ARCH DESIGN IN THE CANADIAN MASONRY DESIGN STANDARD

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ABSTRACT
Clauses for the design of masonry arches have been introduced into the Canadian masonry design standard (CSA S304) for the first time. In the engineered section the clauses are purposefully broad in concept as there are so many possibilities for arch shape, span and depth. Specifics, for example on how many spandrels should be placed in a road or railway bridge are avoided, as is lateral stability of the arch. The clauses therefore are aimed at the design of the arch itself, requiring design as a fixed arch if the abutments are fixed. The alternative is for the designer to allow for estimated relative movement of the abutments or design the arch as a two-pin arch. In the assessment of live-loading, stability is considered allowing a maximum of three hinge locations to be produced. These can be reinforced. For more information on arch analysis and design, the designer is referred to a document available from the Canada Masonry Design Centre. Tables have been developed for the empirical section of the code specifying minimum column widths for low-rise arches of different spans and depths supported on columns. The formulae and basis of the calculations are provided. This is to allow designers to use small span arches in veneers of either brick or block without having to do detailed calculations.

KEYWORDS: arch, design, analysis

INTRODUCTION
Arches are a well established form for masonry, and were widely used in the past. The stock of arches in buildings, bridges and aqueducts in Europe and the middle east is immense, as arches are very robust and usually remain standing under higher loads than envisaged in the original design or construction. A typical example would be Brunel's flat arches over the river Thames at Maidenhead on the Great Western Railway – still standing today carrying heavier and faster trains than allowed for in the design [1, 2]. However, modern engineers, certainly in Canada, are taught little of masonry and even less about how arches work, let alone the design of such structures. The focus in undergraduate structural education is on the design and analysis of reinforced concrete and steel, as little has changed since a detailed analysis performed some ten years ago [3]. The emphasis in what is taught has an obvious consequence in a general lack of appreciation of the value of the arch form, and thus a lack of utilization of arches in design. A second unfortunate consequence is inadequate knowledge when it comes to assessing heritage structures. Indeed, in this respect, Canada is beginning to suffer from the concern of Muir Wood [4] who stated, presumably based on his experience in his own country (the United Kingdom), that "Recent examples of inappropriate analysis of masonry structures, leading to their unnecessary designation as unsafe, suggest modern engineers could benefit from a wider appreciation of elementary principles."

In addition to the weak appreciation and understanding of arches by many modern structural engineers, masons building arches in residential construction have no guidance on what will work and what will not. There is some information available from the Brick Institute of America (BIA) [5, 6, 7] (these documents avoid the issue of bending moments in the arch and the column (abutment) supporting it) and the National Concrete Masonry Association [8] in the USA. Thus, while the rules are conservative in some respects, arches may still not function adequately unless some further design calculations are performed. The BIA documents point out the omission of consideration of the effects of bending and recommend the user to examine this aspect, but without guidance on how to do so. There have been several cases of arches over the entrances to residential garages collapsing in Ontario.

Thus the Canadian Standards Association committee responsible for preparing the standard for the "Design of Masonry Structures" [9] formed a small working group to examine how the design of arches might be explicitly mentioned in that code of practice. The objectives set out for the working group were to produce clauses for engineered design of arches and some tables for inclusion in the empirical section of the standard.

ENGINEERED ARCH DESIGN

In the nineteenth century, the process of design of an arch would have started with defining the approximate span and rise. From these, one or more of the many rules available for estimating the thickness of the arch at the crown would have been used to allow a first estimate of that parameter to be chosen. The intrados could then be sketched, followed by the extrados, allowing for increasing thickness around the arch. The fill would be added if it was a railway or road bridge, or loading from the masonry above if it were an arch in a building. Half the arch and supported material would then be divided into, say a dozen, vertical sections, and the weight of each determined from the density of the masonry and the density of the fill. It would now be possible to find the total load and its line of action on that half of the arch. There will be no shear at the crown in a symmetrical arch, just the horizontal thrust. At the springing point, there would be the vertical reaction to support the weight and a horizontal reaction to balance the thrust at the crown (see Figure 1). With a first assumption of the thrust being at the mid-height of the thickness at the crown (ie, no moment at the crown), moment equilibrium about the springing point would give a first estimate of the magnitude of that thrust. Now the line of thrust could be drawn through the arch starting at the crown, by changing the line of action of the thrust at the midpoint of each vertical section, by adding the weight of that section to the thrust. The ring shape and thickness would be adjusted with the objective of keeping the line of thrust as close to the centre of the arch ring as possible. At some point, the intrados would not be changed in this process, just the thickness of the arch. The consequential small changes in weight would be accounted for in each iteration. The process essentially omits the effects of live loads, especially moving live loads. Only symmetrical live loads can be accommodated, by adding the load to the segment weights appropriately.

Owen [2] describes a variation on this procedure introduced by Brunel in that he took the shear (weight) to be in the form of a quartic polynomial ($Ay + By^2 + Cy^3 + Dy^4$) and chose the constants such that polynomial is correct at four points on the span. The weight is recognized as the vertical component of the thrust, drawn on the arch ring, so simple statics gives the horizontal component. The graphical approach outlined was not perfect – for example, Harvey [10] argues that the way Brunel drew the thrusts from neighbouring spans in multi span bridges to meet in a single block "cannot be defended". Nevertheless, both processes led to the existence of many successful arches.

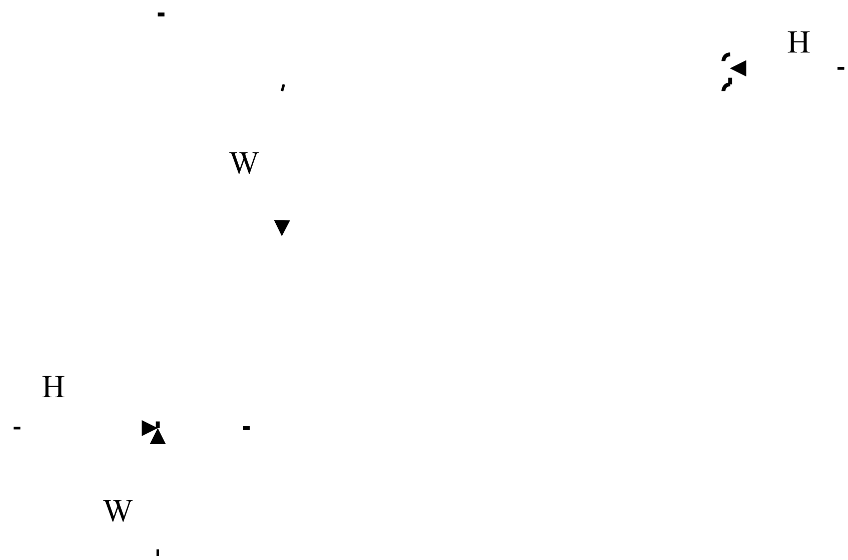

Figure 1: Simple equilibrium for half a symmetrical arch. With the weight and its line of action determined by dividing the arch and fill into several vertical sections, and finding the single equivalent force, moment equilibrium about the springing point gives a first estimate on H – assuming it acts at the centre of the arch ring at the crown.

The same procedure as the first described above can be followed with modern tools, but we can also reinforce arches, so there is perhaps not the need to try so hard to have the line of thrust remain in the kern of the arch. Also, for reasons of economy, we would tend not to overdesign an arch nowadays in the sense that live loads would now be expected to be a much greater proportion of the dead load than the nineteenth century process appeared to assume. Writing code clauses to cover the wide range of possible spans, rises and shapes of arch, given the versatility of modern materials, presented a challenge. Indeed, it is perhaps for this reason that the working group was unable to find the design of arches in any masonry code, except a document from the UK [11]. The preamble to that document states "This document sets the Standard requirements for, and gives advice on, the design of unreinforced masonry arch bridges". Loads are described, particularly wheel loads and how they should be considered. The essential features of the design requirements are to avoid collapse, and compressive and shear failure of the masonry. No guidance is provided on what can cause collapse or how to analyse the structure. Given the results of structural education in Canada, it was thought necessary to provide some suggestion on how an arch structure could be analyzed. The decision therefore was taken to write broad-based clauses that covered the principles of structural safety of arches, rather than be prescriptive about every detail – much like the UK document [11]. As more experience is gained, and feedback received, the clauses can be expanded and adjusted. For example, the UK document contains clauses with respect to temperature variation and allowance for creep and shrinkage, so clauses with respect to these aspects may need to be added. The main objective at this stage was deemed to be alerting designers to the fact that they can design arches, and that there are a few things they need to look out for. It was also decided that a document should be made easily accessible, containing information on arch action, arch collapse mechanisms and arch analysis so that designers would have a base from which to start their design.

Five clauses were therefore proposed as below, following a short preamble:

1. With the span, rise and form (segmental, elliptical, semi-circular, pointed) of the arch specified, the depth (thickness of the ring) shall be estimated and adjusted to the next highest number of courses depending on the construction chosen (soldier vs. running).
2. The arch shall be designed as a fixed arch if the arch is constructed integrally with a rigid (fixed) abutment. Rigid abutments shall not experience relative (lateral or vertical) movement that exceeds 10^{-6} ($span^3$/depth) m. For greater movements, the expected differential displacement of the abutments shall be accounted for in the design, or otherwise the arch shall be designed as having pinned connections to the abutments.
3. For stability, the line of thrust for different load cases shall be determined, and shall lie in the middle third of the arch if no cracking is allowed. Otherwise, no more than three hinge locations in total shall occur in the arch, and the arch shall be reinforced at potential hinge locations.
4. For strength, the maximum normal compressive stress at any section due to the thrust and eccentricity of thrust, assuming plane sections remain plane, shall be less than or equal to $(0.6)\phi_m f'_m$
5. The maximum factored shear force at any section shall not exceed V_R as defined in Clause 7.10.4.1.

The compressive strength and shear requirements are such that there will not be local crushing of the units used, nor shear failure in the ring (this is not usually a problem as it is usually difficult for a unit to slide out of the ring). The restriction on the relative movement of the abutments is derived from one of the analytic solutions of Leontovich [12]. Using the moment and thrust determined for the relative moment of the abutments, and average properties for stiffness of masonry, his formulae can be used to estimate the extreme fibre stress that will develop for a given relative movement of the abutments. Limiting this stress to 0.2 MPa, the minimum flexural bond strength allowed in Canadian standards, allows the maximum permissible relative movement to be derived. The calculation did not include any thrust from dead load other than the arch itself (ie, no fill or overlying masonry) so should be conservative.

The supplementary document that will be made available to help designers understand the terminology with respect to arches, and the failure mechanisms due to stability (for instability, you need four hinges, which is why a maximum of three is allowed in the design (a three-hinged arch is stable)) is currently 30 pages in length. This document has been reviewed by three different design offices, all indicating it is helpful and informative.

EMPIRICAL DESIGN

Members of the code committee [9] requested that tables be prepared for inclusion in the empirical section of the code, to make it easier for small arches to be constructed in residential and small commercial projects. Initial analysis based on the concept that such arches should be fixed, produced abutment lengths (the length of the masonry on each side of the arch to hold it in place) that were unacceptable for the type of construction being considered. Therefore the solutions of Leontovich [12] for parabolic arches were considered. The same solutions had been used to develop the technical notes for arches in the USA [5 – 8]. Tables were developed for rise to span ratios of 0.1 to 0.2, as 0.2 is the maximum ratio where parabolic arches are similar to segmental, and the assumption of constant arch depth introduces an error of less than about 5%, rather than accounting for the depth changing as the slope of the arch increases. Loading above the arch was idealized into an equivalent uniform distribution and the appropriate solution for an arch supported on columns developed in a spreadsheet. Using a maximum bond strength of 0.2 MPa, the necessary minimum length of an abutment could be calculated, allowing for both the moment and thrust that developed. These lengths were rounded into distances that fit dimensions for possible construction with both Canadian and US produced units, such that tables like Table 1 could be presented. Table 1 contains the minimum abutment lengths that must be provided for arches of 1.2 m span, for varying arch depth and varying height of the supporting columns.

Table 1: Typical table for empirical design

Span 1.2 m

Length of abutments/columns

Arch depth			Height of abutments/columns (m)			
mm	**1.0**	**1.4**	**1.8**	**2.2**	**2.6**	**3.0**
200	200	250	250	300	300	300*
300	300	300	300	300	300	300
400	400	400	400	400	400	400

* Minimum rise to span ratio of 0.15

The values obtained were compared to values obtained from the US technical notes [5 – 8], and found to be the same in the mid-range. However, longer abutment lengths were deemed necessary at some of the higher column heights (abutment lengths in the bottom right of some tables), and shorter ones with some low column heights (abutment lengths in the top left of some tables). The difference was thought to be due to the fact that the effects of bending were included in the analysis, whereas this is not the case in the US technical notes – there the designer is advised to check that the design will be able to resist bending effects.

CONCLUSIONS

Arch design clauses are being proposed for inclusion in the Canadian masonry design standard for the first time. As far as we are aware, this is the first inclusion of such clauses in a general masonry design standard. A design document has also been prepared to provide designers with guidance and understanding of arch behaviour, collapse mechanisms and techniques for analysis. It is hoped that these advancements will lead to greater use of arches in masonry construction in the future.

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REFERENCES

1. Pannell, J.P.M. (1964) "An Illustrated History of Civil Engineering" Thames and Hudson, London, 370 pp.
2. Owen, J.B.B. (1976) "Arch Bridges" in "The Works of Isambard Kingdom Brunel" Ed. A. Pugsley, Cambridge University Press, pp. 89-106.
3. Shrive, N.G., Sturgeon, G. (2001) "Masonry Education in Canadian Post-Secondary Institutions", 9th Canadian Masonry Symposium, Fredericton, Canada, 12 pp. on CD-ROM.
4. Muir Wood, A. (2009) "Thomas Young and the Brunels: masters of masonry analysis" Proceedings of the Institution of Civil Engineers, Civil Engineering, Vol. 162, pp 42-48.
5. Brick Institute of America (1995) "Brick Masonry Arches". BIA Technical Note 31, 24 pp.
6. Brick Institute of America (1986) "Structural Design of Brick Masonry Arches" BIA Technical Note 31a, 8 pp.
7. Brick Institute of America (1986) "Structural Design of Semicircular Brick Masonry Arches" BIA Technical Note 31c, 10 pp.
8. National Concrete Masonry Association (1994) "Concrete Masonry Arches" NCMA TEK 14-14, 4 pp.
9. Canadian Standards Association (2004) "Design of Masonry Structures", CSA S304.1-04, Canadian Standards Association, Mississauga, Ontario, 148 pp.
10. Harvey, B. (2012) "Stiffness and damage in masonry bridges" Bridge Engineering, Proceedings of the Institution of Civil Engineers, Vol. 165, BE3, pp 127-134.
11. The Highways Agency, Scottish Executive, Welsh Assembly Government, and The Department for Regional Development Northern Ireland (2004) "Unreinforced Masonry Arch Bridges" Part 14 of Section 2 of Volume 2 of "Design Manual for Roads and Bridges" 31 pp.
12. Leontovich, V. (1959) "Frames and Arches – Condensed Solutions for Structural Analysis", McGraw- Hill, New York, 469 pp.