



CRITERIA OF STABILITY FOR BUILDING
SUBJECTED TO EARTHQUAKE MOTION AND APPLICATION

WenXin Qin¹ ZhenFang Yuan²

ABSTRACT

In this paper, the Liapunov's stability of motion theory is introduced to the structural dynamic analysis. The analytic procedure of stability for nonlinear seismic analysis is presented. The criterion of stability is established for the seismic analysis of SDOF system. Based on the method of normal coordinate transformation and the criterion of stability of SDOF system, the criterion of stability is established for the MDOF system subjected to earthquake ground motion. A simple and effective criterion of stability is also established for the shear model adopted in dynamic analysis of buildings. The relation between the seismic collapse and the stability of motion for buildings is discussed. The collapse definition of buildings subjected to earthquake ground motion is defined. Finally, the collapse response of a fifteen-story reinforced concrete block masonry building subjected to El Centro earthquake ground motion is analysed. The beginning collapsing time of the building is determined by the collapse definition presented.

INTRODUCTION

The theory of stability of motion is an important branch of differential equation theory. The purpose of the stability theory is to study the coefficients of differential equation to determine the stability of the solution. The fundamental theory of stability of motion

¹ Ph. D. and Associate professor, Dept. of civil Engr. , Dalian University of Technology, Dalian 116024, P. R. China

² Senior Engineer, China Northeast Building Design Institute, Shenyang, 110006, P. R. China

was established by Russian mathematician—A. M. Liapunov in last century. In recent several decades, the theory of stability of motion has been widely applied in the fields of autocontrol, space flight, power engineering etc. . In civil engineering, the application of stability of motion has just begun(Qin et al. , 1991, 1992, 1993, 1994). In civil engineering, there are many phenomena which belong to the research scope of stability of motion, for instance, the response of a building subjected to dynamic load, such as earthquake, wind, etc. , is a typical problem of stability. When dynamic load is weak, the building vibrates near the place of equilibrium. This expresses that the building is a stable system. When dynamic load is strong, the response of the building increases abruptly, until the building collapsing, at this time, the building becomes an unstable system. In the view of mathematics, above phenomena are suitable for studying of the theory of stability of motion, therefore, it is necessary to study the buildings subjected to dynamic loads by means of the theory of stability of motion. When structures are subjected to severe earthquake excitation, the equations of motion are nonautonomous nonlinear differential equations. Now, in mathematics, there are no effective methods to analyse the stability of this type of equation. In this paper, based on the basic theorems of stability, and considered the characteristics of structure engineering, by means of numerical method, the stability of nonautonomous nonlinear equations of the structures subjected to dynamic loads will be studied. Consequently, the corresponding analysis method and the criteria of stability will be established. Finally, the relation between the stability and collapse of buildings subjected to earthquake will be discussed, and a new seismic collapse definition of buildings will be proposed. The seismic response of a fifteen-story building will be studied.

CONCEPT AND THEOREM

The definition of stability of motion was defined by Russian Mathematician—A. M. Liapunov. The question of stability may in simple terms, be stated as follows: Given an equilibrium state of a physical system, whose stability we wish to study, we consider a state near equilibrium and ask whether in the course of time the system will tend toward the given equilibrium state. In some what more precise terminology, we shall call the departure from equilibrium a perturbation. The initial value of the perturbation is considered to be given in a problem of stability, and the question is asked whether, starting from this initial value, the perturbation will tend to zero in the course of time. If this is the case the equilibrium state is called stable, it is called unstable otherwise. According to above concept of stability, in structural engineering, the stable structures can exist, the unstable structures can not exist, therefore, it is necessary to study the stability of motion in structure engineering.

There are two major methods to determine the stability of a physical system: one method is called the indirect method which bases on the characteristic equation of differential equation to study the stability, another method is called the direct method which uses the Liapunov's function to determine the stability. In reference 2(Qin,1992), we adopted the direct method to determine the stability of buildings. In this paper, we adopt the indirect method to study the stability of buildings subjected to earthquake ground motion. Firstly, we introduce some theorems about stability of motion. The basic theorem is stated as follows: a system of linear differential equation $\dot{x}=Ax$, its origin stability is determined by following criteria: 1) the origin of system is stable, the

taneously the stability of system. Consequently, we can explore the relation between the varying properties and the stability of the system. The procedure of the method is as follows:

(1) Assuming in each time increment, the properties of the system, m, k and c , do not vary with time. The tangent slopes are defined at the beginning of the time intervals, they are

$$C_{ij}(t) = \left. \frac{df_{Dj}}{dv_j} \right|_t, \quad K_{ij}(t) = \left. \frac{df_{Kj}}{dv_j} \right|_t$$

So that the incremental equilibrium equation for time t :

$$M\Delta\dot{V}(t) + C(t)\Delta V(t) + K(t)\Delta V(t) = -MI\Delta\ddot{u}_g(t) \quad [2]$$

(2) The numerical integral procedure is adopted to solve the Eq. (2) for the incremental response in each incremental time region.

(3) To study the stability of Eq. (2) in each incremental time Δt . Because the Eq. (2) is the problem of non-origin stability, the theorems of stability of motion can not be used, it is necessary to transform Eq. (2) to the problem of origin stability. Now introducing the state variables $X_1 = \Delta V$, $X_2 = \Delta\dot{V}$, then Eq. (2) can be written by the state variables:

$$\dot{X} = AX + \Delta P \quad [3]$$

in which

$$\dot{X} = \begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{Bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad X = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}, \quad \Delta P = \begin{bmatrix} 0 \\ -MI\Delta\ddot{u}_g \end{bmatrix}$$

Assuming $X=G(t)$ is the solution of Eq. (3), let us study the stability of $G(t)$. Introducing the disturbance vector $Y=X-G(t)$, then $X=Y+G(t)$, substituting X into Eq. (3) leads to the following form:

$$\dot{Y} + \dot{G}(t) = AY + AG(t) + \Delta P$$

Because $G(t)$ is the solution of Eq. (3), above equation becomes: $\dot{Y}=AY$, setting $F(Y, t)=\dot{Y}=AY$, then $F(0, t)=0$, therefore

$$\left. \begin{array}{l} \dot{Y} = AY \\ F(0, t) = 0 \end{array} \right\} \quad [4]$$

Through above transforming, the Eq. (2) becomes Eq. (4), Eq. (4) is the origin problem of stability.

Above transformation illustrates that the stability of homogeneous linear equations is the same with that of its nonhomogeneous equations. In the linear analysis of structure, the stability of the free-vibration system is the same with that of the system subjected to any dynamic loads.

CRITERION OF SDOF SYSTEM

In above section, the equation of motion of a structural system has been transformed into the form of origin stability of motion—Eq. (4). Now, let us discuss the stability of motion of Eq. (4). For a SDOF system, $A(t)$ of Eq. (4) is

$$A(t) = \begin{pmatrix} 0 & 1 \\ -k(t)/m & -c(t)/m \end{pmatrix}$$

The characteristic equation of Eq. (4) is $|\lambda I - A(t)| = 0$, that is

$$\lambda^2 + \frac{c(t)}{m}\lambda + \frac{k(t)}{m} = 0 \quad (5)$$

according to Eq. (5), the H matrix can be obtained

$$H = \begin{pmatrix} c(t)/m & 0 \\ 1 & k(t)/m \end{pmatrix}$$

its two principal minors $\Delta_1 = c(t)/m$, $\Delta_2 = c(t)k(t)/m^2$. If we adopt the basic stability theorem and Hurwitz theorem, the stability of this SDOF system can be determined. For demonstrating clearly, we use the characteristic roots to discriminate the stability of motion. the roots of Eq. (5) are

$$\lambda_{1,2} = -\frac{c(t)}{2m} \pm \frac{1}{2} \sqrt{\left(\frac{c(t)}{m}\right)^2 - 4\frac{k(t)}{m}}$$

There are 11 types of solution in mathematics, but in practice structure engineering, the damping $c(t)$ is greater than zero in most cases, so that discussion at here, only assuming damping $c(t) > 0$, therefore, there are 5 types of solution as follows:

(a) when $(c(t)/m)^2 > 4k(t)/m$, then $\lambda_1 < 0$, $\lambda_2 < 0$, the solution of Eq. (4) is

$$y(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \quad \lim_{t \rightarrow \infty} \begin{pmatrix} y(t) \\ \dot{y}(t) \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

that is the system is stable.

(b) when $(c(t)/m)^2 < 4k(t)/m$, λ_1, λ_2 are a pair of complex which real parts are negative $\lambda_{1,2} = -n \pm \omega i$, then the solution of Eq. (4) is

$$y(t) = e^{-nt}(c_1 \cos \omega t + c_2 \sin \omega t) \quad \lim_{t \rightarrow \infty} \begin{pmatrix} y(t) \\ \dot{y}(t) \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

that is the system is stable.

(c) when $(c(t)/m)^2 = 4k(t)/m$, $\lambda_1 = \lambda_2 = -c(t)/2m < 0$, the solution is

$$y(t) = (c_1 + c_2 t)e^{-(c/2m)t} \quad \lim_{t \rightarrow \infty} \begin{Bmatrix} y(t) \\ \dot{y}(t) \end{Bmatrix} \rightarrow \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

that is the system is stable.

(d) when $k(t)/m=0$, $\lambda_1 = -c/m < 0$, $\lambda_2 = 0$, the solution is

$$y(t) = c_1 e^{\lambda_1 t} + c_2, \quad \dot{y}(t) = c_1 \lambda_1 e^{\lambda_1 t} \quad \lim_{t \rightarrow \infty} \begin{Bmatrix} y(t) \\ \dot{y}(t) \end{Bmatrix} \rightarrow \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

the magnitude of $y(t)$ and $\dot{y}(t)$ depends on the initial values. this state is called critical state or limiting state.

(e) When $k(t)/m < 0$, $\lambda_1 > 0$, $\lambda_2 < 0$, the solution is

$$y(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \quad \lim_{t \rightarrow \infty} \begin{Bmatrix} y(t) \\ \dot{y}(t) \end{Bmatrix} \rightarrow \begin{Bmatrix} \infty \\ \infty \end{Bmatrix}$$

that is the system is unstable.

Above discussion indicates the relation between the stiffness $K(t)$ and the stability of system, so that the criterion of stability for SDOF structural system subjected to earthquake ground motion is obtained as follows:

CRITERION1. For a SDOF system, if the stiffness of the system is greater than zero at time t , i. e. , $k(t) > 0$, the system is stable at time t ; if the stiffness of the system is less than zero, i. e. . $k(t) < 0$, the system is unstable at time t ; if the stiffness of the system is equal to zero, i. e. , $k(t) = 0$, the system is in limiting state.

CRITERION OF MDOF SYSTEM

The criterion of stability of a SDOF system subjected to earthquake motion presented in the preceding section can also be obtained by using the basic stability theorem and the Hurwitz theorem. When using the Hurwitz theorem, firstly, the Hurwitz matrix must be formed. For a MDOF system, especially, when the number of degrees of freedom of a system is great, to form the Hurwitz matrix is difficulty. For this reason, in structure engineering, applying the Hurwitz theorem to determine the stability is not convenient. It is necessary to develop another method for the MDOF system in structure engineering.

In this section, based on the method of normal coordinate, a system with N degrees of freedom can be transformed into a set of N independent normal-coordinate equations, then the procedure and criterion presented at preceding section for a SDOF system can be used, and the stability of the MDOF system can be determined.

At time t , for a structural system with N degrees of freedom, its linear incremental equation of motion is

$$M\Delta\ddot{V} + C(t)\Delta\dot{V} + K(t)\Delta V = \Delta P(t) \quad [6]$$

in which the terms $K(t)$, $C(t)$ are stiffness and damping matrixes, they could be evaluated by iteration at time t . When we take the normal coordinate as the mode of vibration, the mode-superposition procedure can be adopted. Using the mode coordinate,

the Eq. (6) is transformed into a set of N independent mode coordinate equations, the i th independent equation of motion is given by

$$m_i \Delta \ddot{v} + c_i \Delta \dot{v} + k_i \Delta v = \Delta p_i(t) \quad (7)$$

in which $m_i = \Phi_i^T M \Phi_i$, $c_i = \Phi_i^T C \Phi_i$, $k_i = \Phi_i^T K \Phi_i$, $\Delta p_i(t) = \Phi_i^T \Delta P(t)$, they are the normal coordinate generalized mass, generalized damping, generalized stiffness, and generalized load for mode i , respectively. Φ_i is the i th modeshape.

The analysis procedure of stability of motion for Eq. (7) is the same with that of a SDOF system. In most cases the damping matrix C is the positive definite, then $c_i = \Phi_i^T C \Phi_i > 0$, therefore, we can only discuss the relation between the generalized stiffness k_i and the stability of the system. In general, the structural stiffness matrix is symmetric, so that the i th generalized stiffness $k_i = \Phi_i^T K(t) \Phi_i$ is a quadrics. If $K(t)$ is positive definite matrix, then $k_i = \Phi_i^T K(t) \Phi_i > 0$. If $K(t)$ is negative definite matrix, then $k_i = \Phi_i^T K(t) \Phi_i < 0$. Therefore, using the criterion of stability of motion is equal to decide whether the quadrics stiffness matrix $K(t)$ is positive definite or negative definite. In the theory of linear algebra, to decide whether a symmetric matrix is positive definite, there is the theorem as follows:

THEOREM. If the symmetric matrix K is positive definite matrix, a necessary and sufficient condition is that the determinant of every right upper submatrix is positive, i. e. ,

$$\Delta_1 = k_{11} > 0, \Delta_2 = \det \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} > 0, \dots, \Delta_n = \det K > 0.$$

According to this theorem, we can discriminate the stability of motion for a MDOF structural system conveniently, and it is unnecessary to find out every modeshape $\Phi_i(t)$ at time t , therefore, the criterion of stability for a N degrees of freedom structural system is obtained, it can be stated as follows:

CRITERION2. For a structural system with N degrees of freedom, its tangent slope linear stiffness matrix at time t

$$K(t) = \begin{pmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \vdots & & & \vdots \\ k_{n1} & \dots & \dots & k_{nn} \end{pmatrix}$$

If $\Delta_1 = k_{11} > 0$, $\Delta_2 = \begin{vmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{vmatrix} > 0, \dots, \Delta_n = |K| > 0$ at time t , then the structure is stable system at time t ; if $\forall \Delta_i < 0, i \in [1, n]$, then the structure is unstable system at time t ; if $\forall \Delta_i = 0, i \in [1, n]$, then the structure is in limiting state at time t .

It must note that the above criterion is derived by using the normal coordinate method, but in the criterion 2, the mode coordinates are not adopted, this illustrates that the criterion 2 is effective to all of the modes of vibration, so that the criterion 2 is accuracy.

CRITERION OF SHEAR BUILDING

The shear model is commonly used in the seismic response analysis of shear buildings. The stiffness matrix of shear model is three diagonal form

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & & & \\ -k_2 & k_2 + k_3 & -k_3 & & \\ & \ddots & \ddots & \ddots & \\ & & -k_{n-1} & k_{n-1} + k_n & -k_n \\ & & & -k_n & k_n \end{bmatrix}$$

in which k_i is the stiffness of the i th story.

In the seismic response analysis of buildings, the Rayleigh damping matrix form is often used, i. e., $C = \alpha M + \beta K_c$, here α and β are real constant, they can be determined by the mode damping ratio ξ_i . M is the mass matrix of building, M is positive definite. K_c is elastic stiffness matrix, K_c is also positive definite. Therefore, the damping matrix C is positive definite too, and the criterion 2 can be adopted to the shear model. As its stiffness matrix is three diagonal form, the simpler criterion of stability can be obtained. We had proved the following statements (Qin, 1992): All the principal minors of the stiffness matrix are greater than zero, a sufficient and necessary condition is that all the story stiffnesses are greater than zero; if any one or more story stiffnesses are less than zero, there must be at least one principal minor is less than zero; if any one or more story stiffnesses are equal zero, there must be at least one principal minor is equal zero.

According to above statements, the very simple criterion of stability for the shear building is obtained, it is stated as follows:

CRITERION 3. If all the structural story stiffnesses are greater than zero at time t , i. e., $k_i(t) > 0$ ($i = 1, 2, \dots, n$), the structure is stable at this time; if any one or more structural story stiffnesses are less than zero, the structure is unstable at this time; if any one or more structural story stiffnesses are equal to zero, the structure is in limiting state.

It is known if the structure keeps on stable, its all story stiffnesses must be greater than zero, but the structure becomes unstable it is good enough that only any one story stiffness is less than zero. This conclusion indicates that shear building is a series system.

DEFINITION OF COLLAPSE

In the collapse analysis of building subjected to earthquake ground motion, the definition of collapse is very important. The very different conclusion may be obtained using the different collapse definition. There are a few of definitions about the collapse of building in the current research or design code for buildings. They are the strength collapse definition, the deformation collapse definition as well as strength-deformation collapse definition. They are all the experience definitions, and lack theoretical foundation. We consider that there is a close relation between stability of motion and collapse for

building. In static problems, unstable structure can not exist, unstable structure means the structure collapsing. In seismic nonlinear problem, situation is different. The direction of earthquake load varies in the course of time, the property of stiffness also varies with earthquake load. For this reason, the stability of building subjected to earthquake ground motion varies with structural stiffness. The instability is the necessary condition of building collapsing, but not the sufficient condition. The sufficient condition of collapse is that the restoring force of building becomes small amount.

According above discussion, now the collapse definition for building subjected to earthquake motion is defined:

DEFINITION. For the building subjected to earthquake ground motion, the necessary and sufficient condition of the building collapsing is that the building is unstable, and the restoring force becomes small value. The beginning collapse time is just the beginning instability time.

APPLICATION

As an example of application, the seismic response of the two main directions of a fifteen-story reinforced concrete block masonry building subjected to El Centro earthquake ground motions (U. S. 1940, NS) is studied on elastic, elastio-plastic and collapse stages. The presented collapse definition is adopted to determine whether the building collapses, and when the building collapses. The input earthquake acceleration record is multiplied by 1.1494, and the acceleration amplitude is amplified to 0.4g which corresponds to the 8 degree of earthquake intensity of China. The plans of the building is shown in Fig. 1. In the analysis, adopted the calculating model is the wall by wall bending-shear model (Qin et al., 1994). The adopted load-deflection (shear-displacement or moment-angle) restoring force model is the trilinear with descending stiffness branch, and its critical points were cut by fuzzy method (Qin et al. 1988) as shown in Fig. 2-2. The results of maximum response in two directions are listed in table 1.

The maximum response values of the building

Table 1

max. value direction	X(cm)	V(cm)	D θ (rad)	θ (rad)	Final state of building	Notes
Transverse	3.535(15)	38.52(15)	0.00150(2)	0.01175(15)	collapse $t=5.048$ sec.	(15)expresses 15th Floor
	3.535(15)	38.52(15)	0.01036(2)	0.01175(15)	collapse	stop time $t=5.248$ sec.
Longitudinal	3.365(9)	26.40(15)	0.00048(9)	0.00482(8)	collapse $t=4.956$ sec	
	4.085(9)	26.40(15)	0.00743(9)	0.00709(8)	collapse	stop time $t=5.056$ sec.

in the table 1, X and V express the maximum floor relative displacement and the maximum floor absolute displacement, respectively. D θ and θ are the maximum floor relative deflection angle and the maximum floor absolute deflection angle, respectively.

In the transverse direction, the maximum floor relative deflection angle took place at

the second floor, its displacement and angle response time histories are shown in Fig. 3. The beginning collapsing time of the building is 5.048 seconds. In the longitudinal direction, the maximum floor relative deflection angle occurred at the 9th floor, its displacement and angle response time histories are shown in Fig. 4. The beginning collapsing time of the building took place at 4.956 seconds. The beginning collapsing time was determined by using the collapse definition proposed in preceding. From Fig. 3 and Fig. 4, we can see that as soon as the building becomes unstable system, the response of the building abruptly increased. The dashed is the response curves after the building collapsing. This numerical analysis indicates that the collapse definition given by authors is very useful to decide when the building collapsing. If we used other traditional collapse definitions, the collapsing time would not be determined.

CONCLUSION

1. In this paper, the stability of motion theory is systematic applied to the structural dynamic analysis. The analytic procedure is not only suitable for earthquake load, but also suitable for other dynamic loads.
2. The criterion of stability for SDOF structural system is proposed. Based on the criterion of SDOF system, and using normal coordinate transforming method, the stability analytic procedure for MDOF structural system is presented, and the corresponding criterion of stability is also established.
3. For commonly used shear model in the structural dynamic analysis, the very simple criterion of stability is established. According to this criterion, we have known if any one story stiffness of shear building were less than zero, the building would become unstable system.
4. The collapse of building depends on the instability of building. Instability is the necessary condition of collapse for structure subjected earthquake ground motion. The stability of building depends on the stiffness of building, so that the stiffness is a very important property, we must pay more attention to the structural stiffness in seismic response analysis.
5. Through the analysis of the fifteen-story reinforced concrete block masonry building, it is demonstrated that the proposed criterion is very effective for determining the stability of motion. The beginning collapsing time has been found exactly using the stability collapse definition.
6. The computational program used in this paper, is designed and compiled by authors. This program can applied to the linear, nonlinear as well as collapse response analysis for buildings subjected to earthquake ground motions.

ACKNOWLEDGMENTS

The financial support by the post doctoral science foundation of China, for which the authors are gratefully acknowledged.

REFERENCES

- Qin, W. X. , Liu, J. (1988), A Fuzzy Method for Dealing with Critical Points of Restoring Force Curve in Nonlinear Dynamic Calculation of Structures, Journal of Harbin Architectural and Civil Engineering Institute.
- Qin, W. X. , Liu, J. , Wang, X. M. (Oct. ,1991), An Analytical Method of Stability of Motion for Structure Subjected to Earthquake ground Motion, Journal of Structure Engineering.
- Qin, W. X. (Aug. , 1992), Application of Liapunov's Stability on Aseismic Structure Analysis, Proc. of 1st Conference on Structure Engineering, the National Science Foundation Committee, Harbin, China.
- Qin, W. X. , Wang, X. M. (Aug. , 1992), An Analytical Method for Nonlinear Liapunov's Stability of Structure Subjected to Earthquake Motion, Proc. of Conference of Engineering Mechanics of North-East of China.
- Qin, W. X. (June, 1993), Methods and Application of Stability of Motion for Analysing Inelastic Structures, Proc. of the First China Post doctoral Academic Congress, Beijing.
- Qin, W. X. , Zhao, J. B. , Wang, X. M. (Sept. ,1994), Seismic Collapse Response Calculation of Reinforced Concrete Block Masonry Building, Journal of Shenyang Architectural and Civil Engineering Institute.
- Qin, W. X. , Wang, X. M. , Zhao, J. B. (Dec. ,1994), A Seismic Analysis Model for Reinforced Concrete Block Masonry Buildings, Journal of World Information on Earthquake Engineering.
- Qin, W. X. , Wang, X. M. (Ang. ,1995), Nonlinear Seismic Response and Collapse Analysis for High-Rise RC Block Masonry Buildings, to be published in Proc. of the 5th International Conference in Engineering Using Small Computers, Masao.
- Wang, L. , Wang, M. Q. (1987), Analysis of Stability for Nonlinear Differential Equation, Harbin Institute of Technology Press.

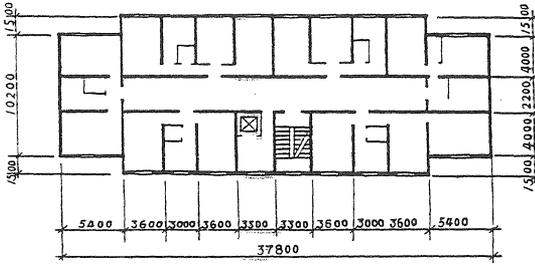


Fig. 1-1 Plan of 1-14 Floors

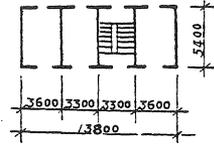


Fig. 1-2 Plan of Top Floor

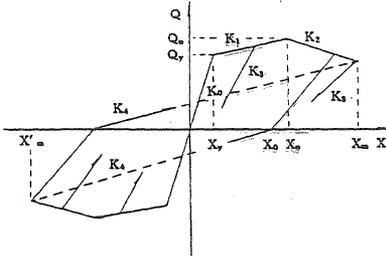


Fig. 2-1 Basic Restoring Force Model

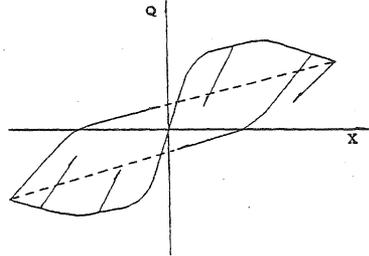


Fig. 2-2 Fuzzy Restoring Force Model

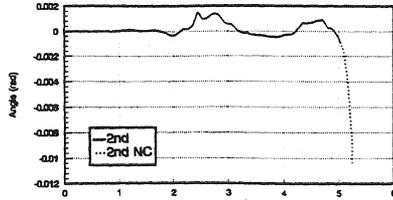
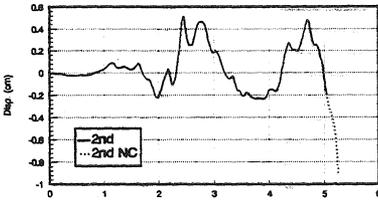


Fig. 3 Displacement and Angle Time Histories in Transverse Direction

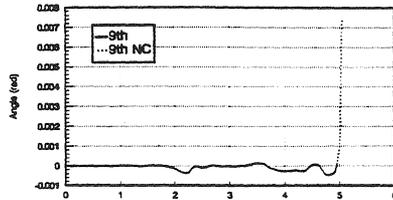
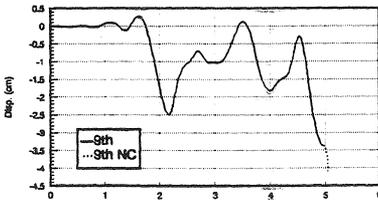


Fig. 4 Displacement and Angle Time Histories in Longitudinal Direction