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INVESTIGATION ON FAILURE CRITERION OF MASONRY UNDER BIAXIAL STRESS

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ABSTRACT

On the basis of available experimental results concerning the biaxial strength of masonry, the failure criterion of masonry under arbitrary biaxial stress is investigated. The influence of bed joint orientation on strength of masonry under biaxial stress is reflected in the criterion by taking account of orthotropic behaviour of masonry strength. The model suggested can be used as the criterion to judge the failure of homogenized masonry structures.

Furthermore, cracking criterion of masonry is suggested according to experimental phenomena. According to different stress state, principal tensile stress and principal tensile strain cracking criteria are suggested. Suggested models are applicable to smeared cracking element model, and the influence of bed joint orientation on crack pattern can be neglected for use of the models.

KEYWORDS: failure criterion, biaxial behaviour, shear strength

INTRODUCTION

The failure criterion of masonry is generally used to judge if masonry structures fail under certain stress states. For special states, such as uniaxial tension, uniaxial compression and pure shear, failure criteria have to be individually established. However, due to the presence of mortar joints, the strength of masonry exhibits significant orthotropic behaviour, and varies with orientation of joints and units. Therefore, the failure criteria of masonry under biaxial stress is unavoidably complicated.

Since 1960, the theories to calculate the shear strength of masonry can mainly be classified into two categories – principal tensile stress failure criterion and Coulomb's failure criterion. According to available experimental data for masonry structures, the principal tensile stress criterion may underestimate the shear strength of masonry because the material behaviour of masonry is not realistically considered. On the contrary, Coulomb's criterion may overestimate the shear strength of masonry because it neglects the fact that the shear strength decreases with normal compressive stress after the occurrence of cracking.

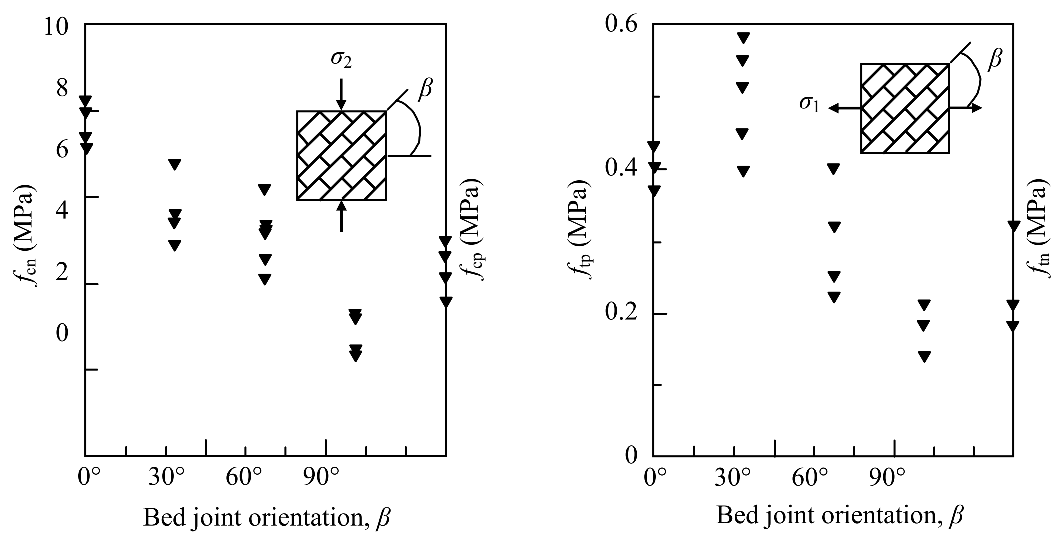
The failure of masonry under plane stress can be classified into three types: (1) slipping failure along the mortar joint; (2) cracking of unit and splitting in mortar joint; and (3) spalling parallel with the surface of panel. The three failure types are respectively expressed in three classic criteria [1], including modified Mohr-Coulomb friction criterion, Saint Venant's maximum tensile strain criterion, and Navier's maximum compressive stress criterion. For practical use to the analysis of masonry structure, the procedure in [1] is somewhat laborious.

A series of test data reported by A. W. Page [2,3], who carried out experimental research on biaxial failure of masonry panels, means the commencing of new time for the investigation on the failure criteria of masonry under biaxial stress.

The purpose of this paper is to develop a realistic, applicable failure criterion of masonry based on available test data concerning the biaxial failure of masonry.

INFLUENCE OF BED JOINT ORIENTATION ON MASONRY STRENGTH

Experimental results for brick masonry reported by Page [3] show that bed joint orientation has a significant effect on the strength of masonry, and part of the results are replotted in Figure 1. It is found that both tensile and compressive strength of masonry vary with the bed joint orientation, β – the angle between the bed joint and the direction of maximum principal stress, σ_1. For the compressive strength corresponding to β=0° and β=90°, notations of f_{cn}, f_{cp} respectively are used; and for the tensile strength corresponding to β=0° and β=90°, notations of f_{tp}, f_{tn} respectively are used. It is noticed from Figure 1 that the ratio of f_{cn} to f_{cp} is close to 2.0 and the ratio of f_{tp} to f_{tn} is also greater than 1.0. Therefore, the orthotropic strength behaviour is significant in masonry, and it is not acceptable to neglect the orthotropic strength behaviour in the failure criterion to be constructed.



(a) Compressive strength (b) Tensile strength

Figure 1 – Experimental Results of Masonry Strength, Replotted. [3]

CONSTRUCTION OF FAILURE CRITERIA

Considering the orthotropic behaviour of masonry strength, failure criterion for the two typical cases of β=0° and β=90° are illustrated in Figure 2, in which coordinates for the typical points represented by circles in Figure 2 are listed in Table 1 and Table 2. The procedure for constructing the criteria is described in detail as follows.

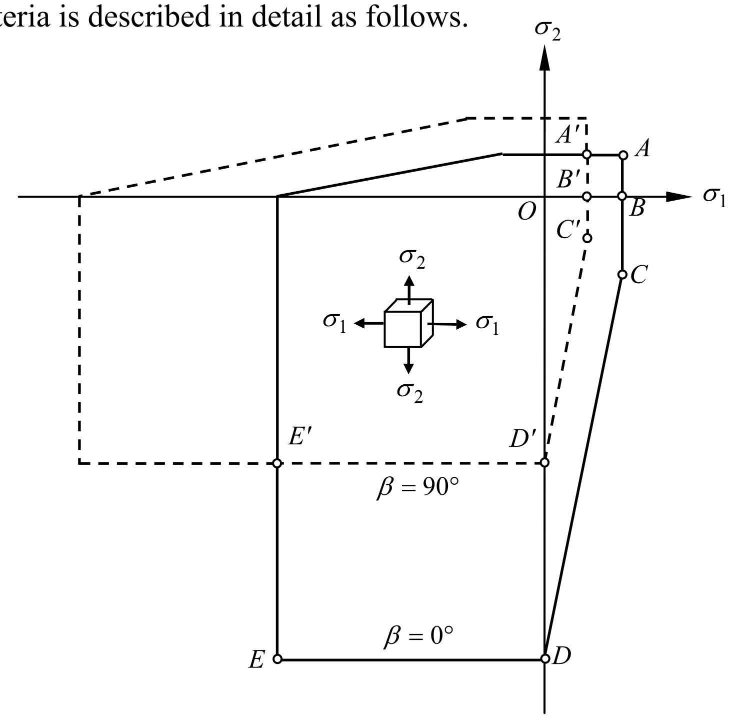


Figure 2 – Suggested Failure Criteria of Masonry

For the case of β=0°, if stress is in tension-tension state, the failure condition without considering the influence of biaxial stress is given in Equation 1 and Equation 2 corresponding to lines *A'A*, and *AB* respectively, as shown in Figure 2.

$$\sigma_2 = f_{tn} \qquad (f_{tn} \le \sigma_1 \le f_{tp}) \qquad \text{Equation 1}$$

or/and

$$\sigma_1 = f_{tp} \qquad (0 \le \sigma_2 \le f_{tn}) \qquad \text{Equation 2}$$

For compression-compression state, if the biaxial effect is neglected, which is conservative, then the failure condition is given in Equation 3 and Equation 4 corresponding to lines *EE'*, and *DE* respectively, as shown in Figure 2.

$$\sigma_1 = -f_{cp} \qquad (-f_{cn} \le \sigma_2 \le -f_{cp}) \qquad \text{Equation 3}$$

or/and

$$\sigma_2 = -f_{cn} \qquad (-f_{cp} \le \sigma_1 \le 0) \qquad \text{Equation 4}$$

Table 1 – Typical points for the criterion corresponding to *β*=0°

Point	Failure Mode	σ_1	σ_2	$\sigma_m = \frac{\sigma_1+\sigma_2}{2}$	$\tau_{max} = \frac{\sigma_1-\sigma_2}{2}$
A	Biaxial tension	f_{tp}	f_{tn}	$(f_{tp}+f_{tn})/2$	$(f_{tp}-f_{tn})/2$
B	Uniaxial tension	f_{tp}	0	$f_{tp}/2$	$f_{tp}/2$
C	Pure shear	f_{tp}	$-f_{tp}$	0	f_{tp}
D	Uniaxial compression	0	$-f_{cn}$	$-f_{cn}/2$	$f_{cn}/2$
E	Biaxial compression	$-f_{cp}$	$-f_{cn}$	$-(f_{cp}+f_{cn})/2$	$-(f_{cp}-f_{cn})/2$

Table 2 – Feature points for the criterion corresponding to *β*=90°

Point	Failure Mode	σ_1	σ_2	$\sigma_m = \frac{\sigma_1+\sigma_2}{2}$	$\tau_{max} = \frac{\sigma_1-\sigma_2}{2}$
A'	Biaxial tension	f_{tn}	f_{tn}	f_{tn}	0
B'	Uniaxial tension	f_{tn}	0	$f_{tn}/2$	$f_{tn}/2$
C'	Pure shear	f_{tn}	$-f_{tn}$	0	$f_{tn}/2$
D'	Uniaxial compression	0	$-f_{cp}$	$-f_{cp}/2$	$f_{cp}/2$
E'	Biaxial compression	$-f_{cp}$	$-f_{cp}$	$-f_{cp}$	0

If $\sigma_1 = -\sigma_2 = \tau$, this corresponds to the so-called pure shear state with shear stress of τ provided the principal stress axis is turned an angle of 45°. Therefore, for the case of *β*=0°, when maximum value of pure shear stress is attained in masonry, the corresponding stress state is

$$\sigma_1 = f_{tp} \;\; \& \;\; \sigma_2 = -f_{tp} \qquad \text{Equation 5}$$

The state expressed by Equation 5 corresponds to point *C* shown in Figure 2.

Suppose sections *BC*, *CD* of the failure criteria are linear lines. They can be expressed in Equation 6 and Equation 7 respectively:

$$\sigma_1 = f_{tp} \qquad (-f_{tp} \le \sigma_2 \le 0) \qquad \text{Equation 6}$$

$$\sigma_1 = \frac{f_{cn}+\sigma_2}{f_{cn}-f_{tp}} \cdot f_{tp} \qquad (-f_{cn} < \sigma_2 < -f_{tp}) \qquad \text{Equation 7}$$

Therefore, the multi-line *A'ABCDEE'* constitutes the full failure criterion for the case of *β*=0° while $\sigma_1 \ge \sigma_2$.

Similarly, for the case of β=90°, the full failure criterion while $\sigma_1 \geq \sigma_2$ can be obtained as the multi-line *A'B'C'D'E'* shown in Figure 2, which is composed by lines *A'B'*, *B'C'*, *C'D'*, *D'E'* which can be expressed respectively as follows:

$$\sigma_1 = f_{tn} \quad (0 \leq \sigma_2 \leq f_{tn}) \qquad \text{Equation 8}$$

$$\sigma_1 = f_{tn} \quad (-f_{tn} \leq \sigma_2 \leq 0) \qquad \text{Equation 9}$$

$$\sigma_1 = \frac{f_{cp} + \sigma_2}{f_{cp} - f_{tn}} \cdot f_{tn} \quad (-f_{cp} < \sigma_2 < -f_{tn}) \qquad \text{Equation 10}$$

$$\sigma_2 = -f_{cp} \quad (-f_{cp} \leq \sigma_1 \leq 0) \qquad \text{Equation 11}$$

The failure criteria corresponding to cases of β=0° and β=90°, while $\sigma_1 < \sigma_2$, shows symmetry with the failure criteria corresponding to cases of β=90° and β=0°, while $\sigma_1 \geq \sigma_2$, with respect to line $\sigma_1 = \sigma_2$. If it is not specifically indicated in the following discussion, $\sigma_1 \geq \sigma_2$ is always assumed for the discussion of failure criteria of masonry.

In Figure 2, only failure criteria for the two cases of β=0° and β=90° are plotted. Actually, an arbitrary bed joint orientation, β, in the range of 0°-90° corresponds to a certain failure criterion, although a large amount of test data are needed to verify the criteria.

In the following discussion, a simplified procedure will be used to construct the generalized failure criteria for $0° \leq \beta \leq 90°$. According to Figure 1, it is acceptable to suppose that linear distribution of masonry strength exists for various bed joint orientations, β. For arbitrary bed joint orientation, β, suppose the tensile strengths in directions of σ_1, σ_2 are f_{1t}, f_{2t} respectively, and the compressive strengths are f_{1c}, f_{2c} respectively. Then

$$f_{1t} = f_{tn} + \frac{90° - \beta}{90°}(f_{tp} - f_{tn}) \qquad \text{Equation 12}$$

$$f_{2t} = f_{tn} + \frac{\beta}{90°}(f_{tp} - f_{tn}) \qquad \text{Equation 13}$$

$$f_{1c} = f_{cp} + \frac{\beta}{90°}(f_{cn} - f_{cp}) \qquad \text{Equation 14}$$

$$f_{2c} = f_{cp} + \frac{90° - \beta}{90°}(f_{cn} - f_{cp}) \qquad \text{Equation 15}$$

For $0° \leq \beta \leq 45°$, failure criteria similar to the criterion corresponding to β=0°, as multi-line *A'ABCDEE'* shown in Figure 2, can be constructed, provided f_{tp}, f_{tn}, f_{cp}, f_{cn}, in Equations 1-7 are replaced with f_{1t}, f_{2t}, f_{1c}, f_{2c}, respectively. For $45° < \beta \leq 90°$, failure criteria similar to the criterion corresponding to β=90°, as multi-line *A'B'C'D'E'* shown in Figure 2, can be constructed, provided f_{tp}, f_{tn}, f_{cp}, f_{cn}, in Equations 1-7 are replaced with f_{2t}, f_{1t}, f_{2c}, f_{1c}, respectively.

APPLICATION PROCEDURE OF SUGGESTED FAILURE CRITERIA

For the criteria constructed above, for $0° \le \beta \le 45°$, criteria similar with the case of $\beta = 0°$ can be applied; and for $45° < \beta \le 90°$, criteria similar with the case of $\beta = 90°$ can be applied. Although the suggested failure criteria seem complicated, simple application procedures can be concluded as follows. For masonry with bed joint orientation of $0° \le \beta \le 90°$, failure occurs if one of following conditions is satisfied:

(1) $\sigma_1 \ge f_{1t}$ or/and $\sigma_2 \ge f_{2t}$, although the latter condition rarely controls for the case of $45° < \beta \le 90°$;

(2) $\sigma_1 \le -f_{1c}$ or/and $\sigma_2 \le -f_{2c}$, although the latter condition rarely controls for the case of $45° < \beta \le 90°$;

(3) if $-f_{2c} < \sigma_2 < -f_{1t}$, and $\sigma_1 \ge \dfrac{f_{2c} + \sigma_2}{f_{2c} - f_{1t}} \cdot f_{1t}$.

f_{1t}, f_{2t}, f_{1c}, f_{2c} can be determined from Equations 12-15.

CRACKING CRITERIA

For uniaxial and biaxial tension, cracking criteria are actually the same as failure criteria. Because the failure is controlled by condition (1) mentioned above, the cracking condition is

$$\sigma_1 \ge f_{1t} \qquad \text{Equation 16a}$$

or/and

$$\sigma_2 \ge f_{2t} \qquad \text{Equation 16b}$$

For $45° < \beta \le 90°$, the latter condition (Equation 16b) almost never controls. This criterion can be called the principal stress cracking criterion. If the principal tensile stress satisfies the criterion, cracking will occur in the plane perpendicular to the direction of the stress.

Experimental results show that the failure pattern of masonry under plane stress state depends on bed joint orientation as well the ratio of two principal stresses. The failure patterns of masonry panels are shown in Figure 3 and are according to results found in the literature [4,5]. For uniaxial tension, cracking is controlled by the principal tensile stress criterion of Equation 16a, but it is found from Figure 3 that the head and bed mortar joints determine the crack patterns and cracks are not ideally perpendicular to the direction of principal tensile stress, however the difference is not greater than 45°. Therefore, a failure crack appearing at the position which is perpendicular to the direction of principal tensile stress σ_1 can be generally assumed.

For uniaxial compression, it is shown in Figure 3 that failure cracks appears in the direction parallel to the principal compressive stress σ_2. Therefore, it is supposed that the principal tensile strain ε_1 ,which is perpendicular to the direction of σ_2, is responsible for the occurrence of cracking, and corresponding cracking criterion is expressed as:

$$\varepsilon_1 \geq \varepsilon_{1cr} \quad \text{Equation 17}$$

where ε_{1cr} is the cracking strain in the direction perpendicular to the direction of σ_2. The criterion expressed in Equation 17 is called principal tensile strain criterion, and cracks will appear at positions parallel to the direction of σ_2. For uniaxial and biaxial tension, the criterion can be used to judge if cracking of masonry occurs. Nevertheless, for biaxial compression with almost equal compressive stress in two orthotropic directions, the failure pattern may alter from in-plane failure to out-plane failure, as shown in Figure 3.

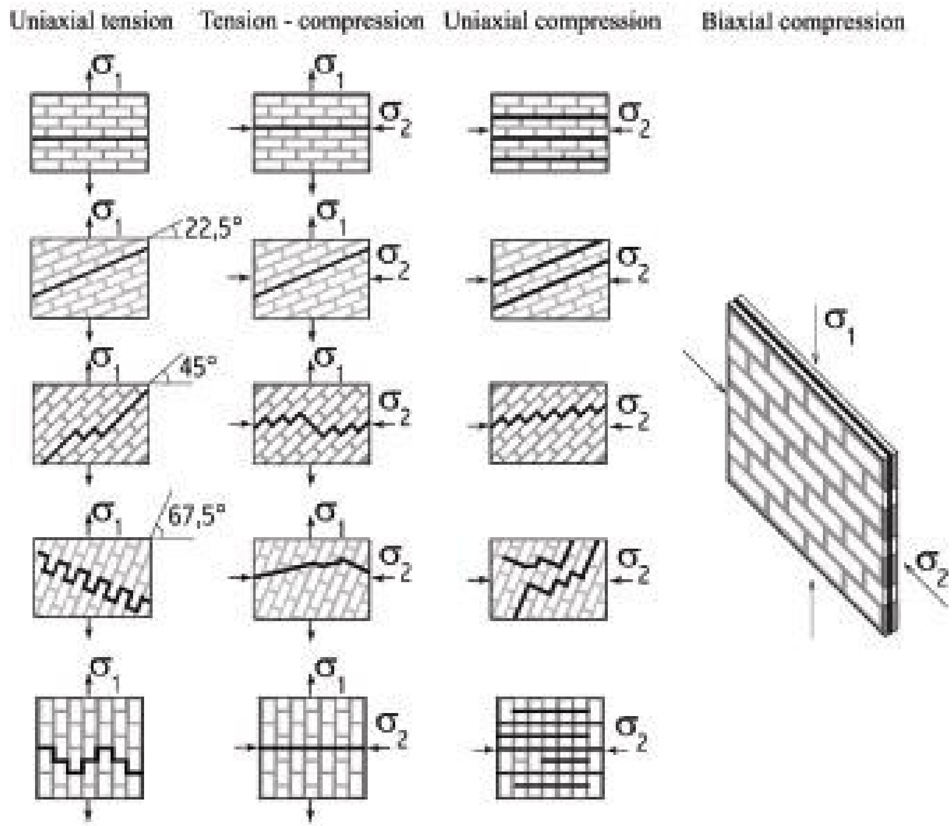


Figure 3 - Typical Failure Patterns of Masonry Panels [4,5]

For tension-compression state, cracking is possibly controlled by principal tensile stress cracking criterion or principal tensile strain cracking criterion, depending on the ratio of principal stresses. Therefore, both the criteria should be used to check if the cracking of masonry occurs.

As for cracking strain ε_{1cr} perpendicular to the direction of σ_2, results were not found in the literature. There are three ways to determine the value of ε_{1cr}, including:

(1) The value of ε_{1cr} is taken to be equal to the strain corresponding to the peak stress of stress-strain relationship for uniaxial tension, although no evidence shows they are necessarily equal. Due to the difficulty to make experiment on the tension of masonry, little test data is reported in literatures.

(2) The value of ε_{1cr} is taken to be equal to the transverse cracking strain when the specimen is in compression. Because the cracking stress is difficult to record, little related data is reported in literature and is not sufficiently accurate if any. It is suggested to take $\varepsilon_{1cr} = \alpha \nu_1 \varepsilon_{20}$ as an approximation, where ε_{20} is the strain corresponding to the peak stress in stress-strain relationship for uniaxial compression, ν_1 is the Poisson's ratio in the transverse direction, and α is a reduction factor taken as 0.2-0.3.

(3) Numerical procedures may be used to determine the value of ε_{1cr}. As an analytical parameter, a reasonable value of ε_{1cr} can be determined by enormous amount of numerical calculation, but the results obtained will be influenced to some extent by the modelling of constitutive relationships.

If the influence of bed joint orientation is considered, the determination of ε_{1cr} tends to be more complex. Therefore, a feasible method to obtain the value of ε_{1cr} is still under investigation.

CONCLUSION

Based on available experimental results concerning the biaxial strength of masonry, the failure criteria of masonry under biaxial stress are investigated. By considering the orthotropic behaviour of masonry strength, the influence of bed joint orientation is reflected in the criteria. Suggested criteria are applicable to homogenized masonry. The criteria are close to classical failure criteria of masonry in style, and their application procedure is simple.

Furthermore, cracking criteria of masonry are suggested in this paper, which are suitable to the smeared cracking element model. The influence of bed joint orientation on crack pattern is neglected in using the cracking criteria. For the cracking controlled by principal tensile stress, explicit criterion to judge the cracking of masonry is suggested in the paper; but for the cracking controlled by principal tensile strain, explicit expression of the criterion needs further investigation.

For the nonlinear analysis of masonry structure, in addition to failure criteria, a set of constitutive relationships are necessary. They are out of the context of this paper.

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