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EFFECTIVE FLEXURAL RIGIDITY OF CONCRETE MASONRY WALLS

Liu, Y, Aridru, G. G., and Dawe, J. L.
Department of Civil Engineering
University of New Brunswick, Fredericton, New Brunswick, E3B 5A3

ABSTRACT

An experimental program consisting of seventy-two full scale wall specimens was carried out to investigate behaviour of masonry walls under eccentric loading. Two equations were proposed to evaluate the flexural rigidity immediately prior to failure for reinforced masonry walls with various eccentricities. For reinforced walls tested in the range $0 \le e/t \le 0.18$, an appropriate value of EI_{eff} was found to be $0.70E_mI_0$. In the range $0.18 < e/t < 0.5$, it was found that $EI_{eff} = 2.7E_mI_0 \exp(-7.5e/t) \ge E_mI_{cr}$. It is reasonable to conclude that the values applied in the Canadian masonry code are conservative by comparison with those obtained from the experimental results.

INTRODUCTION

In addition to gravity loads, masonry walls are often subjected to lateral forces resulting from wind and earthquake. Construction methods also frequently create load eccentricities. In general, a rational design procedure for plain and reinforced walls must consider the combined effects of axial load and bending.

Primary moment caused by lateral and eccentric loads creates initial lateral deflection. Axial loads acting over the deflected profile produce additional bending referred to as secondary moment. The Canadian masonry code (CSA S304.1) suggests a load - displacement method or a moment magnifier method to account for this secondary effect. In calculating displacement for the load - displacement method, buckling load for the moment magnifier method, and the ultimate moment capacity, an effective flexural rigidity, EI_{eff}, is employed. For a masonry wall subjected to compression and significant lateral bending, reductions in the modulus of elasticity, E_m, and the moment of inertia, I, occur simultaneously with increased loading. As load increases, the development of cracks in a masonry wall reduces the net cross-sectional area resulting in a higher stress level and a reduced moment of inertia. The high stress level results in a reduction in modulus of elasticity due to non-linearity of the stress vs. strain relationship of masonry. Therefore, any attempt to accurately evaluate the effective flexural rigidity of a masonry wall must consider the simultaneous interaction of each of these changing phenomena.

An experimental program was conducted to investigate the behaviour of masonry walls under eccentric loading. Two equations were proposed to evaluate the flexural rigidity for reinforced masonry walls under various eccentricities. Furthermore, the EI value provided by the current Canadian masonry code (CSA S304.1 - M94) is compared with results generated through experimental testing.

LITERATURE REVIEW

Various proposals have been made for evaluating flexural rigidity of masonry walls. Yokel et al. (1971) proposed the following equation to account for the flexural rigidity of masonry walls at failure:

$$EI = E_m I_0 [0.2 + \frac{P}{P_0}] \le 0.7 E_m I_0 \qquad (1)$$

where EI is the effective flexural rigidity, E_m is the initial tangent modulus of elasticity of masonry determined from masonry prism tests, I_0 is the uncracked moment of inertia of the section, P is the compressive load at failure, and P_0 is the axial capacity derived from prism tests with flat support conditions. This equation was the result of a study based on the capacity of brick masonry walls tested under various combinations of flexure and axial compression.

The accuracy of the above equation for short sections of brick walls, and possible application to concrete masonry walls was investigated by Fattal and Cattaneo (1976) who tested eccentrically loaded short walls. They argued that for short walls, the secondary load - displacement moment produced by a vertical load, P, acting through a transverse deflection, Δ, is negligible compared to the primary moment, Pe, where e is the applied load eccentricity at the ends of the wall. Using a moment-curvature relationship for short wall specimens bent in constant single curvature, Fattal et al.(1976) calculated the flexural rigidity, EI, from the following relationship:

$$EI = \frac{Pet}{\varepsilon_1 - \varepsilon_2} \qquad (2)$$

where P is the axial load, t is the wall thickness, e is the load eccentricity, and ε_1 and ε_2

designate the strains on the compression and tension surfaces, respectively. The values of EI obtained in this manner for brick and concrete masonry specimens were plotted by Fattal and compared to the equation proposed by Yokel et al.(1971). It was noted that values of EI for concrete block prisms were underestimated (Fattal, S. G. and Cattaneo, L. E. 1976).

Hatzinikolas and Warwaruk (1978) performed extensive tests on eccentrically loaded plain and reinforced concrete masonry walls. Using the solution for the elastic deflection curve of a wall, they showed that the moment of inertia of a cracked wall section can be approximated by the following expression:

$$I = 8\left[\frac{1}{2} - \frac{e}{t}\right]^3 I_0 \tag{3}$$

where all the terms are defined as before. It should be noted that load - displacement (P-Δ) effects were not taken into account in either of Equations (2) or (3).

The current Canadian masonry code, CSA Standard S304.1 - M94 (1994), recommends the following equation for calculating the effective flexural rigidity for plain grouted and hollow concrete masonry walls:

$$(EI)_{eff} = 0.40 E_m I_0 \tag{4}$$

E_m is the modulus of elasticity taken as 850 f'_m where f'_m is the compressive strength of masonry, and I_0 is the uncracked moment of inertia of the effective cross-sectional area. For reinforced masonry, the effective flexural rigidity is evaluated as:

$$(EI)_{eff} = E_m\left(0.25I_0 - \left(0.25I_0 - I_{cr}\right)\left[\frac{\left(e - e_k\right)}{2e_k}\right]\right) \tag{5}$$

$$\text{and } E_m I_{cr} \le (EI)_{eff} \le 0.25 E_m I_0$$

where I_{cr} is the moment of inertia of the cracked section taken at yield and neglecting axial load effects, and e_k is the kern eccentricity. In both equations, P-Δ effects are not included.

Although the Canadian masonry code provides equations for calculating EI_{eff} for both plain and reinforced masonry walls, it does not provide convincing proof or satisfactory correlation with testing data. In fact, there are few comprehensive studies available in the published literature on the subject of flexural rigidity of laterally or eccentrically loaded masonry walls. Available equations have been either empirically or semi-empirically founded with questionable agreement between experimental and theoretical results. Based on these observations, the gathering of additional information on the determination of appropriate values of $(EI)_{eff}$ is justified.

METHODOLOGY

Lateral out-of-plane loads and eccentric loads cause flexural cracking within the wall section decreasing its effective cross-sectional depth and moment of inertia. This results in a reduced flexural rigidity accompanied by increased lateral deflection and moments which further deepen flexural cracks. Simultaneously, the reduced net cross-sectional area elevates internal stresses thereby lowering E_m which typically results from a non-linear stress - strain relationship. In the model presented herein, the simultaneous change of E and I is considered by evaluating EI as a single quantity using the moment - curvature relationship:

$$EI = \frac{M}{\Phi} \qquad (6)$$

where M is the moment taking into account the load - displacement effect and Φ is the measured curvature of the section. For single curvature and pinned support conditions, cracking in a wall usually starts at mid-height where the total moment reaches maximum, while EI_{eff} is minimum. In this research, strain measurements and lateral deflection at the mid-height of wall specimens are used to compute EI_{eff}. In this case, M is expressed as $P(e+\Delta)$ where e is the applied eccentricity and Δ is mid-height deflection measured at the end of each load increment. Φ can be determined as $\frac{\varepsilon_1 \quad \varepsilon_2}{t}$ where ε_1 and ε_2 are the strains on the compression and tension faces of a wall specimen respectively, and t is the wall thickness at the point of strain measurement. Therefore, the flexural rigidity may be obtained as:

$$EI = \frac{P(e+\Delta)t}{\varepsilon_1 - \varepsilon_2} \qquad (7)$$

Strains at the compression and tension faces of masonry walls and deflection at mid-height were measured and recorded continuously during each test. The moment - curvature relationship was developed on the basis of values calculated using Equations (6) and (7) where,

$$M = P(e+\Delta) \qquad (8)$$

Three important limit states (Hart, G. C. 1995) can be identified on the moment vs. curvature graph of a concrete masonry wall. These are the moment at which cracking occurs, M_{cr}, the moment at yielding, M_y, and the ultimate moment, M_u. As shown in Figure 1 for each limit state, effective flexural rigidities, $(EI)_{eff}$, can be determined using Equation (6). As presented herein, values of $(EI)_{eff}$ corresponding to $(EI)_3$ in Figure 1 were determined and compared with values recommended in the current Canadian masonry code (CSA S304.1 - M94)

TESTING PROGRAM

Test Specimens

An experimental program was developed purposely for the determination of the effective flexural rigidity of masonry walls. Seventy-two full scale wall specimens at various load eccentricities were tested. They included fifteen plain grouted and fifty-seven reinforced concrete masonry walls with various grout patterns as illustrated in Figures (2) and (3). The load eccentricity ratios, e/t, as applied in this study were 0.0, 0.18, 0.27, and 0.36. Standard two-core concrete masonry blocks of nominal thicknesses, 150 mm and 190 mm, were used for building the wall specimens which measured 800 mm long by 1200 mm high. Along with each test specimen, auxiliary tests were conducted to evaluate the material properties of masonry constituents, including blocks, mortar, grout and reinforcement.

Experimental Procedure

Figure 4 shows a self-equilibrating test frame consisting of two supporting columns and upper and lower reaction beams. Vertical compressive loading was applied using an 1800 kN hydraulic ram secured to the top reaction beam.

For walls loaded vertically at various load eccentricity ratios, three linear strain converters (LSCs) were mounted across a mortar joint on the compression face as well as the tension face of each specimen. Lateral mid-height deflection was continuously monitored using an endless dial gauge mounted on an independently supported frame. Electrical wire leads from the load cell and linear strain converters were connected to a data acquisition unit controlled by a computer. At each load increment, load cell and continuous dial gauge readings were taken. Data acquisition and saving rates were set to 3 seconds. At about 90 per cent to 100 per cent of ultimate loading, LSCs were removed to prevent damaging them while the wall specimen was loaded to failure.

ANALYSIS AND DISCUSSION OF TEST RESULTS

A typical graph of moment vs. curvature for wall specimens is shown in Figure 5. The graphs were best fitted using a second degree polynomial taking into account the maximum moments measured during testing. EI_{eff} was defined as the slope of a line drawn from the origin to the maxima of moment vs. curvature curves.

Wall specimens tested at e/t equal to 0.18 developed only slight flexural tension. Consequently, failure occurred primarily by crushing of the masonry in compression. A slight reduction in flexural rigidity was attributed to non-linearity of the stress vs. strain curve of concrete masonry in compression. Wall specimens tested at e/t equal to 0.27 developed some horizontal tension cracks at mortar bed joints. Those tested at e/t equal to 0.36 developed significant tension cracks at mortar joints at relatively low vertical compressive loads. Reduction in flexural rigidity for both groups was due to a combination of flexural cracking and non-linearity of the stress vs. strain relationship of concrete masonry. The crack depth did not reach the mid-section for wall specimens with e/t ratio of 0.27. However, crack depth was beyond the mid-depth of wall thickness for those tested at e/t ratio of 0.36.

The reduction in EI with increasing vertical compressive loading, P, is evident in Figure 6. Since the stress vs. strain relationship of concrete masonry is linear up to about 50 per cent of ultimate load, E_m is constant within this range of loading. Referring to Figure 6, for P/P_0 ratios less than about 0.3, it is reasonable to conclude that the reduction in flexural rigidity is mainly due to cracking. For P/P_0 ratios between 0.30 and 0.75, the applied loading has little effect on the EI_{eff}/E_mI_0 ratio which remains relatively constant. Further increases in the vertical compressive loading and a reduced net area cause an increase in the stress level at the wall section. The increased stress level causes a reduction in the modulus of elasticity due to non-linearity of the stress vs. strain relationship. Consequently, for P/P_0 ratios equal to or greater than 0.70, the reduction in EI_{eff} is due to a combination of flexural cracking and a reduced modulus of elasticity. At high P/P_0 ratios, the reduction of flexural rigidity is mainly due to a reduction in the modulus of elasticity of masonry where non-linearity of the stress vs. strain relationship is dominant.

Typically, experimental values of EI used for curve-fitting were the minimum obtained in each group of four identical tests. In most cases, EI was determined experimentally at a point within 95 per cent of the ultimate load. The proposed equation is an exponential function and is expressed in terms of uncracked flexural rigidity, E_mI_0, and load eccentricity ratio, e/t. From experimental results of reinforced wall specimens tested within the eccentricity ratio range of $0 \leq e/t \leq 0.18$ (kern eccentricity), EI_{eff} can be expressed as:

$$EI_{eff} = 0.70E_mI_0 \qquad (9)$$

For eccentricity ratios, e/t, greater than 0.18 the effective flexural rigidity, EI_{eff}, can be expressed as:

$$EI_{eff} = 2.70E_mI_0 \exp\left(-7.5\frac{e}{t}\right) \geq E_mI_{cr} \qquad (10)$$

where e/t is the applied load eccentricity ratio, and I_{cr} is the cracked moment of inertia of the wall section at the location of maximum moment. Equations (9) and (10), which apply to reinforced walls, are plotted together with equations recommended by CSA S304.1-M94 in a non-dimensional form and presented in Figure 7. For comparison purposes, results for plain walls are also plotted.

For e/t less than or equal to 0.18 a constant value of effective flexural rigidity is assumed since both plain and reinforced wall sections fail by crushing of masonry in that range.

For plain walls tested at e/t equal to 0.18, the experimental value of 0.74 E_mI_0 is higher than the code value of 0.40 E_mI_0. However, for e/t equal to 0.36, the test value of 0.43 E_mI_0 is close to the recommended code value. For reinforced wall specimens tested at e/t between 0.18 to 0.36, the test values remain substantially higher than the maximum code value of 0.25 E_mI_0, despite being based on the least values. Although no wall specimens were tested at e/t greater than 0.36, Equation (10) can be used.

Table 1 shows a comparison of EI_{eff} test values with those suggested by CSA S304.1 - M94 (1994) and those predicted by Equations (9) and (10) for a fully grouted wall section with two layers of reinforcement and a fully grouted plain wall section. The values of EI_{eff} computed using the equations recommended by CSA S304.1 - M94 remain low except for values of e/t near 0.50 where all values are approximately the same. This is expected since E_mI_{cr} is calculated without taking into account the effects of axial loading in a wall section.

CONCLUSION

The evaluation of effective flexural rigidity of both plain grouted and reinforced concrete masonry walls under eccentric loading has been investigated. The P-Δ effect is included in the determination of EI_{eff}. Based on experimental results, exponential equations are proposed to compute EI for reinforced masonry walls with various eccentricities. By comparing the values obtained from these equations with the values used in the Canadian masonry code, it is noted that the code suggests more conservative values to estimate effective flexural rigidity of masonry walls. It is anticipated that more research will be carried out on this subject to acquire a better understanding of the behaviour of masonry load bearing walls.

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Table 1 Experimental, proposed, and and Canadian masonry code $EI_{eff} / E_m I_o$ values of masonry walls

e/t	0.18	0.27	0.36	0.50
Test (Plain Wall)	0.74	N/A	0.43	N/A
Test (Reinforced)	0.72	0.33	0.20	N/A
Eqns. 9 & 10	0.70	0.31	0.18	0.11
CSA S304.1 - M94 (Plain Wall)	0.40	0.40	0.40	0.40
CSA S304.1 - M94 (Reinforced)	0.25	0.23	0.17	0.11
Cracked EI	0.11	0.11	0.11	0.11

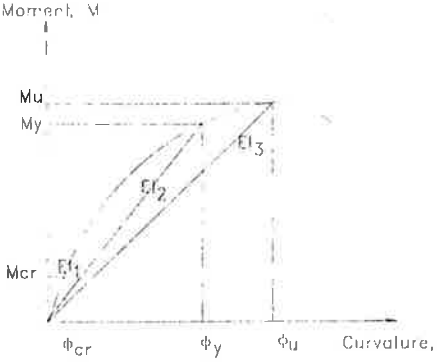


Figure 1 -Moment - curvature diagram

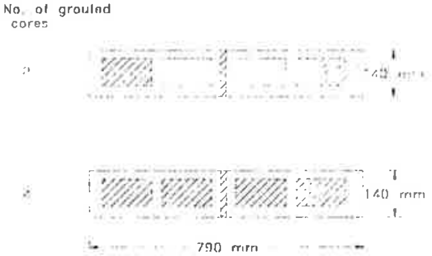


Figure 2 - Grout patterns for plain concrete masonry walls

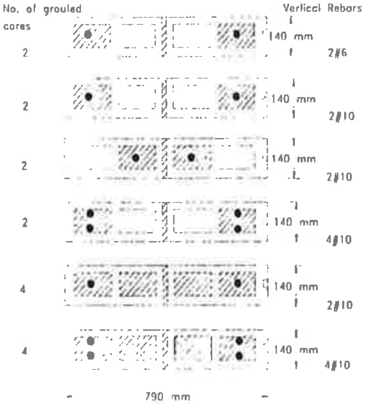


Figure 3 - Grout and reinforcement patterns for reinforced specimens

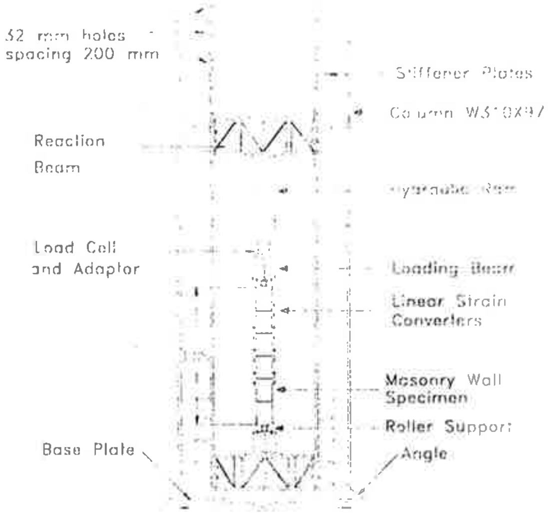
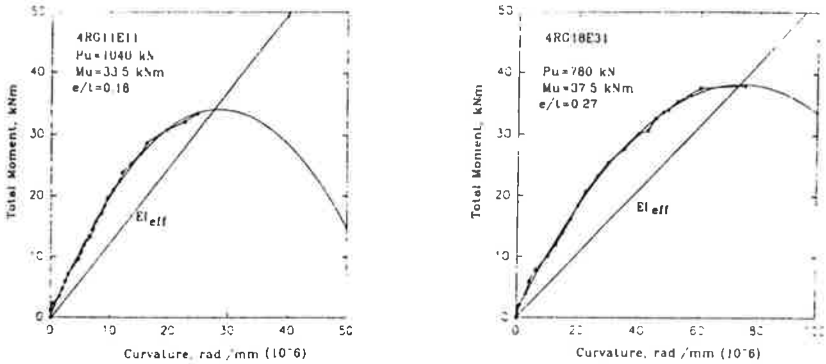
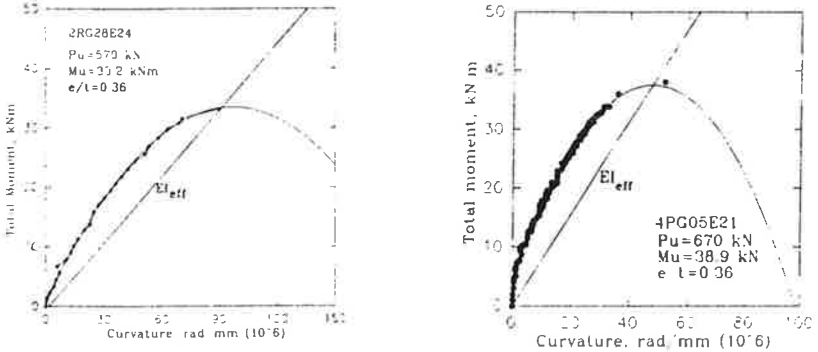


Figure 4 - Front elevation of test frame



(a) Moment vs. curvature for wall specimens with a central layer of steel reinforcement

(b) Moment vs. curvature for wall specimens with two layers of steel reinforcement



(c) Moment vs. curvature for wall specimens with two layers of steel reinforcement

(d) Moment vs. curvature for plain wall specimens

Figure 5 - Moment vs. curvature curve

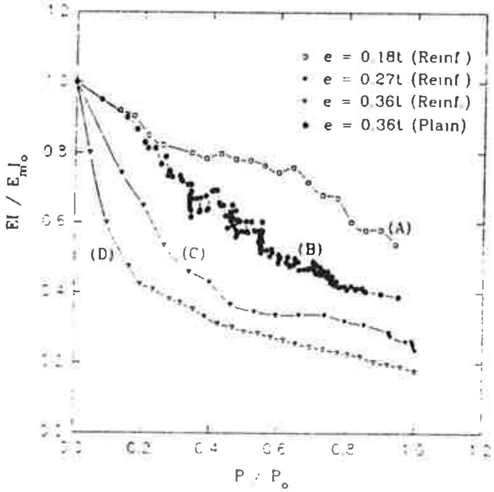


Figure 6 - Typical EI_{eff}/E_mI_0 vs. P/P_0 for plain and reinforced walls.

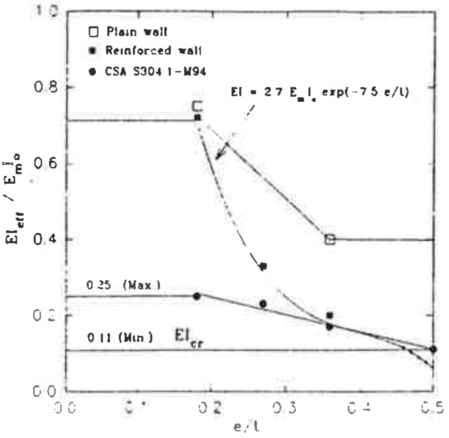


Figure 7 - Graph of EI_{eff} vs. e/t for masonry walls at ultimate loading.