THE INFLUENCE OF SLENDERNESS ON CAPACITY OF MASONRY WALLS

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ABSTRACT

The importance of the influence of slenderness on masonry wall capacity is becoming more widely recognised. Contemporary codes for masonry structures (Eurocode6 [1], ACI530-02 [2], CSA S304.1 [3]) allow the design of very slender masonry walls with effective height/thickness ratio exceeding 20. However, a relatively small number of experiments for verification of relationships determining the influence of slenderness on masonry wall capacity have been performed worldwide.

This paper presents the results of tests carried out by the author on brick masonry walls. These together with the results of tests on the masonry walls performed in USA, Germany, China, among others, have provided a basis for a discussion on the relationships given in [1, 2, 3].

To define the effect of slenderness on the reduction of masonry wall capacity, a numerical analysis has also been carried out using a model of a masonry wall compressed eccentrically. In the presented model a non-linear stress-strain relationship and the influence of the second order effects were taken into account. The results of the numerical analysis were compared with tests and results obtained using the simplified methods given in codes, in which the realistic stress-strain relationship for different types of masonry is disregarded.

KEYWORDS: slender walls, unreinforced masonry, tests, effects of 2nd order

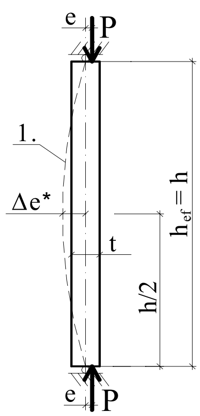
INTRODUCTION

Relationships for determining the influence of slenderness on the capacity of masonry walls and columns can be estimated from theoretical analysis taking into account a real material model for masonry and second order effects. In [4] Angervo presented a solution for a linear model of a material with no tensile strength. Non-linear models for masonry were assumed in the analyses made by Kukulski-Luges [5], Kirtschig [6], and Graubner et al. [7]. In engineering computations, simplified functions based on theoretical analyses are preferred. Such a method was used for determining the relationships proposed in Eurocode 6 [1].

Relatively simple empirical relationships can also be obtained when an appropriate database is available, which is illustrated by the American code ACI 530-02 [2]. The general application of such relationships depends on the material range investigated in tests.

This paper presents code recommendations for determination of the influence of slenderness in computations of load capacity of masonry walls given in European code EC6 [1], American code

ACI 530-02 [2] and the Canadian code CSA S401.1 [3]. The criterion of selection of codes was the different methods used in these codes for the solution of the problem in question.
The discussion of code recommendations is based on tests performed on masonry walls and columns including tests carried out by the author at the Institute of Building Materials and Structures of the Cracow University of Technology. From the published results of experiments only tests on members with hinged ends and loaded with the same eccentricity at both ends were selected. This relates to the model of a wall under eccentric load, developed by the author. The model was applied in numerical computations, which enabled us to carry out an analysis of the influence of different forms of the stress-strain relationship on the capacity and deformability of slender walls. The basic scheme of the masonry wall and symbols used in the paper are presented in Figure 1. This paper is only concerned with unreinforced masonry.



1. – deflected axis of wall
Δe^* - maximum deflection of wall
e - eccentricity of load

Figure 1 - Basic Scheme for Determining the Influence of Slenderness On Masonry Wall Capacity

CODE RECOMMENDATIONS

The new European code for masonry structures, Eurocode 6 [1], recommends the following function for determining the influence of slenderness and eccentricity on masonry wall capacity:

$$\Phi_1 = [1 - 2\frac{e_{mk}}{t}]\exp\left(-0.5\frac{\frac{h_{ef}}{t}\sqrt{\frac{f_k}{E_m}} - 0.063}{0.73 - 1.17\frac{e_{mk}}{t}}\right) \qquad \text{Equation 1}$$

where:

f_k characteristic compressive strength of masonry
E_m elasticity modulus of masonry
e_{mk} eccentricity of load equal to sum of static eccentricity, additional eccentricity and eccentricity caused by creep, not less than 0.05t.

According to [1] the slenderness ratio of a masonry wall should not be greater than 27.

To reduce load capacity of compressed members [2] the following forms are recommended:

$$\Phi_2 = 1 - (h_{ef} / 140r)^2 \quad \text{for} \quad h_{ef} / r \leq 99 \qquad \text{Equation 2}$$

$$\Phi_2 = (r / h_{ef})^2 \quad \text{for} \quad h_{ef} / r > 99 \qquad \text{Equation 3}$$

where:
r radius of gyration

Equation 2 is based on an analysis of the results of axial load tests performed on clay and concrete masonry elements.
For a solid rectangular section, $r = \sqrt{t^2 / 12}$ and:

$$\Phi_2 = 1 - 0.000612(h_{ef} / t)^2 \quad \text{for} \quad h_{ef} / t \leq 28.58 \qquad \text{Equation 4}$$

$$\Phi_2 = 0.0833(t / h_{ef})^2 \quad \text{for} \quad h_{ef} / t > 28.58 \qquad \text{Equation 5}$$

Additionally, [2] gives a formula for critical compressive load as follows:

$$P_e = \left(\frac{\pi^2 E_m I_n}{h_{ef}^2} \right) (1 - 2e / t)^3 \qquad \text{Equation 6}$$

where:
I_n uncracked moment of inertia.

ACI 530-02 [2] recommends design of unreinforced masonry walls in such a way that they will remain in an uncracked state.

The new Canadian code CSA S.304.1 [3] allows one of two methods of calculation to be used: Pδ (load displacement) method and moment magnifier method. In the latter, the increase of section bending moment due to horizontal deflection of the wall, for the scheme of members presented in Figure 1, is given by Equation 7:

$$\eta = \frac{1}{1 - \frac{P}{P_{cr}}} \qquad \text{Equation 7}$$

According to [3] the value of P_{cr} can be determined by the formula:

$$P_{cr} = \frac{\phi_e}{1 + 0.5\beta_d} \frac{\pi^2 0.4 E_m I_n}{h_{ef}^2} \qquad \text{Equation 8}$$

In calculations of P_{cr} on the basis of Equation 8 the impact of long-term load (ratio β_d), and resistance factor ϕ_e (=0.65) are taken into account. According to [3] the slenderness ratio of a wall should not be greater than 30, however, a minimum load eccentricity of not less than 0.1t should be assumed in calculations.

The comparison of code recommendations given in [1, 2, and 3] is presented graphically in Figure 2. The comparison was made for short-term loads using an elastic modulus of $1000f'_m$ (f'_m – specified compressive strength of masonry determined by prism tests).

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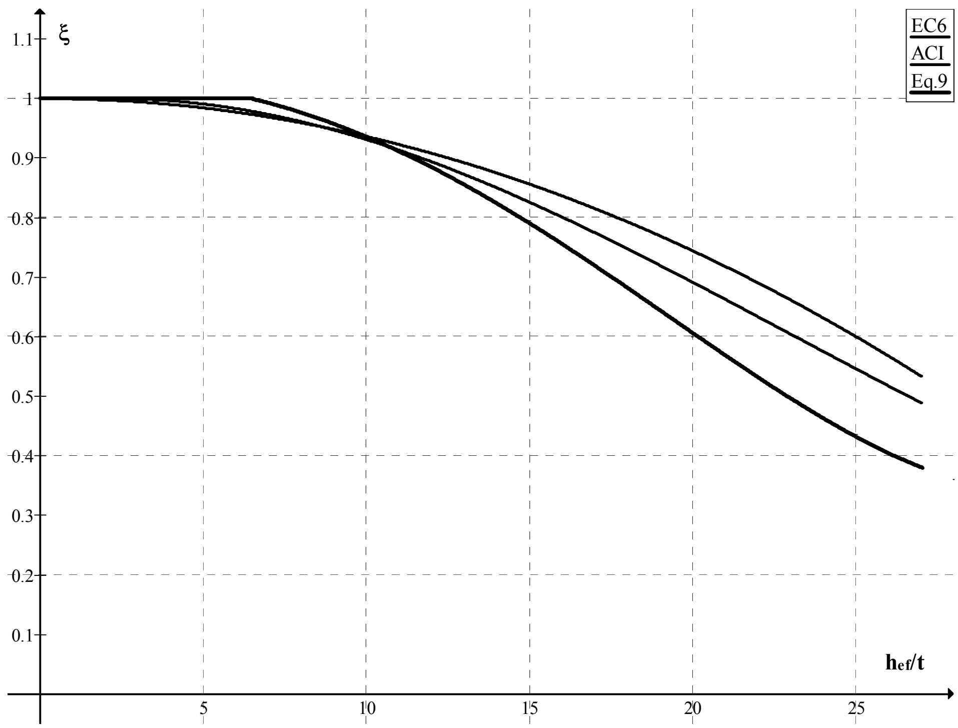


Figure 2 - Comparison of Code Recommendations for e/t = 0.1

The precise comparison of code recommendations is difficult, because in [3] the influence of slenderness depends on P/P_{cr}. Ratio. The dotted curve in Figure 2 represents the solution of equation:

$$\Phi_3 = \left(1 - 0.2\frac{1}{1 - \frac{(h_{ef}/t)^2 \Phi_3}{313}}\right) \qquad \text{Equation 9}$$

with reference to the case of a short wall for which $\Phi_3 = 0.8$ (for e/t=0.1).

For a wall with slenderness < 15 the differences between ξ values are within the range of 10%. For very slender masonry walls the differences are greater, but they do not exceed 27%. For walls with large slenderness the values of ξ obtained from Equation 9 are the smallest; it should be remembered, however, that in wall stiffness reduction ($0.4E_mI_n$) the creep effect is also taken into account. For lower levels of wall stiffness reduction and lower P/P_{cr} ratios the value of $\xi(h_{ef}/t)$ from Equation 9 will be much closer to the solutions given in [1] and [2].

TEST RESULTS AND THEIR COMPARISON WITH CODE RECOMMENDATIONS

In a commentary on [2] the results of axial load tests performed on clay and concrete masonry specimens were presented. Test results were compared to specified compressive strength for masonry (f'_m). Taking into account the test results for a masonry wall with $h_{ef}/t<5$ a relationship can be determined as follows: $f_{mean}/f'_m=1.03\div1.7$ (f_{mean} – mean compressive strength for masonry determined on short walls). The relationship $f_{mean}/f'_m=1.05$ was used in the present paper.

A comparison of experiments performed in various research centres with code recommendations given in [1], [2] is presented in Figure 3. The comparison covers only experiments on members with hinged ends. Each point on the graph represents an average value of 2 or 3 tests.

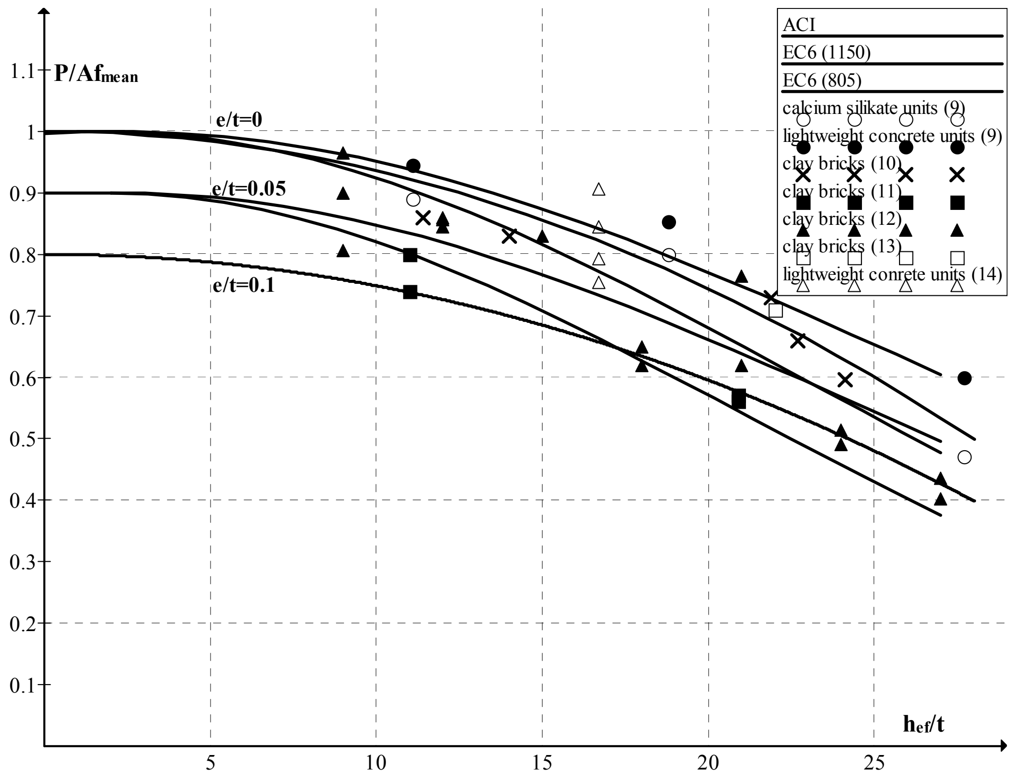


Figure 3 - Comparison of Code Recommendations with Test Results

The conformity of the test results and relationships proposed in the analysed codes is satisfactory. Far better conformity and safer estimation can be obtained when actual relationships between the modulus of elasticity and compressive strength of masonry are employed. For relations $f_{mean}/f'_m=1.05$ and $f_{mean}/f_k=1.2$ the curves representing recommendations given in [1] and [2] are very close. The values of P/Af_{mean} ratio calculated according to [1] were obtained for $E_m/f_k=1150$ and $E_m/f_k=805$. If accidental eccentricity of 0.1t is included, safe estimations with regard to the presented test results are produced.

The comparison presented in Figure 3 includes the results of tests carried out by the author on brick masonry. The tests were performed on members of 120x520 mm cross-section made from cement, lime-cement and lime mortars. The average compressive strength of the masonry determined on walls with slenderness ratio of 3-5 was 3.1-9.98 MPa. The compressive strength of masonry made from lime mortar (compressive strength of mortar 0.5 MPa) was 3 times smaller than the compressive strength of masonry made from cement mortar (compressive strength of mortar 9.6 MPa). The results of tests on masonry with lime mortar, due to the specific properties ($E_m/f_{mean}\approx 200$) were not marked in Figure 3. The reduction of load capacity for slender walls made from lime mortar was: $P/f_{mean}A = 0.55$ for $h_{ef}/t=11$, and $P/f_{mean}A = 0.35$ for $h_{ef}/t=21$. In Figure 4 failure modes for the brick walls are presented.

a) b)

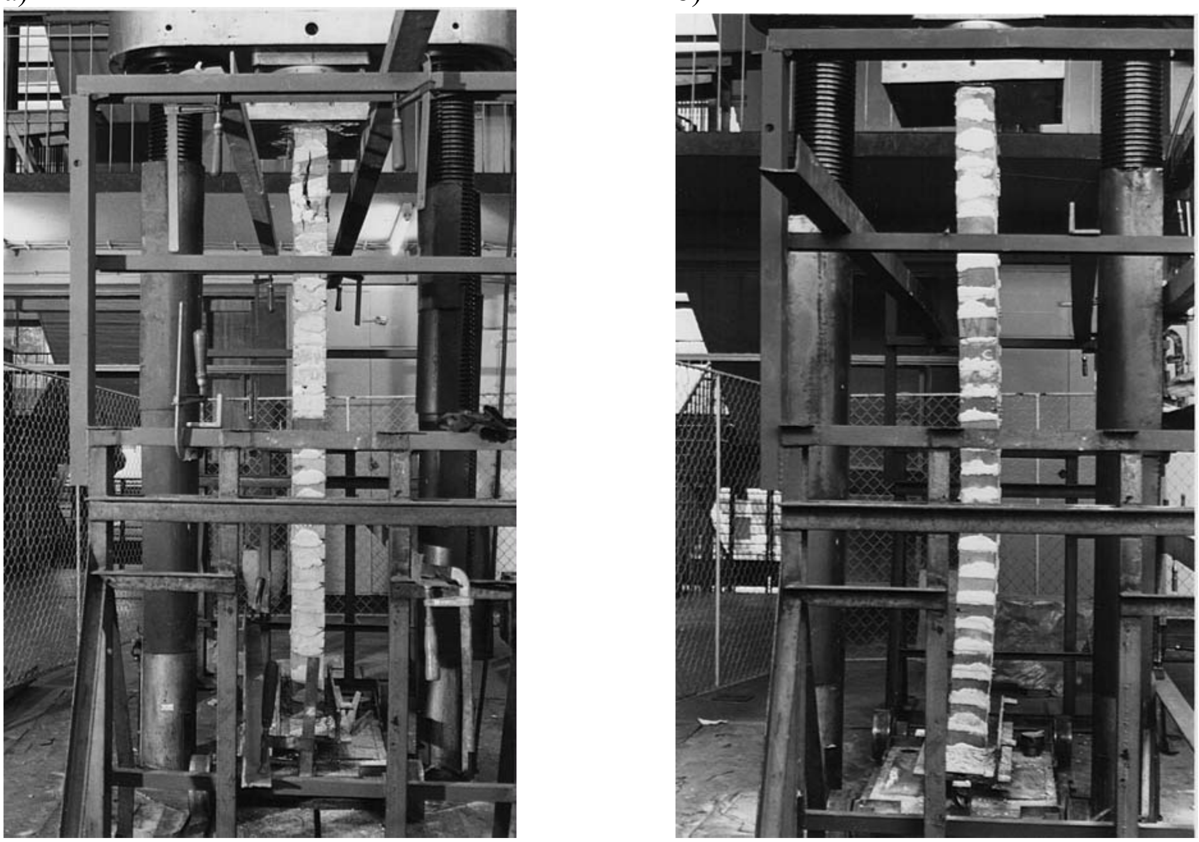


Figure 4 - Failure Modes For Brick Walls Under Concentric Compression
a) Wall Made From Lime Mortar, b) Wall Made From Cement Mortar

Slender walls made from lime mortar failed due to bending and loss of stability (the maximum deflection in tests was 23.5mm); however, slender walls made from cement and cement-lime mortar crushed typically like short walls – cracking in half of the thickness (the maximum deflection not exceeding 3 mm). In the experiments the vertical strains along the whole height of the wall were measured. Considerable material non-uniformity was found – differences in the modulus of elasticity for fragments of the wall reached 30%.
The value of accidental eccentricities that appeared in the masonry tests was estimated following the strain recordings. The accidental eccentricities computed assuming the planar cross-section principle did not exceed 0.04t.

The results of tests performed with load eccentricities of 0.33t are presented in Figure 5. These tests were far fewer than axial load tests.

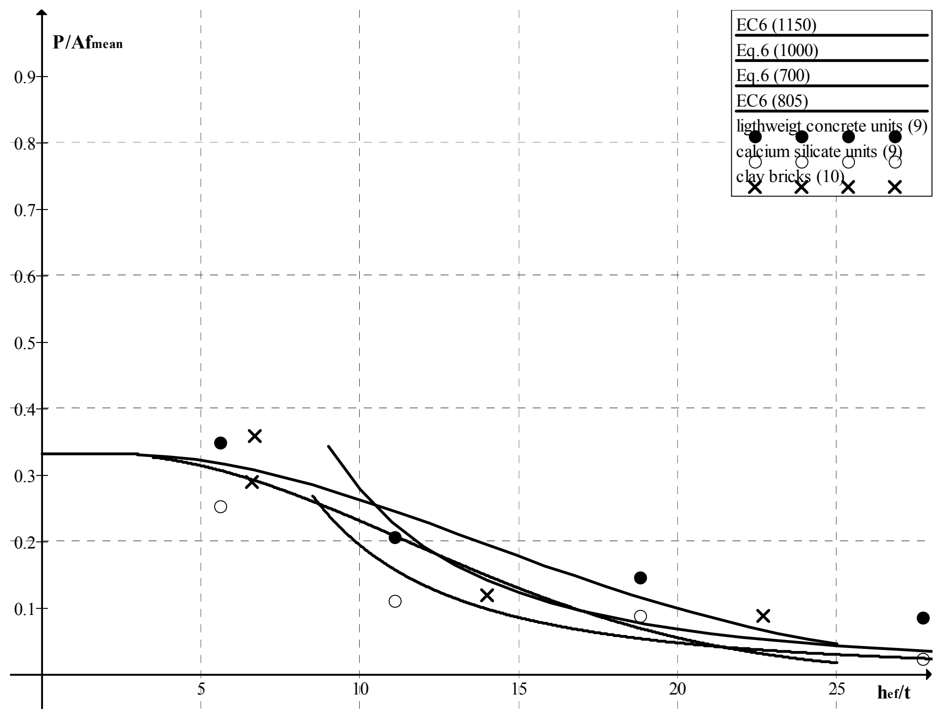


Figure 5 - Comparison of Code Recommendations with Test Results For e/t=0.33

The conformity of test results and the relationships proposed in [1] for some masonry is by no means satisfactory. For walls with slenderness ratio >10 safer estimation can be obtained using Equation 6.

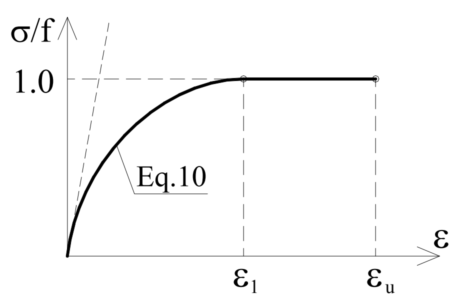
MODEL FOR ECCENTRICALLY COMPRESSED MASONRY WALL

A detailed description of the model is given in [11] and [15] (scheme according to Figure 1).
The stress-strain relationship for masonry is taken in the form:

$$\frac{\sigma}{f} = \alpha_1 \cdot \varepsilon_1 \left[1 - (k_1 - 1) \cdot \frac{\alpha_1 \varepsilon_1}{k_1^2} \right] \quad \text{for } \sigma \le f \qquad \text{Equation 10}$$

$$\frac{\sigma}{f} = 1 \quad \text{for } \sigma > f \qquad \text{Equation 11}$$

where the symbols are according to Figure 6.



$k_1 = E_0\varepsilon_1/f$; $k_1 = 1 \div 2$
$k_2 = \varepsilon_u/\varepsilon_1$
$\alpha_1 = E_0/f$
E_0 – initial modulus of elasticity

Figure 6 - Stress-Strain Relationship for Masonry Used In Wall Model

The shape of a wall under bending is assumed according to the formula proposed by Haller [16]:

$$\Delta e(y) = \Delta e^* \cdot \sin \frac{\pi}{h_{ef}} y \qquad \text{Equation 10}$$

In the computer program developed during the research, the solution in the middle cross-section of the wall is sought in an iterative way. The results of numerical computation (for $E_0/f=1000$) were compared with the recommendations given in [1] (see Figure 7).

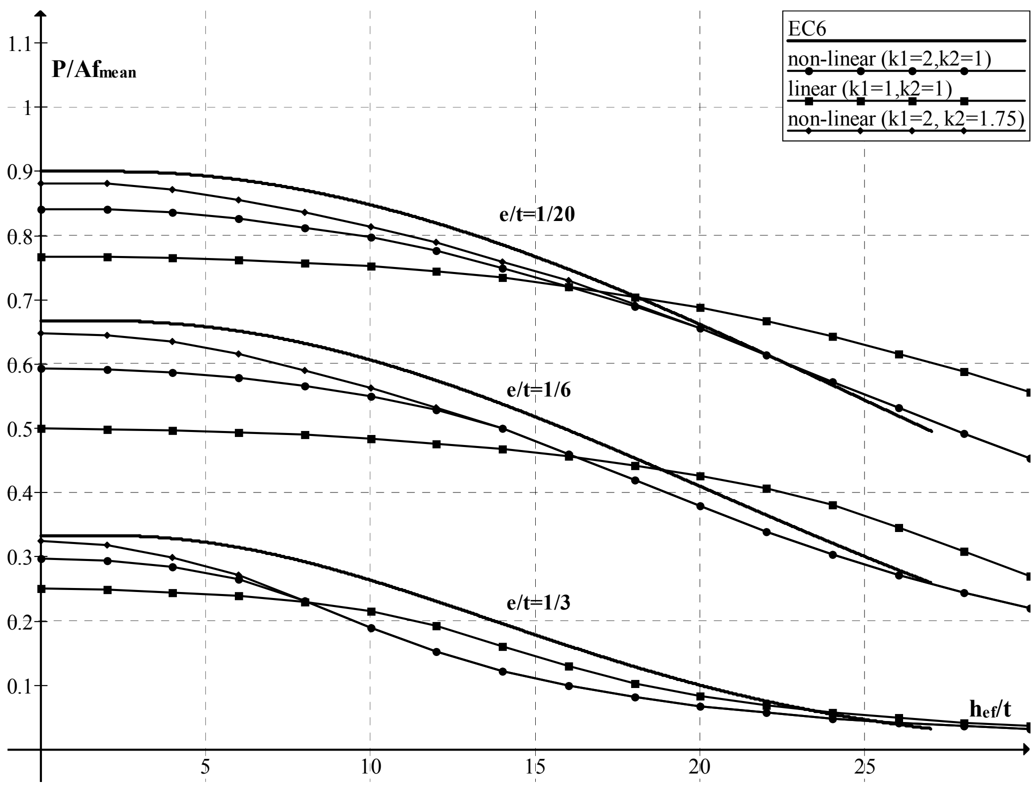


Figure 7 - Function $\frac{P}{f_{mean}A}\left(\frac{h_{ef}}{t}, \frac{e}{t}\right)$ **for Different Models of Materials**

The influence of parameters describing masonry deformability on the values of Φ ratio is very significant. For close estimation of the influence of slenderness on masonry wall capacity it is necessary to take into account the ratio of the modulus of elasticity to the compressive strength of masonry and the parameter describing non-linearity of the stress-strain relationship (k_1). In the computation of load capacity of masonry with slenderness ratio >7 the plastic plateau ($k_2 \neq 1$) can be neglected.

For the linear model ($k_1=1$, $k_2=1$) the impact of slenderness on capacity of a compressed member obtained from numerical analysis is the smallest.

The results calculated on the basis of the proposed masonry wall model (including realistic stress-strain relationships for masonry) are close to those obtained in tests – differences in the analysed cases do not exceed 20%.

To determine the reduction factor Φ a simplified relationship is proposed

$$\Phi = \Phi^* \left(1 - 0.007 \sqrt{\frac{k_1^2}{\alpha_1}} (1 + 6e/t) \left(\frac{h_{ef}}{t} \right)^2 \right) \qquad \text{Equation 11}$$

where:

Φ^* coefficient of reduction due to eccentricity.

The comparison of numerical calculations with the simplified solution is presented in Figure 7 The agreement for a wall compressed with eccentricity ≤ 0.166t is very satisfactory.

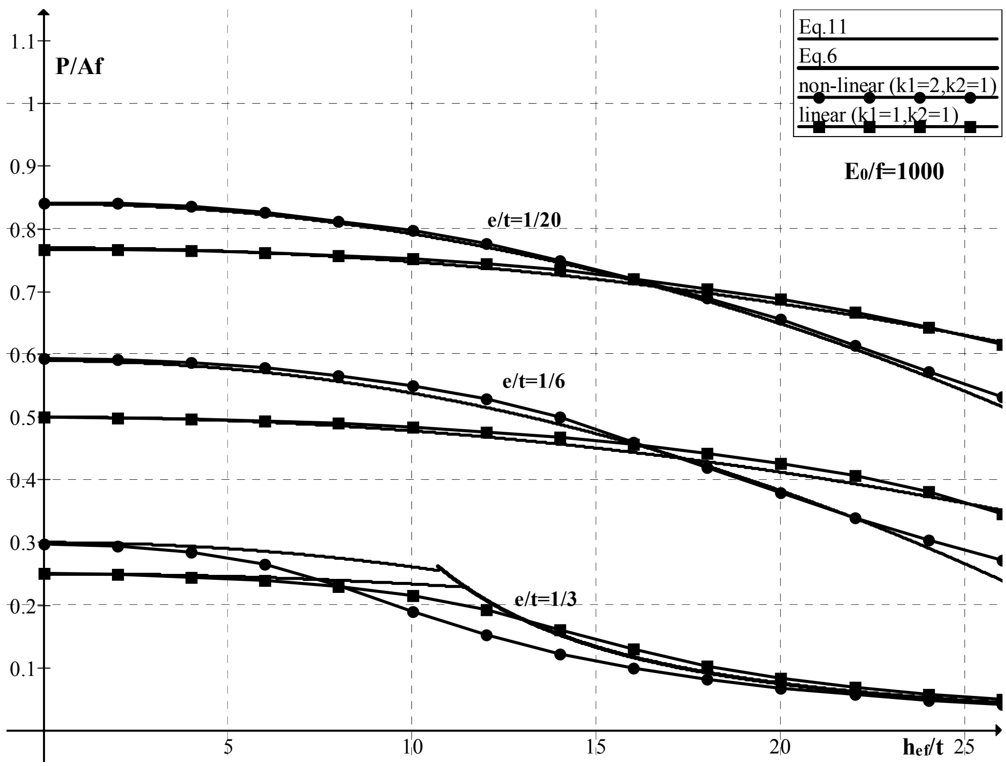


Figure 7 – Comparison of Numerical Calculations with Simplified Formula

SUMMARY

The simplified relationships recommended in the analysed codes for assessment of the influence of slenderness on capacity of quasi-axially compressed masonry walls show a good agreement with the results of tests on walls from cement or cement-lime mortar. Better agreement can be achieved by including the actual ratio of the modulus of elasticity to the masonry compressive strength and accidental eccentricities. The assumption of minimum load eccentricities on the level of 0.1t gives, in general, very safe estimations of the capacity of quasi-axially compressed masonry walls. For walls compressed with large eccentricities estimations stemming from the simplified code relationships give less satisfactory results. From the comparison made in this paper it appears that the differences for some types of masonry are a few times larger than for quasi-axial compression. The computations on the basis of the presented wall model point out that in such cases, besides the ratio of the modulus of elasticity to the compressive strength of masonry, it is also necessary to take into account the parameters describing the level of non-linearity of the stress-strain relationship for masonry in compression. Relevant formulae have been given in the paper.

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