A FINITE ELEMENT ANALYSIS OF A 17TH CENTURY WELSH BRIDGE USING ABAQUS

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ABSTRACT

The bridges known from the Welsh as *Pont-Y-Prydd* were built by Mr Edwards in 1755 for wagon traffic. The first bridge was washed away in a flood, the reconstructed bridge failed during construction when the centring was removed, whilst standing successfully at the third attempt with lighter haunches. Baker wrote that the work was constructed by an un-educated mason, although the Welsh consider Mr Edwards one of their early architects. The construction is unusual in the steepness of the roadway, the relatively thin arch elements and the circular elements used to reduce the mass of the haunches in the third bridge. The paper presents a finite element analysis of the second *Pont-Y-Prydd* using the data presented by Baker and other researchers. The finite element analysis using Abacus on the TAMU supercomputer provided the data to confirm the deflected shape of the completed bridge. A comparison of the results and a commentary is provided between the collapse of the second bridge and the success of the third bridge. The third bridge still stands.

KEYWORDS: historic masonry arches, masonry analysis, finite element analysis

INTRODUCTION

The first *Pont-Y-Prydd* (Translation from the Welsh: *Bridge by the earthen house)* was washed away in a flood. Mr Edwards constructed a new bridge across the River Taff in 1755 for wagon traffic. This second bridge failed when the supports were removed, whilst the lighter third bridge stands today (Figure 1)[1]. Baker [2] notes that it is twenty third in terms of size in the list of sixty one large voussoir arch bridges built prior to 1903. The bridge is the subject of ongoing investigations by a team from Cardiff University [3] who constructed a model of the third bridge, which was tested in a centrifuge. The purpose of this research is to investigate numerically the structural aspects of the second bridge, specifically the cause of the progressive bridge collapse.

The commentary on the second bridge failure describes a progressive collapse over a six-week period after removal of centering. This type of progressive collapse of a masonry arch has been observed elsewhere [4]. Heyman [5] defines this type of short-term loss as a design failure. A flood washed the first structure away, an environmental loading failure. The third bridge has stood for a quarter of a millennium, whilst not definitive, does suggest an adequate third design and a final failure mode of a material failure, this mode is suggested by Heyman as a potential fourth stage of the life cycle of a structure. There is no reason short of a major environmental load that Edward's third bridge will not last as long as other sandstone structures in Wales.

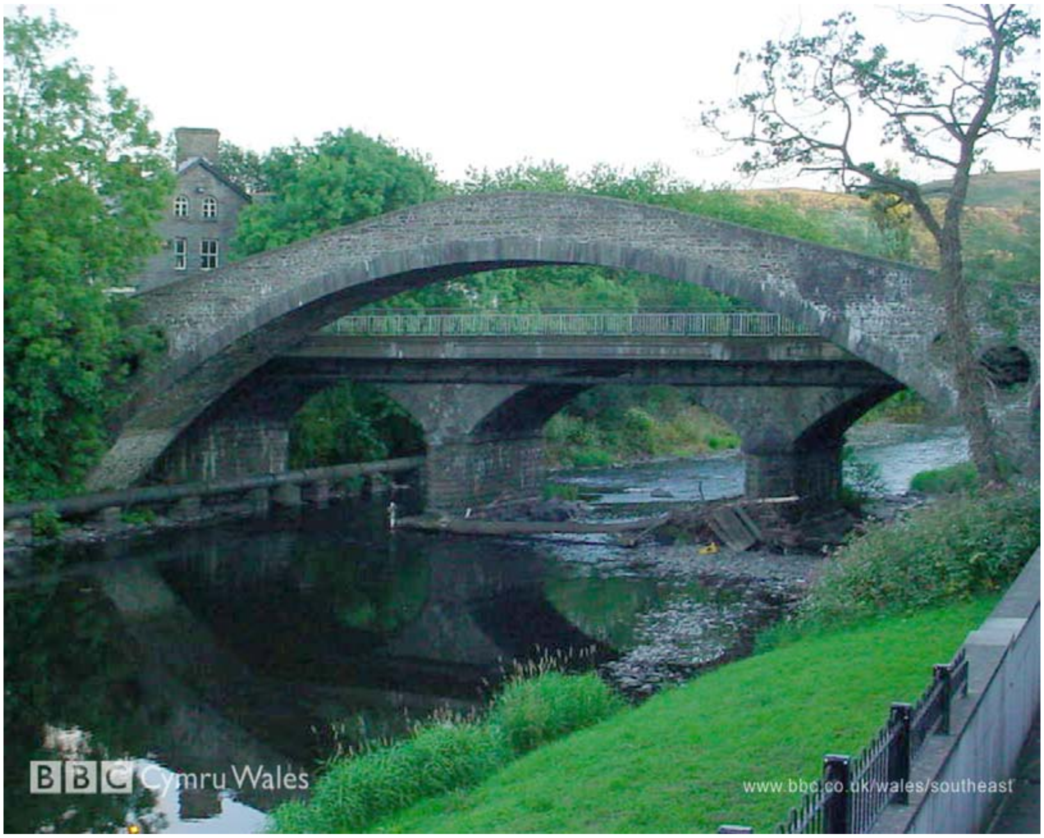

Figure 1 – Pont-y-Prydd (Courtesy BBC Wales)

The third bridge has very steep roadways and has been replaced for daily traffic use by a conventional flat structure using multiple masonry arches as shown in Figure 1. The style of the *Pont-Y-Prydd* makes it ideal for construction on navigable rivers in Great Britain to allow the free flow of river traffic as it is significantly easier to "shoot" a bridge in a yacht with a lowered mast with this clear open span than the flat narrow multiple arched bridge type [6].

LITERATURE REVIEW

Baker [2] noted the technical details for the *Pont-Y-Prydd* a circular intrados, a radius of the crown of 26.8 m (88 ft), a span of 42.67 m (140 ft), a rise of 10.668m (35 ft), a crown thickness of 0.457 m (1.5 ft) and a springing of 0.457 m (1.5 ft). He provided the following commentary on the construction of the sandstone bridge:

> *"This is a remarkable bridge. It was built by an "uneducated" mason in 1750; and although a very rude construction, is still in perfect condition. A former bridge of the same general design at the same place fell, on striking the centers, by the weight of the haunches forcing up the crown; and hence in building the present structure the load on the haunches of the arch was lightened by leaving horizontal cylindrical openings through the spandrel filling. The cylindrical arches extend from the face of one parapet wall to that of the other. In addition, the filling immediately over the*

arch and around the cylinders was charcoal. This is among the first applications of this method of lessening the load on the haunches[1]. Between the surface of the roadway and the extrados is rubble masonry laid with horizontal joints. The outer, or showing, arch stones are 2.5 feet deep, and that depth is made up of two stones; and the inner arch stones are only 1.5 feet deep, and but from 6 to 9 inches thick. The stone quarried with tolerably fair natural beds, and received little or no dressing. It is a wagon-road bridge, and has almost no spandrel filling, the roadway being very steep. A stress sheet of the arch shows that the line of resistance remains very near the center of the arch ring. The maximum pressure is about 1,025 pounds per square inch. It is an example of very creditable engineering."

A sketch by Baker of the bridge is shown in Figure 2. Baker used a standard stress arch calculation sheet in determining that the resultant of the force system remained within acceptable limits in the arch. A pressure of 1025 psi is equivalent to 7.1 MPa, which is about 17 percent of the typically measured sandstone crushing strength [7] of 40 to 120 MPa. Hughes et. al. [3] provide a summary of a similar analysis for the solid arch showing acceptable engineering results.

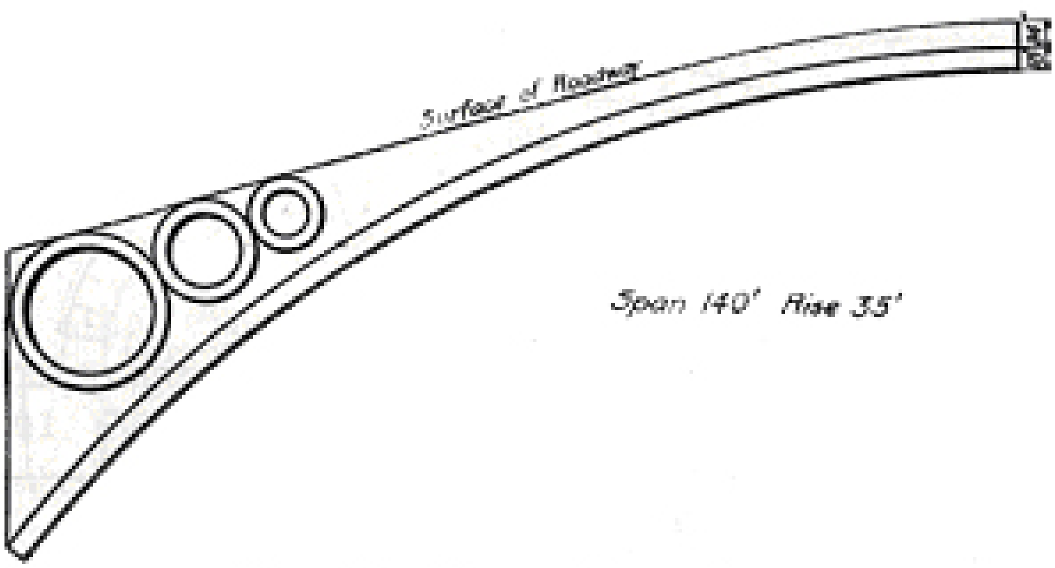

FIG. 220. PONT-Y-PRYDD ARCH.

Figure 2 – Pont-y-Prydd – Sketch in Baker (1914)

The bridge is currently being investigated by a team at the Cardiff University – School of Engineering [3] using finite element techniques and three-dimensional models in a centrifuge. The characteristics of the bridge determined by the Cardiff group are presented in Table 1. The dimensions provided by Baker are noted also in Table 1. Differences exist in the dimensions reported between Hughes article of 1998 and Baker's book of 1914. These differences in dimensions are noted in the table. Baker's dimensions are used for the analysis, as they are a

[1] This paper will use Baker's term haunch for the large areas of stone that form the bridge abutments.

closer approximation to the metric equivalents for the original units of feet shown on the available plans, and provide a better error close on the plan dimensions.

Table 1 – Pont-y-Prydd Details

Description	Hughes Dimensions (metres)	Baker Dimensions (metres)
Radius	27	26.8
Span	42.67	42.67
Rise	10.67	10.668
Arch thickness	0.76	0.457
Springing	-	0.457
Spandrel Wall thickness	0.5	-
Cylindrical Hole Sizes	2.7, 1.7, 1.0	-
Surface Slope	20%	-
Stone	Pennant Blue Stone	Sandstone
Mortar	Lime	Lime
Width	5	-

The dimensions given by Baker have been drafted to provide a geometric check on the consistentency of the published information. The survey misclose on the Baker information is a few millimetres, and these dimensions have been adopted for the analysis where the dimensions differ from Hughes dimensions as noted earlier. An AutoCAD drawing of the bridge is shown in Figure 3, showing the composite information.

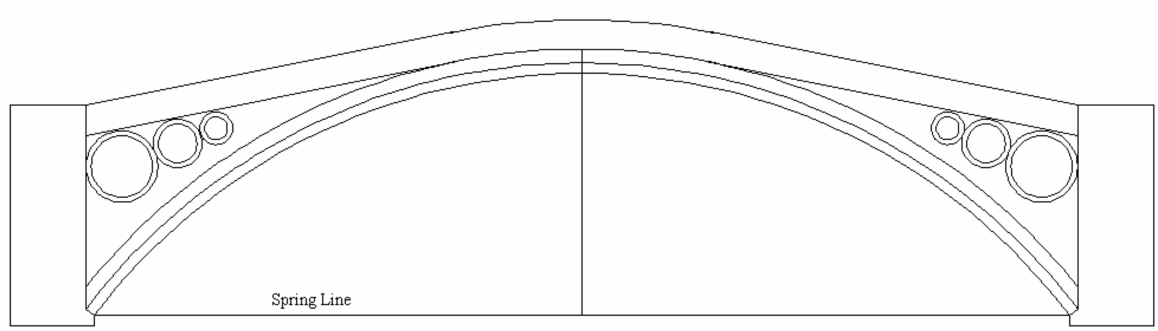

Figure 3 – Composite Drawing of the Bridge

ANALYSIS OF THE PLAIN ARCH

The first stage in the analysis of the *Pont-Y-Prydd* was to create an Abaqus [8] model of the innermost arch element. A picture of the plane arch ABAQUS model is shown in Figure 4. The purpose of this model is to check the structural adequacy of the plane arch. The typical mechanical property values for the sandstone are Young's Modulus 20 GPa and a Poisson's ratio of 0.2 was assumed for the analysis [7]. The Young's modulus for the sandstone masonry is assumed to be equivalent to a nominal 20 GPa for plain sandstone. A simple linear elastic finite element analysis was used to analyse the plain arch under self-weight and a small live load to represent human foot traffic on the bridge. The analysis assumed a standard specific density for the sandstone of 2.15 to 2.4 [7], and fixed ends to the structure. The finite elements are 8-node *3D stress linear bricks* with incompatible modes from the Standard Abaqus library.

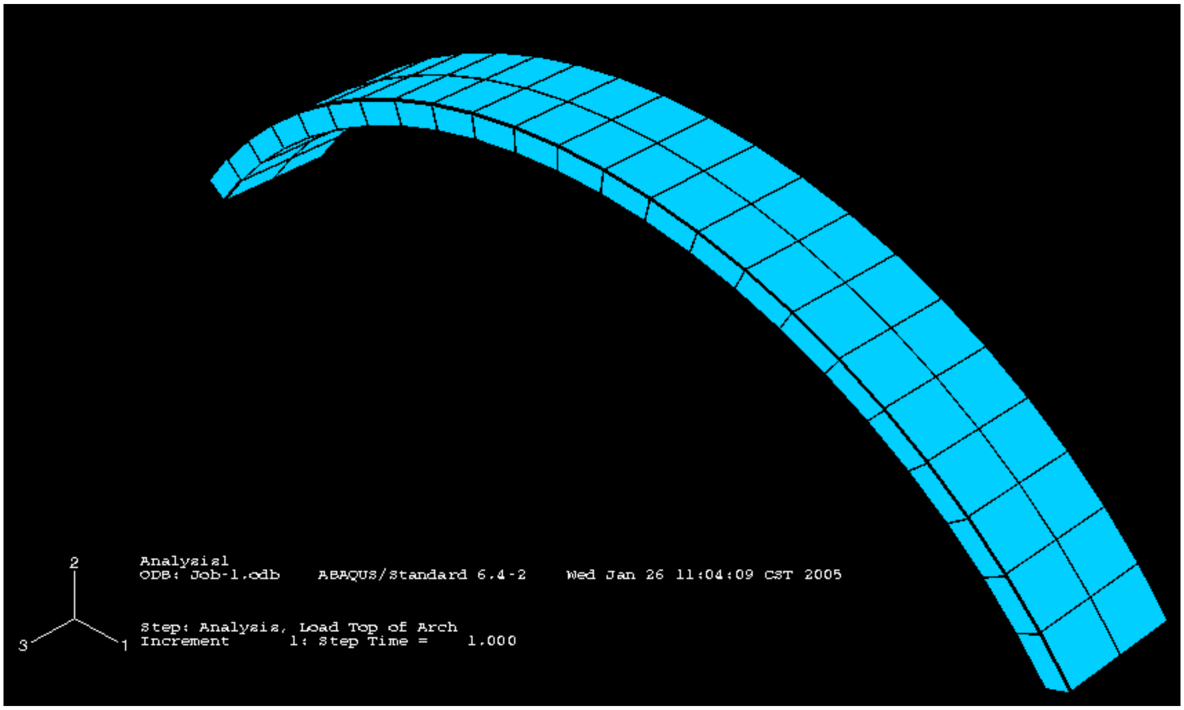

Figure 4 – Bridge Arch Model

The graphical results for the arch are presented in Figure 5 for the principal stress components.

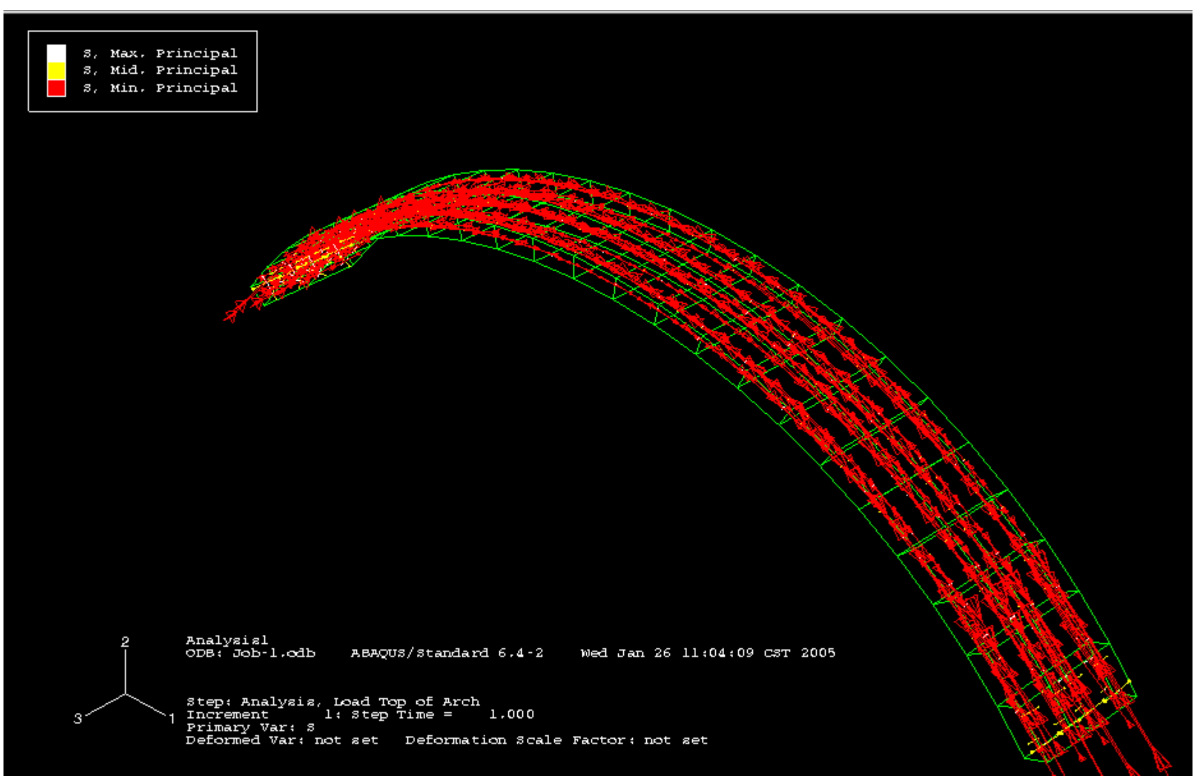

Figure 5 – Results of the Plain Arch Analysis

The results show, as expected, that Edward's plain bridge arch is capable of carrying its self-weight and light foot traffic with the resultant for the resultant forces remaining within acceptable limits for a stone arch, and the deformation of the structure remaining within tolerable limits. There is nothing surprising in this simple linear elastic analysis, but it does provide confirmation on the use of ABAQUS for this limited analysis technique for this stone masonry assuming homogeneous properties for the finite elements.

FINITE ELEMENT ANALYSIS OF THE SECOND BRIDGE

The second bridge collapse over a six-week period probably represents a design failure, rather than a construction failure [5]. A linear finite element analysis provides some guidance as to the deflections causing the distress at various points in the structure. The classic assumptions of hinges forming in the stone masonry and linear analysis are a very real simplification of a complex process of cracking and sliding, but this simplifying assumption still provides a clear guide as to the areas in the design that will cause problems in the field [5, 9]. The assumptions for this analysis were the structure is pinned along the edge of the arch, the remaining faces were free to move and rotate, the loading was self-weight and a light load to replicate foot traffic. A picture of the Finite Element Model is shown in Figure 6. The assumptions as for all structural analysis limit the applicability of the results; however, the analysis is designed to show the deflected shape of the second bridge. The deflected shape data for an elastic structure provides information to consider the failure mechanism for the second bridge. The deformed shape from an Abaqus analysis is shown in Figure 7.

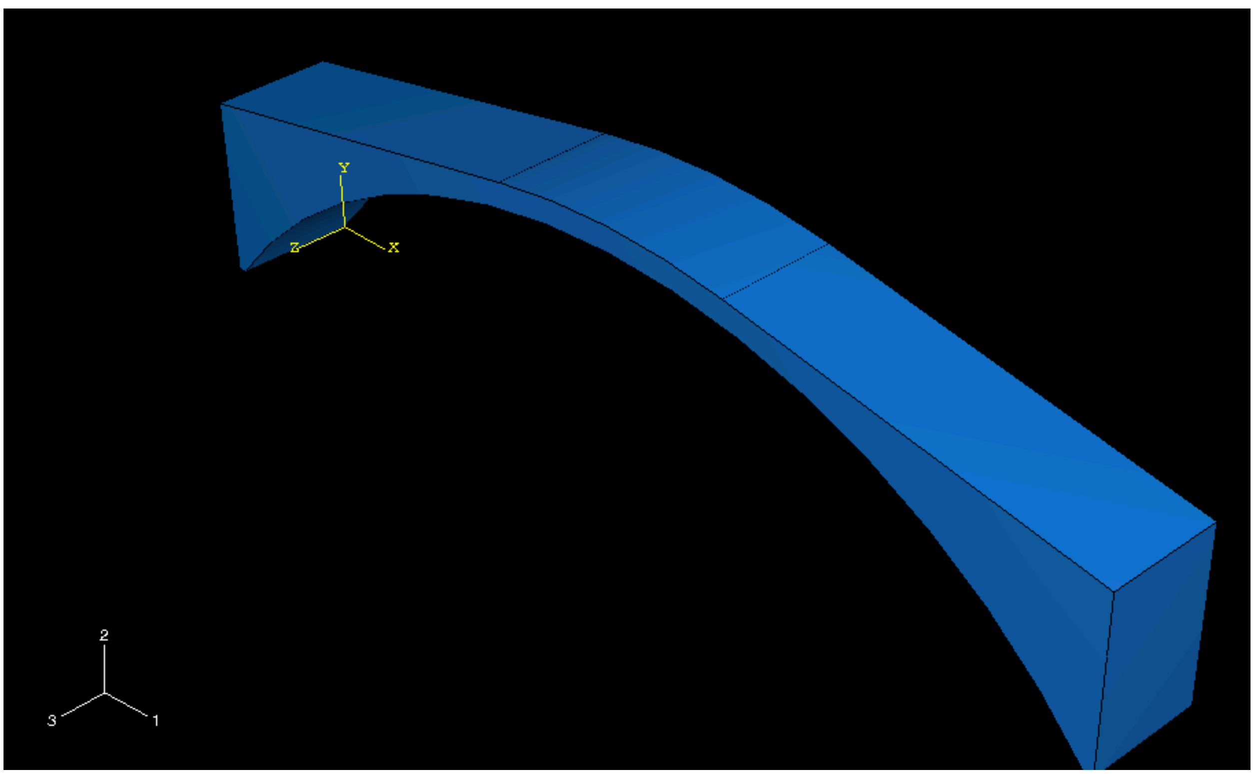

Figure 6 – Second Bridge FEM Model

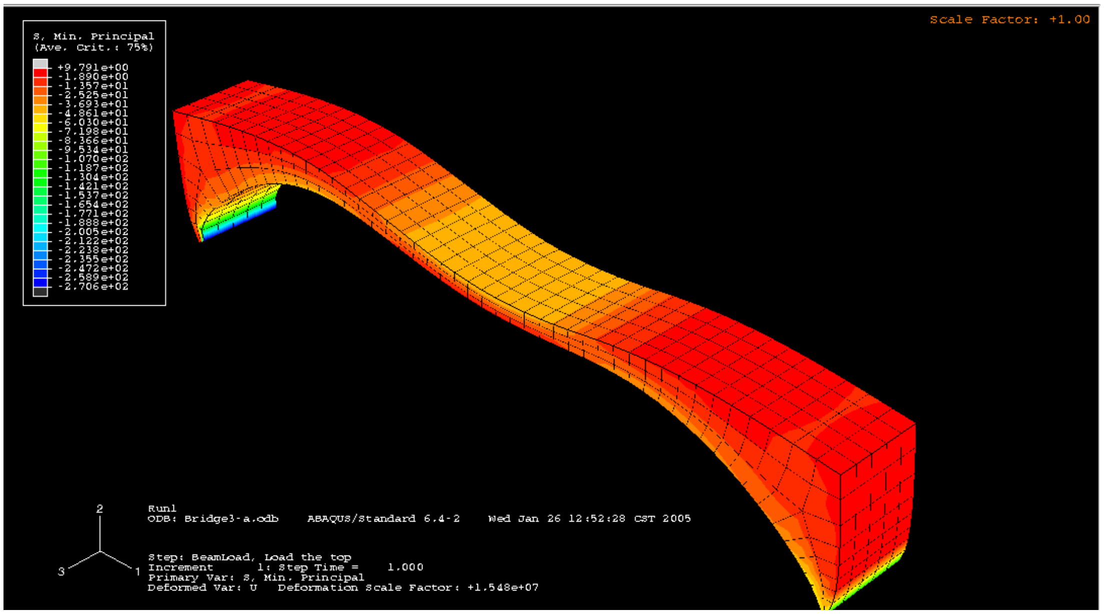

Figure 7 – Deformed Shape – (Elastic Analysis)

Figure 7 shows the vertical downwards deflection of the structure based on a uniform loading and the horizontal displacement of the haunches. Abaqus provides a simple method to determine the shape of the first harmonic for the structure. The results must be used with caution as the structure has probably zero tensile capacity and limited moment capacity, not withstanding this limitation. The vertical up component from the Abaqus harmonic shape program is shown in Figure 8.

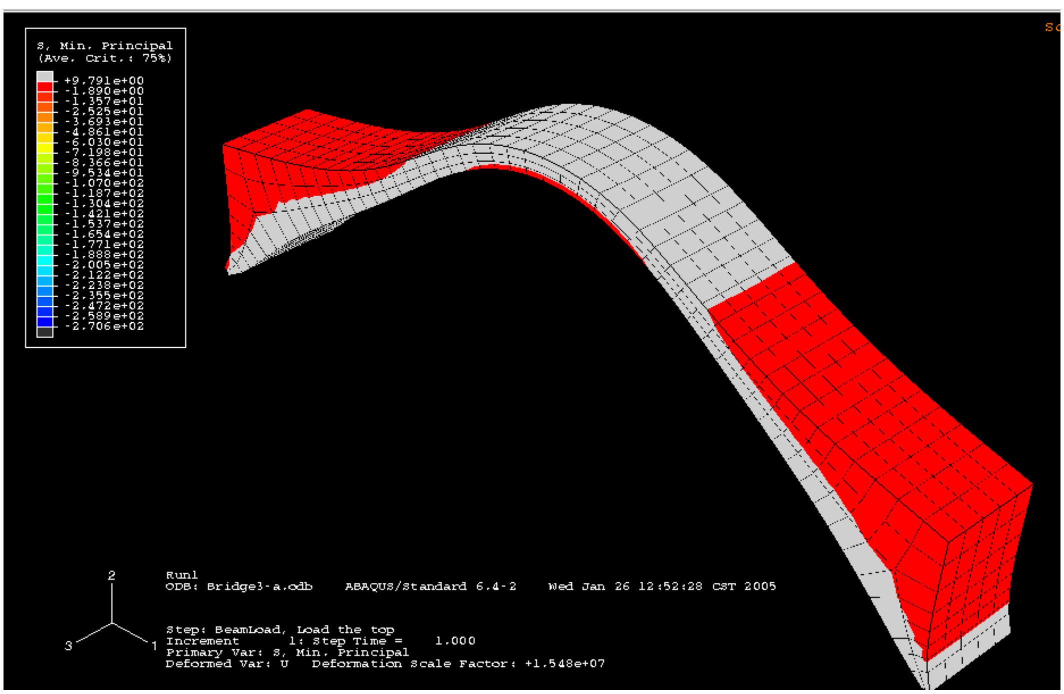

Figure 8 – Deformed Shape of Fundamental Mode (Vertical Up)

FAILURE MODE OF THE SECOND BRIDGE

Baker noted the second bridge was observed to fail by rotation of the haunches and a rising of the centre of the arch prior to collapse [2]. The critical data from this current analysis was:

- the central third of the bridge arch has a limited vertical depth providing a relative slender element to resist the horizontal thrust and overturning moment of 21000 tonne metres from each of the haunches of the bridge
- the total mass of a stone bridge is 7500 tonnes with approximately 12 percent of the mass in the central third of the bridge

The Abaqus analysis results shown in Figures 7 and 8, coupled with the manual calculations, and from observations of the failure of progressive failure of damaged arches provide a guide as to the likely design flaw that led to the collapse of the structure.

Four failure modes are considered in this paper:

1. deflection failure as shown in Figure 7 for the central third of the bridge
2. a vibration loading with vertical deflection of the bridge
3. failure of the central third of the bridge
4. a composite failure mode

The first failure mode would require outward deflection of the abutments. If this outward deflection had occurred, then the bridge given the tensile principal stresses shown in Figure 7 would have failed. It is a possible failure mode, but one that cannot be proven.

The second failure mode represents a vibration of the fundamental mode. The removal of the centering may have caused damage to the bridge although this cannot be proven. The Abaqus analysis suggests that the central third of the arch is compressed, as expected under all forms of analysis, but due to the slenderness and most likely the low self-weight relative to the haunches the central third rises or buckles vertically as shown in Figure 8. This movement may have been quite slow at first as the bridge was loaded and unloaded with live load. The progressive cracking adds to the distress and causes further degradation leading to failure over a several week period. The author has observed this type of failure in two masonry structures, where progressively increasing damage accumulates under self weight alone over a period of several weeks.

The third failure mode is essentially a static failure mode. The haunches will deflect inward towards the centre of the arch as the supports are removed. The central slender third of the arch has to resist this force using compression, shear, and self weight. The shear stress across the central third of the arch would be as high as 3 MPa from a simple moment analysis. The high shear stress means that the compression capacity is compromised because of the slenderness of the central third as shown in the principal stress mapping. It is suggested that the self-weight of the central third of the arch, which is 12 percent of the total mass of the structure, is insufficient to maintain the arch in a stable configuration combined with the limited compression capacity of a slender arch and the high shear stress.

The fourth failure mode is a composite of the second and third failure modes. The analysis is based on relatively straightforward elastic finite element modelling techniques. However, the analysis points to the conclusion the bridge failed probably due to the rotation of the haunches not being able to be resisted in the long term by the relatively thin central arch segment. The failure and cracking was probably a progressive failure of the bridge as small increments of damage accumulate in the arch. This is consistent with other observed arch failures.

The fourth failure mode is considered most likely given the progressive collapse over a six-week period. The final observation is that as engineers and builders learn from their mistakes, and in this case, of failure, there is no anecdotal evidence that anyone died in the collapse, which is not always true.

COMMENTS ON THE THIRD BRIDGE DESIGN

Edwards rebuilt the bridge for a third time. The design of the third bridge is shown in Figure 9.

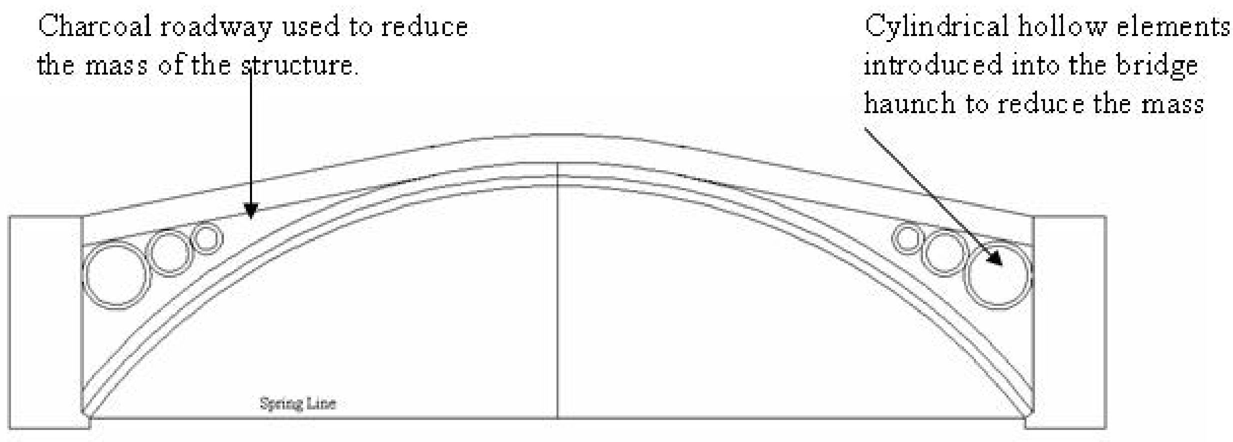

Figure 9 – Third Bridge Modifications

Edwards used two methods to significantly reduce the weight of the bridge from 7500 tonnes for the second bridge to about 4000 tonnes for the third bridge, ensuring that the bulk of the mass was in the arch element and not in the haunches. The first method was to introduce cylindrical hollow elements, six in total with three in each haunch, to reduce the mass. The second technique was to use charcoal to make up the roadway element. Charcoal has a specific gravity of about 0.2, which less than the 2.15 to 2.4 for sandstone. Charcoal is also easier to procure and place. The bridge has remained standing for a quarter millennia in this configuration.

CONCLUSIONS

Structural engineering has evolved as a field of study over the last four millennia as the rules for the analysis of structures have slowly been determined by observation of the natural and built environment. However, structures were being built and used long before the observational rules were determined and shown to be approximately correct. Mr. Edwards tried to build a structure with a thin central arch segment, which had steep roadways at about 20 percent. The construction of the second bridge after the failure of the first in a flood ended in failure of the second bridge upon removal of the centering for the construction support. Heyman has classified this type of failure as a design failure.

An elastic finite element analysis using Abaqus shows that the arch ring alone is capable of standing as a separate structure, but the addition of large sandstone haunches induces a considerable deformation in the slender central third segment of the arch resulting in the failure of the bridge in 1755. The fundamental mode of vibration appears to match the failure mode which was observed to be a rise in the central segment followed by excessive movement of the haunches. This type of failure can occur over a period of several weeks as damage accumulates in the masonry from small movements gradually increasing. The bridge traffic may have been sufficient to exacerbate damage in the fundamental mode.

Edwards learnt from his mistake, as engineers do to this day, and constructed a successful third bridge. He correctly decided to remove mass from the haunches reducing the mass of the structure from 7500 tonnes to 4000 tones, using the methods of air spaces and charcoal. The third bridge was noted by Baker as being is twenty third in terms of size in the list of sixty one large voussoir arch bridges built prior to 1903, and it has stood for 250 years. There is no observable reason that the bridge will not last for a considerable time unless a significant environmental event overloads the structure.

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