{\bar{s}}(s)$	Theoretical $F_{\bar{s}}(s)$
$\bar{\dot{e}}_v$ = **0.08e - 005**	35.5%	20.3%
$\bar{\dot{e}}_v$ = **1.5e - 005**	13%	3.2%

This behaviour is connected with the presence of a high strain-rate value in the experimental data (Figure 10) that brings the Weibull distribution to present a high dispersion and a long tail. As explained above, the problem of the tail is always present: to decide to “cut off” the “strange” values from the data sets is not a solution because it is not on the safe side. It is correct to keep them because they are representative of the high non-homogeneity of the studied masonry behaviour.

Since the curve used to fit the experimental data has been chosen also on the basis of knowledge of the physical phenomenon, and the fitting is optimised on the basis of all the data, we can consider its behaviour reliable, therefore, the risk of an underestimation of prediction around $\bar{s}$ = 3 N/mm^2 can be accepted (error quantifiable around the 10-15%).

In view of preserving historical heritage, this type of prediction allows the evaluation, for instance, of the results of a monitoring campaign on a massive historic building subjected to persistent load and to judge whether the creep strain indicates a critical condition in terms of safety assessment [6]. Of course, pre-emptive recognition of a critical state allows one to design and implement a strengthening intervention to prevent total or partial failure of the construction.

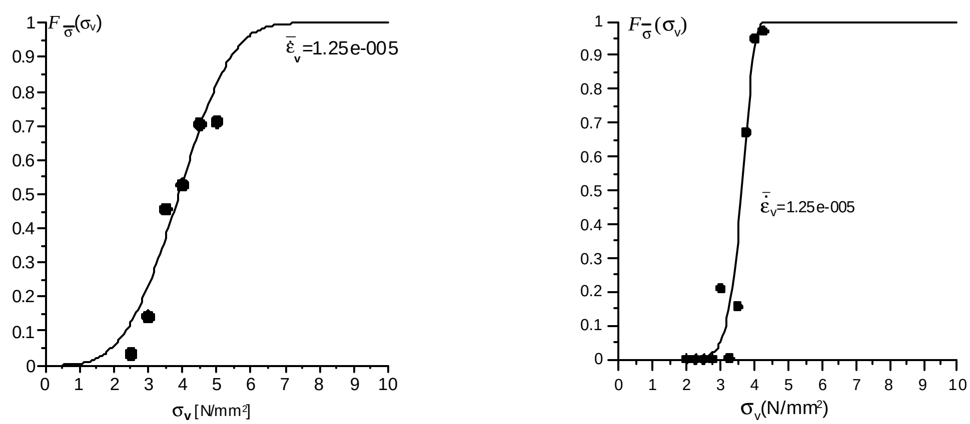
COMPARISON BETWEEN TWO DIFFERENT EXAMPLES OF HISTORIC MASSIVE MASONRY

The procedure proposed has been applied, as said previously, to the crypt of the Cathedral of Monza and then compared with the results previously obtained on the Civic Tower of Pavia [6].

The fragility curves built for the same threshold $\bar{\dot{e}}_v = 1.25e-005$ on the basis of experimental data recorded during the laboratory creep test show a different behaviour (Table 2, Figure 13): for the crypt of Monza the fragility curve has a more rapid increase than for the Tower of Pavia; it seems that, reached a stress level close to 2.65 N/mm^2, the probability to exceed the strain rate $\bar{\dot{e}}_v = 1.25e-005$ increases rapidly for low increments of the stress value. This behaviour is related to higher strain rates recorded on the Monza samples than on the Pavia's samples, due to the above mentioned worse quality of the masonry of Monza.

Table 2 – Probability to exceed $\bar{\dot{e}}_v$ for different s_v

$\bar{\dot{e}}_v$ = 1.25e - 005		
	Civic Tower of Pavia	**Crypt of Monza**
Exceedance prob. of $\bar{\dot{e}}_v$	s_v **(N/mm^2)**	s_v **(N/mm^2)**
10%	2.35	2.65
63%	4.30	3.72
90%	5.40	3.88



Civic Tower of Pavia **Crypt of Monza**

Figure 13 - Experimental (·) and theoretical (¾ ¾) fragility curves

CONCLUSIONS

A probabilistic model has been applied to study the time-dependent behaviour of masonry specimens subjected to pseudo-creep tests in the laboratory. The chosen model seems to interpret appropriately the experimental results whilst also capturing the differences between two samples of masonry with different characteristics. The satisfactory results indicate an interesting research direction toward the collection of more data in view of achieving a tool for predicting the service life of masonry under particular states of stress.

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