TRI-LINEAR FORCE-DISPLACEMENT MODELS REPRESENTATIVE OF OUT-OF-PLANE UNREINFORCED MASONRY WALL BEHAVIOUR

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ABSTRACT
Behaviour of full-scale unreinforced masonry (URM) walls subjected to out-of-plane uniform loads was investigated by testing three brick walls. Uniform loading was applied on the surface of the walls using a system of airbags. The walls, having a constant width of 1200 mm, had a height varying from 2000 mm to 4100 mm. The walls were two-leaf thick and had slenderness ratios of 9, 16 and 19. All of the tests were performed using simply supported boundary condition, and test specimens behaved as ideal one-way bending elements. Tri-linear force-displacement models were constructed based on the experimental curves recorded during the testing. It was found that the wall geometry and axial load influenced the shape of the models. General recommendations were made for tri-linear modelling of out-of-plane URM walls based on this finding.

KEYWORDS: out-of-plane, unreinforced masonry wall, force-displacement curve, tri-linear model, airbag testing.

INTRODUCTION
Several reports of past earthquakes have identified out-of-plane collapse of URM walls as one of the predominant modes of failure. This suggests that existing unretrofitted out-of-plane URM walls may be vulnerable to future earthquakes, and therefore they should be investigated for their seismic resistance. Considerable effort has been made by several researchers in the past to understand the seismic behaviour of this type of elements. A wide range of approaches from analytical modelling [1, 2] to extensive experimental and numerical methods have been used. In the area of numerical modelling micro-models [3] as well as multi-degree-of-freedom (MDOF) macro-models [4-6] have been used. Generalized SDOF modelling is also a popular form of macro-modelling which can be used for analysis of mechanical systems with one or nominally one degree of freedom. SDOF generalisation of a system consisting of an out-of-plane wall connected to rigid diaphragms or 2DOF generalisation of an out-of-plane wall connected to a flexible diaphragm are the most effective methods from a practical point of view. Ignoring the
effects of corners on the behaviour of URM walls, a single horizontal crack is expected to form at about the mid-height of the wall, when subjected to out-of-plane forces. The wall segments will then start to rock about the pivotal points formed at top, bottom and the cracked joint. For simplicity, the wall behaviour in this state can be modelled as a SDOF system. Several researchers [6-9] have studied out-of-plane URM walls using this method. In fact, the huge nonlinearities associated with the material and construction practice of the URM walls, and uncertainties involved in selection of appropriate parameters for a complex numerical analysis may render the micro-modelling ineffective. In this paper the force-displacement behaviour of simply-supported (SS) out-of-plane URM walls modelled as a SDOF system is studied.

An extensive experimental study performed in the 1970s [10] was among the first to show that the applied force and maximum displacement of the wall had a curvilinear relationship. Instead of the real curvilinear model, many researchers assumed the behaviour of a wall past the cracking point to follow the behaviour of rigid bodies. Bi-linear models have been developed based on this assumption and have been used in numerical modelling [4, 6]. While being simple and suitable for performing fast analyses, a bi-linear model fails to capture the effects of the elastic deformation of the wall. In addition, modelling of the cracked parts of a URM wall as rigid blocks is generally accompanied by ignoring the effects of material crushing at the cracked joint. It also implies an infinite strength for the mortar at the cracked bed-joint. All of the above-mentioned shortcomings contribute to this recognition that a more realistic force-displacement model should be used in development of a SDOF model for seismic assessment of out-of-plane URM walls. A tri-linear model was introduced [7] to represent the actual curvilinear behaviour for one-way out-of-plane URM walls with known mass, boundary conditions, overburden and dimensions. Two values ($\Delta_{1}/\Delta_{\text{ins}}$ and $\Delta_{2}/\Delta_{\text{ins}}$ ratios) are used in conjunction with the bi-linear rigid body model of the wall to construct the tri-linear model. $\Delta_{\text{ins}}$ is the maximum stable displacement which can be obtained from static equilibrium of the cracked wall at the point of incipient instability, and the values of $\Delta_{1}$ and $\Delta_{2}$ should be selected according to results from prior testing of wall specimens. As a tri-linear model matches more closely the real curvilinear force-displacement relationship of an out-of-plane wall, characterisation of a suitable tri-linear model for URM walls in New Zealand has been performed.

Airbag testing, which is among the quasi-static methods with frequent applications in uniform loading tests, has been used for the above-mentioned characterisation study. The method is recommended in ASTM E72-02 standard for testing of URM panels subjected to out-of-plane uniform loading [11]. As earthquakes generate an approximately uniform loading on the surface of out-of-plane walls [1], results from airbag testing can be used in subsequent numerical modelling for seismic analysis. Researchers have widely used the airbag method for performing both small-scale tests [12, 13] and full-scale or in-situ tests [14-17]. Several researchers have used airbags together with an in-plane loading system to study the bi-directional behaviour of masonry infill panels [18-20]. More recently in Australia the method has had applications on testing of both new [21] and retrofitted [22] two-way URM walls. The airbag testing method has also had frequent applications for investigation of one-way bending behaviour of walls. A series of out-of-plane airbag tests was performed on 192 one-way walls subjected to axial compressive forces [10]. Airbags have also been employed to apply uniform forces on half-scale one-way URM walls [23]. A similar method with water bags applying out-of-plane loads has also been used to perform tests on a large one-way URM wall panel [24]. Other loading methods used
elsewhere are point/line loads or three-point/line loads, which are all considered to be less satisfactory substitutes for the wall’s inertial loads when compared to pressure applied by a system of airbags.

In the following the testing programme is first presented. Then the results from the tests are given and discussion on the results is provided. Finally the generic shape of the tri-linear model is derived from the test results.

TESTING PROGRAMME
The airbag testing was performed using the setup shown in Figure 1 on 3 full-scale URM walls described in Table 1. In this table H, w, and t_{nom} are the total height, width, and nominal thickness of the walls, respectively. Wall specimens were tested with simply supported boundary conditions, and with and without pre-compression according to the specification given in Table 2. In Table 2, y is the overburden ratio defined as the ratio of the axial load to the total weight of the wall. The simply-supported condition was achieved by restraining the horizontal movements of the walls at top and bottom using steel angles. As shown in Figure 1(a), the bottom end of the walls was propped at the opposite side of the airbag loading. This allowed wall to freely rotate around its base while deflecting out-of-plane. In addition, Figure 1 shows top end of the walls confined by two steel angles connected to an overburden leverage assembly [21]. The overburden leverage assembly was consisted of a horizontal steel channel which was pinned at one end and was free to move at the other end, hence permitting the test wall to rotate freely about the top of the wall. The overburden loads were applied by hanging weights from the far end of the overburden leverage assembly (Figure 1(a)). The leverage mechanism transferred the weights to the top of the wall by a factor of 3. More details of the testing procedure can be found elsewhere [25].

<table>
<thead>
<tr>
<th>Wall</th>
<th>Test</th>
<th>Axial stress, (kPa)</th>
<th>Overburden ratio, y</th>
<th>Max. applied uniform load, kN</th>
<th>Corresponding walls in real buildings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-A</td>
<td>0</td>
<td>0</td>
<td>2.89</td>
<td>Top-storey or 1-storey wall without parapet</td>
</tr>
<tr>
<td></td>
<td>1-B</td>
<td>20</td>
<td>0.31</td>
<td>4.05</td>
<td>Top-storey or 1-storey wall with parapet</td>
</tr>
<tr>
<td></td>
<td>1-C</td>
<td>40</td>
<td>0.63</td>
<td>5.40</td>
<td>Ground storey of a 2-storey wall without parapet</td>
</tr>
<tr>
<td>2</td>
<td>2-A</td>
<td>0</td>
<td>0</td>
<td>3.46</td>
<td>Top-storey or 1-storey wall without parapet</td>
</tr>
<tr>
<td></td>
<td>3-A</td>
<td>0</td>
<td>0</td>
<td>2.66</td>
<td>Same as 1-A</td>
</tr>
<tr>
<td>3</td>
<td>3-B</td>
<td>75</td>
<td>1.02</td>
<td>7.98</td>
<td>Ground storey of a 2-storey wall with parapets and large storey height</td>
</tr>
</tbody>
</table>

Table 1: Wall specification

<table>
<thead>
<tr>
<th>Wall Geometry</th>
<th>H (mm)</th>
<th>w (mm)</th>
<th>t_{nom} (mm)</th>
<th>H/t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3500</td>
<td>1150</td>
<td>220</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>1150</td>
<td>220</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>4100</td>
<td>1150</td>
<td>220</td>
<td>19</td>
</tr>
</tbody>
</table>

Table 2: Test matrix and summary of the results
Solid bricks and weak ASTM type “O” mortar were used to build the test walls. Bricks were used in a “common bond” pattern, with headers every fourth course. Material testing was performed according to ASTM standards on a sufficient number of mortar cubes and masonry prisms, and the data given in Table 3 were acquired. Out-of-plane loads generated by means of inflated airbags were transferred from the backing frame to the reaction frame using four S-type 10 kN load cells. Frictionless steel plates covered with a film of grease were used underneath the plywood backing frame to minimize the transfer of load from the backing timber frame to the ground. The displacements were measured by a linear variable displacement transducer (LVDT) placed at the wall centre at about the mid-height of the walls.

![Image](image1)

(a) Airbag testing setup; (b) Test 3-A

**Table 3: Material properties**

<table>
<thead>
<tr>
<th>Test Method</th>
<th>Wall 1&amp;2</th>
<th>Wall 3</th>
<th>Wall 1&amp;2</th>
<th>Wall 3</th>
<th>Wall 1&amp;2</th>
<th>Wall 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM C 780 – 02 [26]</td>
<td>3.95</td>
<td>6.49</td>
<td>6.91</td>
<td>0.44</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>ASTM C1314 – 03b [27]</td>
<td>0.13</td>
<td>0.037</td>
<td>0.038</td>
<td>0.19</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>
OBSERVATIONS OF THE TESTS

As shown in Figure 1(b), a single horizontal crack occurred at a mortar bedjoint at about the mid-height of each wall. The location of the crack was at 60%, 52% and 61% of the wall height for Wall 1 to 3, respectively. Neglecting the results for Wall 2 which was a squat wall, it can be assumed that 60% of the wall height is a sufficiently accurate assumption for the location of the crack. During the first half-cycle of Test 3-B, a high level of overburden caused severe material crushing at the cracked joint, which included spalling of the adjacent bricks. Masonry crushing did not occur in any other test. Table 2 shows maximum applied horizontal uniform load calculated using load cell recordings.

RESULTS

Typical force-displacement curves obtained from the testing are shown in Figure 2 for Tests 3-A and 3-B. Force is calculated as sum of all readings from four load cells attached to the backing timber frame, and displacement is measured at the mid-height of the test walls. Figure 3 gives a summary of the results from all of the tests. In Figure 3 only the response envelopes are shown.

![Figure 2: Force-displacement histories; a) Test 3-A, b) Test 3-B](image)

![Figure 3: Response envelopes](image)
Comparing the results from tests performed on Wall 1, which are shown in Figure 3, one can see that axial load significantly increased the strength of the masonry wall but did not have a noticeable effect on its initial stiffness. The same trend can be observed for Tests 3-A and 3-B performed on Wall 3. Real force-displacement curves for out-of-plane URM walls approach to rigid-body push-over force-displacement plots. This relationship can be seen in Figure 4. Rigid-body plots can be achieved using Equations 1 and 2 [5]. These equations can be obtained by equating overturning and resistant forces acting on the cracked wall immediately before instability. These equations have been re-written below for the appropriate boundary conditions of the performed tests.

\[ F_{\text{max}} = (4t.W/H).(1+1.5y) \]  

\[ F = F_{\text{max}}.(1-D/D_{\text{ins}}) \]  

In the above equations, \( W \) is the total weight of the wall, and \( y \) has been defined before. The values of \( y \) are given in Table 2 for the tested walls. \( D_{\text{ins}} \) is the maximum displacement that the cracked wall can sustain before a collapse mechanism is formed. This value can be calculated from the equilibrium of the cracked wall at the onset of instability, and is very close to the thickness of the wall. Figure 4 shows that the difference between the real force-displacement curves and the bilinear models is greater for tests performed on walls with higher overburden ratios. This implies that the wall resistance may be overestimated if bilinear models are used for analysis of walls located in the lower storey of a building.

![Figure 4: Real force-displacement curves vs. bilinear models](image-url)
Figure 5 shows the approximate tri-linear force-displacement curves for selected tests. The experimental curve for each test is also shown together with the tri-linear model. Values of $D_1$ and $D_2$ for each test have been assigned so that the resulting tri-linear model has an approximately equal dissipated energy to that of the real curve. Table 4 summarises these calculations from all of the tests. The ratios of $D_1/D_{ins}$ and $D_2/D_{ins}$ can be interpolated or extrapolated for walls with reasonable aspect ratios and overburden ratios using data available in Table 4. Figure 6 shows a plot of $D_2/D_{ins}$, with $D_{ins}$ assumed to be equal to the effective wall thickness, $t$. This assumption was made based on the experimental curves summarised in Figure 3. The wall effective thickness can be obtained from Equation (3) [5], with $t_{nom}$ equal to 220 mm for a two-leaf wall.

$$t = t_{nom}(0.975 - 0.025y)$$

Based on the information from Table 4 and from Figures 5 and 6, it can be concluded that both slenderness ratio and overburden ratio affect the shape of the tri-linear model, and that assuming constant values for $D_1/D_{ins}$ and $D_2/D_{ins}$, as done in [7], is not valid for all URM walls.

Figure 5: Tri-linear representation of the force-displacement curves; (a): Test 1-B, (b): Test 1-C, (c): Test 3-A, (d): Test 3-B
Table 4: Recommended tri-linear model parameters

<table>
<thead>
<tr>
<th>Slenderness ratio</th>
<th>y = 0</th>
<th>y = 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D_1/D_{ins}</td>
<td>D_2/D_{ins}</td>
</tr>
<tr>
<td>9</td>
<td>0.01</td>
<td>0.25</td>
</tr>
<tr>
<td>16</td>
<td>0.01</td>
<td>0.37</td>
</tr>
<tr>
<td>19</td>
<td>0.01</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Figure 6: Suggested values of $D_2/D_{ins}$ for a two-leaf URM wall

CONCLUSION
Results from six out-of-plane airbag tests performed on URM walls were presented in the paper. Tri-linear models were constructed using rigid-body push-over plots to match the real curvilinear behaviour. The results from the obtained tri-linear models were compared with an earlier research, and it was found that both overburden and slenderness ratios have effects on the configuration of the model. General recommendations were made for constructing tri-linear models for URM walls, as a function of the wall slenderness ratio and the overburden ratio (ratio of axial load to wall weight).

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