BUCKLING SAFETY OF MASONRY WALLS UNDER CONSIDERATION OF NONLINEAR STRESS-STRAIN-RELATIONSHIPS

Jäger, Wolfram[1)], Pflücke, Torsten[2)]

ABSTRACT

As part of a research project carried out in 1999/2000 the solution for the verification of walls subjected to vertical loads under consideration of the buckling effect as contained in the European Masonry Standard ENV 1996-1-1 (EC 6) was intended to be reviewed. The basis of the assessment was, within the scope of the project, the solution of buckling problem of masonry under consideration of non-linear stress-strain-relations without restriction of the course of the curve.

The results clearly show the strong, load bearing capacity-reducing influence of load eccentricities and slenderness in the region of second order effects as well as the fullness of the stress-strain-relationship in the region of material failure. The approximation of the theoretical solution of the EC 6 worked out by KIRTSCHIG analogue to the way of HALLER was compared with the exact solution of the differential equation. It represents a safety risk for certain application parameters. With the result of the research project now a solution of the problem exists which may serve as the basis for the formulation of a practical approximation, explained here in principle. It has the numeric determination of the integration constants after solution of the differential equation of a centrically respectively eccentrically vertical loaded wall as its basis, whereby the geometrical and physical non-linearities have been considered.

The task was also to study the influence of large eccentric loads by means of centric and eccentric compression tests of masonry specimens. The experimental research was focused on tests with small test samples (4-stone units) under a large eccentric application of loads. The evaluation of the obtained data showed that, particularly in case of mortars with low strength (M 1 to M 5) and at large eccentricities ($e > 0.4 \cdot t$), the assumption of a rectangular stress block as a basis of assessment may lead to an overestimation of the load bearing capability. This can be attributed to the significant reduction of stress at failure under eccentric load in contrast to the centric compressive strength to be assumed in form of the rectangular stress block.

Keywords: masonry walls, buckling safety, non-linear stress distribution, load-bearing capacity, differential equation, experimental investigation, articulation

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INTRODUCTION

With the elaboration of a regulation effective in all of Europe for the design of masonry buildings, the Eurocode 6 (EC 6), several design approaches regarding their application specifically for the national conditions within Germany had to be reviewed.

During the last two years, the authors have especially been involved in the subject of the buckling safety of bearing masonry walls. There, elementarily theoretical considerations as well as experimental investigations into the buckling behaviour of walls of different stone-mortar-combinations that are frequently used in Germany have been carried out.

The theoretical basis for the determination of the buckling safety of masonry structural elements for the European Standard EC 6 (EN 1996-1-1) and the German Standard DIN 1053-1 is principally composed by the same non-linear differential equations. However, differences arise from the approach of the stress-strain-relation in the compressed area of the wall sections and the way of the solution of the problem. Here, the EC 6 arranges a non-linear and the DIN 1053-1 a linear function.

With a research project, that compared both standards and had been already finished in 1998 (REEH and JÄGER 1998), it could be shown that the approximate solution of the EC 6 for the evaluation of the buckling stability of walls with small and medium load eccentricities better corresponds to the actual bearing behaviour as well as to the theoretically exact solution. In comparison with the German Standard DIN 1053-1, the use of the EC 6 leads to more economically efficient structures. However, no statement regarding safety relevant considerations for large static eccentricities could be made at that time. For this, a special research project (JÄGER, PFLÜCKE, BAIER et al.) has been carried out by the authors which was intended to enable the estimation of the load-bearing capacity of walls under large eccentric loading and slender walls.

Moreover, a further phenomenon that appears at the buckling of masonry units and has not yet been investigated, that is the moving in of the turning point from the edge of the cross section further to the inner part of the wall, had to be looked into. When the proof is furnished on the level of the failure state an overestimation of the bearing safety will be the result if the eccentricity exceeds a wall thickness (t) of t/3, i. e. ($e > t/3$).

Here, the application of an orthogonal stress block (plastification) according to the European regulation for masonry, the EC 6, at an eccentricity that exceeds 0.4 times the wall thickness ($e \geq 0{,}4 \cdot t$) has to be verified. Transferring the bending moments at the cross-wall-junction due to the connection of the floor slabs with the wall is the reason for a high eccentric action. This fact is schematically given in Figure 1:

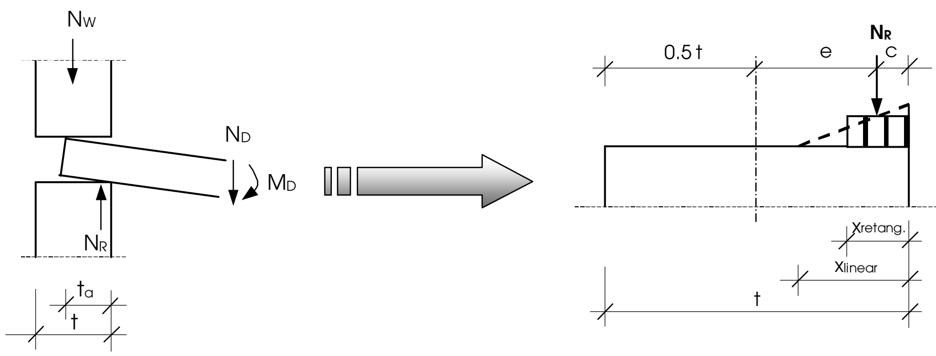

Figure 1 Arrangement of the stress block for the determination of the design load of the strength

So far, the German Masonry Standard DIN 1053-1 did not allow the use of the rectangular stress block for the verification of the eccentric action. Rather the eccentricity has been limited to e = t/3 which can be equivalently comprehended with the cracking of the cross section of the wall up to half of the wall thickness (t). Using these regulation of DIN 1053-1 special measures have to be considered in the structural detailing.

Theoretical Solution of the buckling problem

The State of Research

The centric or eccentric loaded masonry compression member shows in case of a closed solution a number of so far unsolved problems as regards to taking into account physical non-linearities. It is to differentiate between totally compressed and cracked cross-section of the wall. When cracks occur the bending stiffness decreases and the non-linear proportion of second order effects of the total moment grows accordingly higher with the increasing bending. In the past there has been several approaches for solving this problem theoretically or empirically.

In 1949 HALLER first regards this stability problem by approximately introducing a sine curve for the deformation in the deflected state of the wall. For this problem he could derive a solvable extreme value calculation for non linear material behaviour from looking at deformation, compressed length of the cross-section and the point of application of the resultant of the compression stresses.

Afterwards, assuming a linear-elastic material behaviour of masonry with a cracked cross-section, i. e. without consideration of the tensile strength, ANGERVO succeeded in finding the exact solution of the differential equation of a cracked compression member endangered of buckling. He gave factors for the reduction of the bearing load capacity due to buckling.

MANN analysed the procedure of ANGERVO comprehensively. He looked at the uncracked as well as the cracked cross-section. On the assumption of a linear stress distribution he used the equilibrium state on the deflected system in order to consider the terms of the second order theory. The solution given by MANN finally was the bases of the approximation formula for the determination of the buckling safety of slender masonry walls presently given in the German Masonry Standard DIN 1053-1.

KUKULSKI and LUGES were based on the state of knowledge of ANGERVO and introduced a logarithmic material law for masonry which although it allows to transfer certain tensile stresses in the cross-section of the wall (reserves of load-capacity). The disadvantage of this scientific work is, that the real course of the stress-strain-diagram of different types of masonry can not be reflected realistically.

FÜHRER also introduced like HALLER a sinusoid deformation on the eccentrically compressed masonry member for solving the buckling problem. However, he uses a parabolic description of the stress distribution in the compressed cross-section area. Moreover, he compares his results with the reduction values calculated by ANGERVO. There FÜHRER comes upon deviations between the two solutions whose reason he sees in the application of the deformation arrangement in form of a sine curve.

KIRTSCHIG has regarded the procedure of HALLER comprehensively and made it applicable, by the introduction of a stress distribution in form of a parable that has been derived from analyses of test results. However, with KIRTSCHIG's description of the stress-strain-relationship received by one fixed formula always the same degree of fullness (α_0) and the same distance of the resultant form the edge of the compressed cross-section (a_s) are used. The result is that it can only be corresponded to the different types of masonry by considering the values of the modulus of elasticity and this has proven to be insufficient. The works of KIRTSCHIG are included in the European norm EC 6 as the base for the determination of the reduction factor due to the buckling problem of masonry walls.

In the field of reinforced concrete structures there is a further procedure of approximation for the determination of internal forces according to the second order theory by KORDINA and QUAST. This procedure, assuming moment-curvature-relations, also takes the equilibrium at the deflected system as well as the occurring curvature of the elongated beam in the maximal compressed area as a basis.

The theoretical solution for big eccentricities

The basis for the theoretical contemplation of the stability problem of slender masonry walls is given by the differential equation, written down for the deflected stage on an eccentrically loaded compressed masonry member according to the second order theory.

Equivalently to the approach of MANN and ANGERVO the following basic equation for the compressed masonry member can be given:

$$E \cdot I(z) \cdot w(z)'' + P \cdot a(z) = 0 \,. \qquad (1)$$

This formula is valid under the provision of a linear moment-curvature-relation with

$$\kappa = \frac{1}{r} = \frac{M}{E \cdot I} \text{ and } M = P \cdot a(z) \,. \qquad (2)$$

The cracked cross-section of the wall is variable along the 'rod axis' as a result of the acting bending moment. Therefore, the 'rod axis' is slightly curved. Possible imperfections (w_0) have to be considered, according to the regulations of the EC 6, constantly along the rod's length so that the static eccentricity (e_0) and the accidental eccentricity (e_a) can be summarised to:

$$e = e_0 + e_a \qquad (3)$$

Depending on the assumption of the stress distribution we obtain the distance between the resultant and the centroid of the compressed area with

$$a(z) = a_0 \cdot x(z) \,. \qquad (4)$$

However, for the evaluation of the differential equation also the distance between the line of acting force and the totally compressed edge of the cross-section will be needed. This one can be determined with

$$c = w(z) + a_s \cdot x(z) \qquad (5)$$

where a_s represents the actual distance from the resultant of the compressed area to the outer edge of the compressed cross-section in the deformed state. The deflection of the rod has been introduced with w(z) (compare with Figure 2).

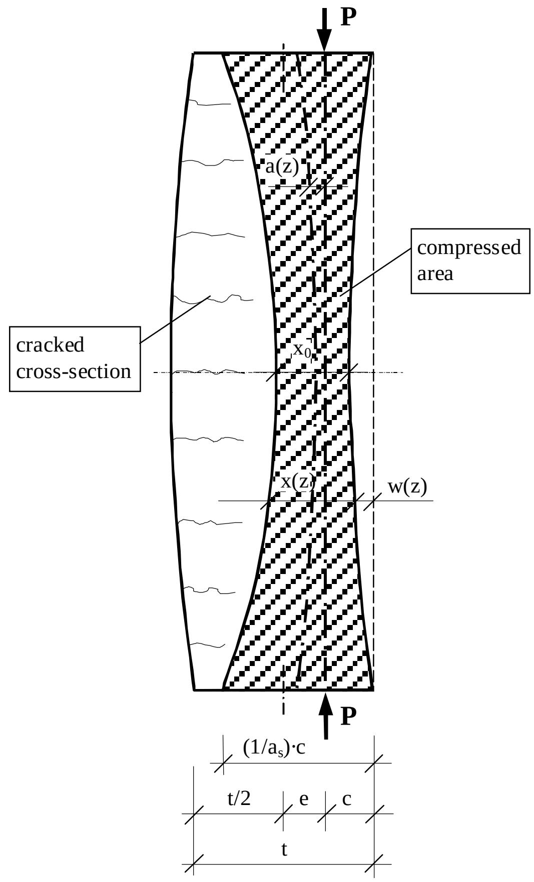

Figure 2 Eccentrically loaded masonry member with cracked cross-section

For the consideration of a general non-linear stress distribution over the compressed cross-section of the eccentric loaded masonry member the necessary parameters of the equations (4) and (5) can be introduced as follows:

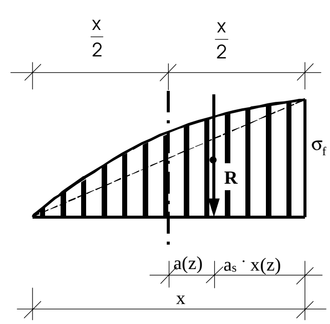

Figure 3 General non-linear stress distribution over the compressed cross-section of the wall

Depending on the variable flexural stiffness of the cross-section

$$E \cdot I(x) = E \cdot \frac{b \cdot x^3}{12} \qquad (6)$$

and on the distance of the resultant of the compressed area to the rod's axis

$$a(z) = \frac{1}{2} - a_s \qquad (7)$$

the following differential equation for the eccentrically loaded masonry member was obtained from equation (1):

$$E \cdot \frac{b \cdot x(z)^3}{12} \cdot (-a_s \cdot x(z)'') + P \cdot \left(\frac{1}{2} - a_s\right) = 0 \,. \qquad (8)$$

After the conversion the equation can be written in the following form:

$$x(z)^2 \cdot x(z)'' = 12 \cdot \frac{P \cdot \left(\frac{1}{2 \cdot a_s} - 1\right)}{E \cdot b} \,. \qquad (9)$$

The solution of this common second-order differential equation is

$$\pm z \cdot C_1^3 \cdot \frac{24 \cdot P \cdot \left(\frac{1}{2 \cdot a_s} - 1\right)}{E \cdot b} = C_1 \cdot x \cdot (C_1 \cdot x - 1) + \ln(C_1 \cdot x + C_1 \cdot x - 1) + C_2 \,. \qquad (10)$$

Where C_1 and C_2 are integration constants. Thanks to symmetry reasons of the system, the constant C_2 can be set to

$$C_2 = 0 \qquad (11)$$

However, for the evaluation of the solution further boundary conditions have to be found as the determination of the second integration constant C_1 is not quite simply possible.

For solving the problem at first P_0 will be introduced as centric failure load and secondly $P_1 = P$ as the failure load under eccentric action in the middle of the rod. Therefore, the reduction of the bearing capacity caused by the buckling of the masonry wall can be given through the relation between the eccentric limit load and the purely centric one that is determined without taking into account the influence of stability.

$$\eta = \frac{P_1}{P_0} \qquad (12)$$

Where $\eta = \phi$ according to EC 6.

Now, it is necessary to introduce where a second condition for the determination of the integration constant C_1 or rather of the transformed constant D the material failure in the middle of the rod without consideration of stability. So, the whole problem can be reduced to a numerically solvable extreme value equation:

$$\eta = \max(\eta_s, \eta_m). \qquad (13)$$

The two calculation equations for the numerical solution of the extreme value equation can be given as follows:

$$\boxed{\eta_s = \frac{\zeta}{6 \cdot \left(\frac{1}{2 \cdot a_s} - 1\right)} \left[\frac{(3-m)^2}{36 \cdot a_s{}^2 \cdot \overline{\lambda^2}}\right] \left[\left[\frac{(3-m)}{6 \cdot a_s} \cdot D\right] \left[\sqrt{1-D} + D \cdot \ln\left(\frac{1+\sqrt{1-D}}{\sqrt{D}}\right)\right]\right]^2} \qquad (14)$$

$$\boxed{\eta_M = \frac{\alpha_0}{6 \cdot a_S} \cdot (3-m) \cdot D \cdot \frac{\sigma_1}{f_k}}. \qquad (15)$$

where

f_k	characteristic compressive strength
σ_1	maximum compressive stress under eccentric load
$\zeta = \frac{E}{f_k}$	ratio of the modulus of elasticity to the compressive strength
$m = 6 \cdot \frac{e}{t}$	relative non-dimensional eccentricity
$\overline{\lambda} = \frac{h}{t}$	wall slenderness
$D = \frac{1}{\frac{1}{a_s} \cdot c \cdot C_1}$	transformed integration constant
$\alpha_0 = \frac{\int_0^{\varepsilon_b} \sigma(\varepsilon)\, d\varepsilon}{f_k \cdot \varepsilon_b}$	degree of fullness of the stress distribution
a_s	distance between the resultant of the stress diagram and extreme compressed fibre.

The stress-strain-relationship of EC 6 and DIN 1053-1 are different. That fact leads to different results of the reduction factor considering the buckling and the material failure.

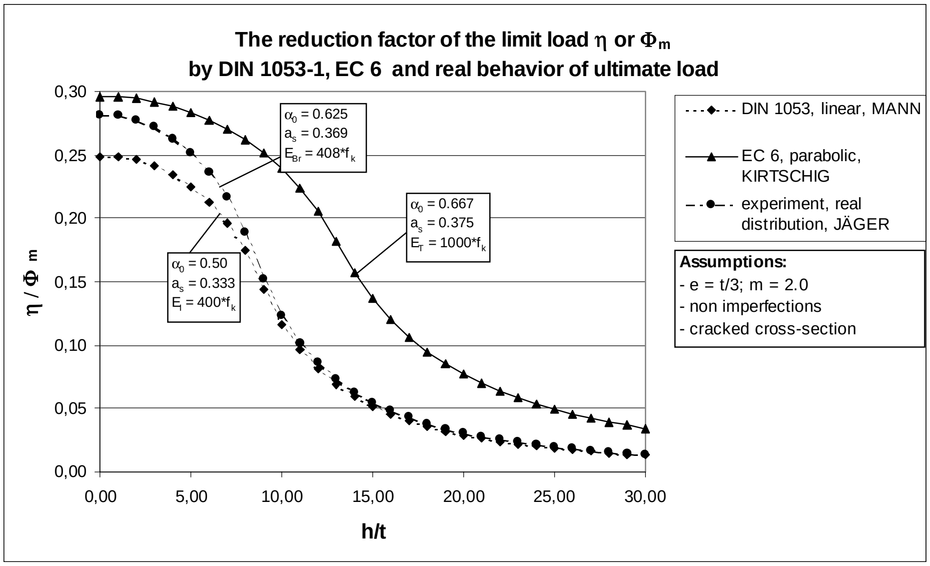

Figure 4 Comparison of the reduction factor according to DIN 1053-1 and to EC 6 with the solution of the differential equation on the example of masonry made from vertical core clay bricks (Hlz) and general purpose mortar M 5.0 (MG II) for an eccentricity of e = t/3

In Figure 4 the dependence of the solution of the buckling problem on the respective assumption of the stress-strain-functions as well as on the application of the corresponding modulus of elasticity can be seen clearly. With the introduction of the degree of fullness (α_0) and the distance between the resultant from the compressed extreme fibre (a_s) to the solution of the differential equation its application is possible for all kinds of stress-strain-courses.

EXPERIMENTAL INVESTIGATIONS

The experiments carried out so far were mainly dedicated to the verification of the ultimate limit state in the area of the material failure. There it had to be ascertained that the calculation equations according to EC 6 partially brought very big deviations of the load-carrying capacity compared to the test results. With the use of general purpose mortar and stone units with a relative smooth surface (e. g. calcium silicate units) noticeable overestimations of the bearing capacity compared to the EC 6 have been worked out. The reason for this could basically be found in the stress redistribution in the ultimate limit state. There, it is assumed that the characteristic compressive strength of masonry, which has been evaluated from centric tests, will be different from that one of a strong eccentric loaded masonry. It is rather the fact that the two-axial mortar strength will be placed in the foreground with ‘soft’ or rather ‘lowly-solid’ mortar. The reasons for this can be found in the reduction of the compressed cross-sectional area available for the transfer of the compressive stresses.

Figure 5 Ultimate limit state resulting from arising of a hinge in the mortar joint of masonry type CS-M 5,0 (MG II)

The assumption of a centric masonry compressive strength as a basis for the compressed residual area as a rectangular stress block leads to a clear overestimation of the bearing capacity for masonry units with a smooth surface in combination with general-purpose mortar of low strength (see also PURTAK). Therefore, it would be better to apply a reduced strength instead of the entire centric masonry compressive strength on this basis when considering the ultimate limit state of eccentrically loaded masonry structures.

CONCLUSIONS

All in all it can be concluded that the buckling safety of slim masonry walls depends substantially on the stresses that arouse from the second-order theory. These in turn are substantially influenced by the deformation behaviour of the material, i. e. by the shape of the stress-strain-relation. Consequently, the actual bearing capacity of slender walls of different materials (brick, calcium silicate units, aerated concrete etc.) can not be described close to reality by the idealizing under the assumption of a linear-elastic material behaviour (MANN, DIN 1053-1) as well as under the assumption of a quadratic parabola with only varying the module of elasticity (KIRTSCHIG, EC 6).

Moreover, when looking at the ultimate limit state in the area where the material failure occurs it is not always justified to arrange centric masonry compressive strength as an entire rectangular stress block resulting from a plastification of the remaining compressed cross-section of a wall. The real bearing capacity of the ultimate limit state rather seems to be dependent on the strength relations between stone and mortar. With mortar of low compressive strength the increase of the eccentricity of the load and by and by the failure of the mortar resulting from the emerging of a hinge in the bed joints will be placed in the foreground.

Further experimental and theoretical considerations especially regarding slender wall constructions under eccentric load are intended to solve the contradictions that have already been found out and to enable that further statements with regard to the actual bearing capacity of such wall construction can be made.

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