LOAD-BEARING CAPACITY OF FLEXURAL UNREINFORCED MASONRY WALL PANELS

L. Richter[1], E. Brehm[2] and C.-A. Graubner[3]
[1] Bilfinger Berger Nigeria GmbH, Wiesbaden, Germany, lars.richter@bilfinger.de
[2] Ph.D. candidate, Institute of Concrete Structures and Materials, Technische Universität Darmstadt, Darmstadt, Germany, brehm@massivbau.tu-darmstadt.de
[3] Professor, Institute of Concrete Structures and Materials, Technische Universität Darmstadt, Darmstadt, Germany, graubner@massivbau.tu-darmstadt.de

ABSTRACT
The design of unreinforced masonry walls subjected to uniformly distributed lateral loading and simply supported on more than two edges is based on the yield line method or a modification thereof (e.g. British Standard, European Standard, Canadian Standard). In the yield line method, plastic behaviour of the material is assumed as shown by steel and reinforced concrete. The analysis of the load-bearing behaviour of masonry has to be divided into two directions. Normal to the bed joint, the plastic behaviour depends on the ratio of the axial load to the corresponding flexural tensile strength of masonry. The transfer of horizontal bending moment depends on the available flexural strength of the units and torsional shear strength of the bed joints. Therefore, two different failure modes with unequal plastic behaviour have to be considered. On the one hand, the unit can fail due to flexural tensile stress (brittle failure), and no residual load-bearing capacity after cracking exists. On the other hand, failure can occur due to the torsional failure in the bed joints, and a plastic moment can transfer depending on the level of normal stresses in the bed joints. The complex load-bearing behaviour of masonry shows that the analysis of flexural wall panels using the yield line theory depends on the axial load and the material strength of the units and mortar, as well as their bond. To consider these different residual load–bearing capacities, a new non-linear method of analysis was developed. This paper will present a simplified design model for non-load bearing walls based on the new calculation method.

KEYWORDS: masonry, load bearing capacity, flexural tensile strength, bending, biaxial

INTRODUCTION
Non-structural flexural wall panels are required to resist out-of-plane loads due to the action of wind. The capacity of unreinforced masonry walls subjected to uniformly distributed lateral loading has been investigated by many authors over the past several decades. Some research has been included in the design methods in masonry codes in Australia [1], Britain [2], Canada [3] and Europe [4]. In general, the provisions for the analysis and design of non-load bearing walls supported on more than two edges are based on the “failure-line”, “fracture-line”, or “yield-line”

methods. In these methods, the failure lines represent the cracking pattern of the failed masonry wall. For the analysis of the load bearing capacity, different assumptions of the moment resistance after cracking are used in the failure lines.

Figure 1 shows the normalized load-bearing capacity Y_w depending on the wall aspect ratio λ_w (see Equation 2) according to various design methods for masonry walls which are continuously supported on all sides. For a given load –bearing capacity q, which again depends on the ratio μ_t of the flexural tensile strength normal and parallel to the bed joint, the factor Y_w can be calculated by Equation 1.

$$\mathrm{Y}_w = \frac{q \cdot A_w}{f_{t2} \cdot t^2} \text{ with } A_w = H \cdot L \tag{1}$$

$$\lambda_w = \frac{H}{L} \tag{2}$$

with

q	uniformly distributed lateral load
H	effective height of the wall
L	effective length of the wall
t	thickness of the wall
f_{t2}	flexural tensile strength parallel to the bed joint

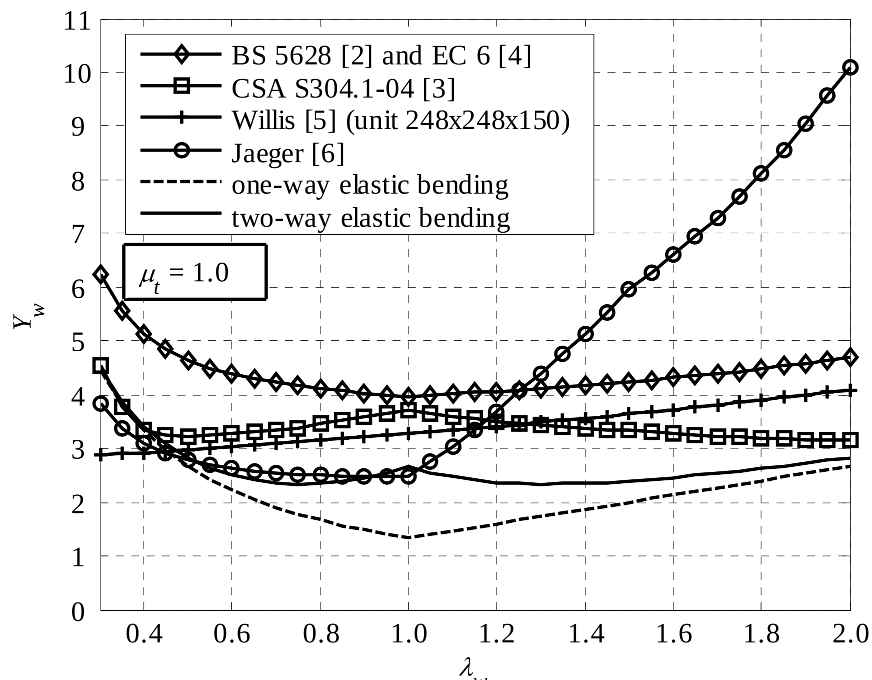


Figure 1: Normalised load-bearing capacity Y_w for continuously supported wall panels (μ_t = 1.0)

In addition to the British Standard BS 5628 [2], the European Standard EC 6 [4] and the Canadian Standard CSA S304.1-04 [3], Figure 1 also shows the results of the research done by Willis [5] and Jaeger [6]. The model of Willis [5] is evaluated for a unit size of l/h/t = 248/248/150 mm and is a further development of the virtual work method according to

Lawrence & Marshall [7] which is provided in the Australian masonry standard. The results of a numerical investigation using the finite-element method are shown by Jaeger [6]. For comparison, the normalised load capacity for walls assuming elastic material behaviour (Equation 1) were analysed in one-way and two-way bending and are also plotted in Figure 1. Flexural walls with aspect ratios of $\lambda_w < 0.5$ or $\lambda_w > 2.0$ supported on all sides generally show one-way bending behaviour. In these cases, the best approximation of the load bearing behaviour, as shown in Figure 1, is achieved by using the CSA [3], whereas the other methods normally overestimate the load bearing capacity. In particular, the assumption of plastic material behaviour in the yield line method used in the design methods of the British Standard [2] and Eurocode [4] cause the overestimation.

The purpose of this paper is to provide a new simplified design method for biaxial bending of masonry wall panels. New material models were developed to describe the biaxial bending behaviour of masonry and were integrated in a nonlinear numerical analysis using the finite element method (FEM). Subsequently, the load bearing capacity of masonry walls were analysed for different material properties, wall dimensions and support conditions using the new numerical model. For a simplified design in practise, an approximation was developed and is provided in terms of a design equation with the tabulated parameters.

NUMERICAL MODEL FOR ANALYSIS

Most masonry walls subjected to uniformly distributed lateral loading will form an initial crack, which generally does not cause failure of the wall. The wall can still carry a larger load than the load causing the initial crack. In a cracked cross section, a little bending moment still can be transferred. Therefore, the remaining moment resistance in the cross section after first crack should be considered in the analysis. Based on this fact, the flexural wall panels were modelled as a beam grid. After cracking of the cross section, a second node with the same coordinates as the original node was placed in the cracked section and the two nodes were connected by vertical and horizontal springs (see Figure 2).

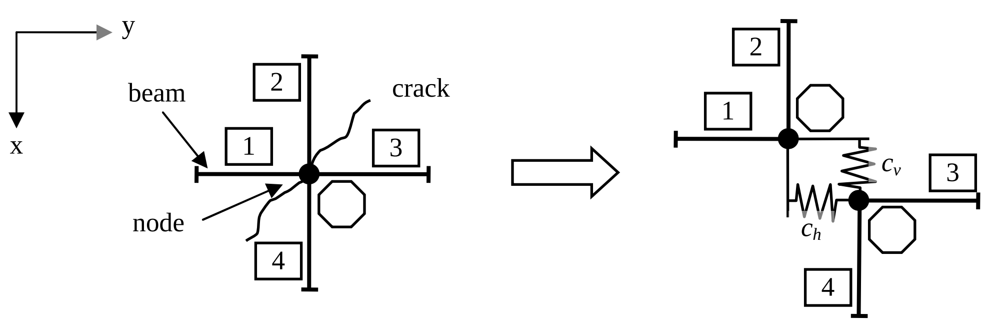


Figure 2: Implementation of springs in cracked zones

The statical system of the masonry wall can be described as a system of crossed beams with springs employed in cracked zones. Fixed supports of walls can be also described by employing springs, such as at the bottom of the wall or at the lateral support for continuous walls. The moment resistance after cracking can be attained by modification of the stiffness c of the springs. The stiffness of the springs depends on the rotation θ_{cr} in the cracked section combined with the material properties of masonry. The anisotropic and non-homogeneous material behaviour of masonry results in different properties for the horizontal and vertical springs which are characterized by the moment-rotation-relationship (M-θ_{cr}-relation). Due to the non-linear M-θ_{cr}-relationship, the iterative Newton-Raphson-method was used to solve the non-linear system of equations (stiffness matrix). The determination of the load-bearing capacity of the wall was accomplished by incremental addition of small loads until the load bearing capacity was reached. In every load step, a non-linear iteration was carried out. The finite element program developed is able to provide the load-deflection diagram for flexural wall panels and the distribution of the internal forces and moments. The FE model was verified with experimental data from Jaeger [6], which show a maximum discrepancy of 7%.

The accuracy of the prediction of the load bearing capacity of biaxial bending masonry walls using the new program developed is based on realistic specification of the material behaviour of masonry. For non-linear structural analysis, it is necessary to define both failure conditions for uncracked and residual load-bearing behaviour for cracked masonry sections. A short introduction to the assumed material model is shown in the following.

MATERIAL MODEL

The finite-element program developed consists of elastic beams and non-linear springs which characterise the bearing behaviour of cracked masonry sections using the M-θ_{cr}-relationship. For realistic characterisation of the cracked masonry section using the moment-rotation-relationship, the analysis of the load-bearing behaviour of masonry has to be divided into two directions. Normal to the bed joint, the plastic behaviour depends on the ratio of the axial load to the corresponding flexural tensile strength of masonry, while the transfer of horizontal bending moment depends on the available flexural strength of the units and torsional shear strength of the bed joints. Therefore, two different failure modes with different plastic behaviour have to be considered. On the one hand, the unit can fail due to flexural tensile stress (line failure) and no residual load-bearing capacity exists after cracking. On the other hand, failure can occur due to the torsional failure in the bed joints (stepped failure) and a plastic moment can transfer depending on the level of normal stresses in the bed joints.

Finally, the influence of the interaction of vertical and horizontal bending moments, as well as the twisting moment in bending plates had to be analysed. Therefore, an approach for the load-bearing behaviour of masonry stressed only by twisting moments was developed. The resulting equations for twisting in masonry were extended to the analytical investigation of bending in the vertical and horizontal directions. The analytical results describe the combined load-bearing behaviour of biaxial bending in masonry and provide equations both for failure conditions of uncracked masonry and load-bearing capacity of cracked masonry subjected to the internal forces and moments in plates n_x, m_x, m_y and m_{xy}. Some final results are presented below. Further and more detailed explanations can be found in Richter [8].

Load-bearing behaviour normal to the bed joint

Based on the moment-curvature-relationship (m_x-κ-relationship) of masonry and the assumed curvature distribution in the cracked zone shown in Figure 3, the angle of rotation θ_{cr} for cracked masonry can be determined by Equation 3.

$$\theta_{cr} = (\kappa_{cr} - \kappa_{un}) \cdot \tan\beta \cdot x_{cr} \qquad (3)$$

The curvatures for the cracked section κ_{cr} and for the uncracked section κ_{un} depend on the moment $m_x = n_x \cdot e$ and were analysed by the m_x-κ-relationship. A linear-elastic stress-strain relationship was assumed, because the compressive stresses normal to the bed joint of non-load bearing walls don't reach the plastic curve of the stress-strain relationship. Figure 4 shows the m_x-θ_{cr}-relationship evaluated for cracked masonry. For simplified and general representation, the moment, as well as the angle, are normalised according to Equation 4.

$$\theta_{n,cr} = \frac{E \cdot t}{n_x} \cdot \theta_{cr} \qquad \text{and} \qquad e/t = \frac{1}{t} \cdot \frac{m_x}{n_x} \tag{4}$$

with

E modulus of elasticity for masonry normal to the bed joint (x-direction)

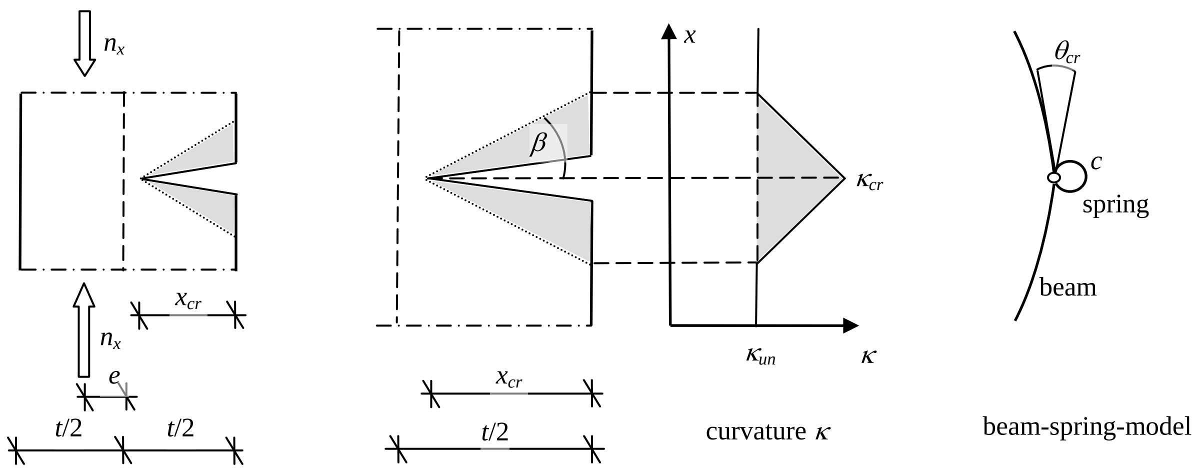


Figure 3: Curvature distribution in the cracked zone and associated beam-spring-model

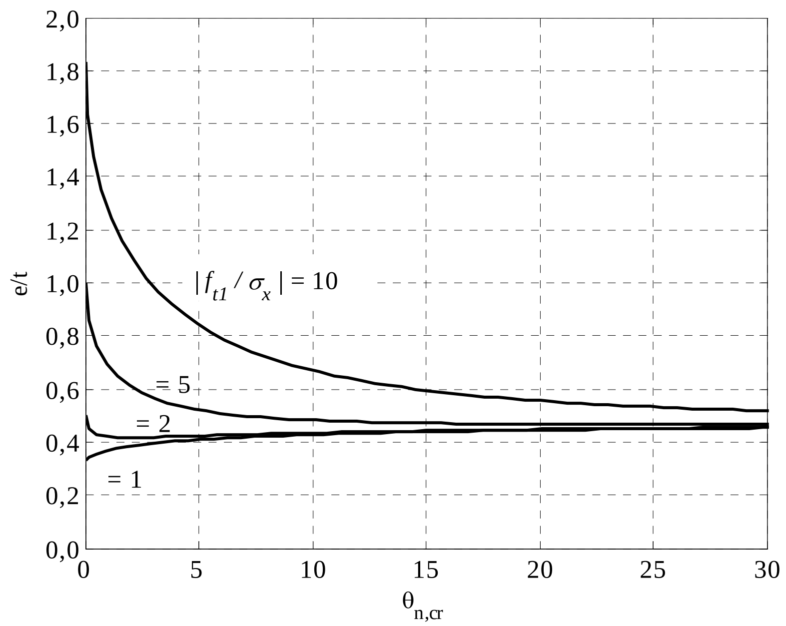


Figure 4: Moment-rotation-relation of vertical flexural masonry for different ratios of flexural tensile strength f_{t1} and compressive stress σ_x normal to the bed joint

Load-bearing behaviour parallel to the bed joint

The transfer of horizontal bending moment depends on the available flexural strength of the units and torsional shear strength of the bed joints. As already described, two different failure modes have to be regarded. Only in the case of stepped failure due to torsion in the bed joints, a plastic moment can be transferred after cracking. The knowledge of this residual moment resistance m_r is important for realistic prediction of the load-bearing capacity of flexural wall panels. Figure 5 shows the m_y-κ-relationship and the corresponding m_y-θ_{cr}-relationship, which is used for the analysis of the load-bearing capacity of flexural wall panels. An analytical model is given by Equation 5.

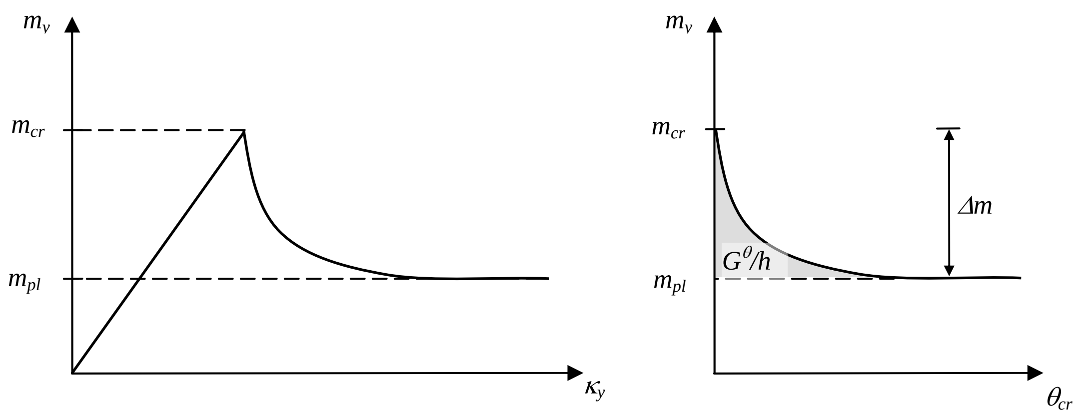


Figure 5: Moment-curvature-relation (left) and moment-rotation-relation (right) for horizontal flexural masonry due to torsional failure in the bed joint

$$m_r = m_{pl} + (m_{cr} - m_{pl}) \cdot \Omega_m = m_{pl} + \Delta m \cdot \Omega_m \quad (5)$$

with

$$m_{cr} = m_{t2} + m_c \quad (6)$$

$$\Omega_m = \frac{m}{\Delta m} = e^{-\frac{\Delta m \cdot h}{G^\theta} \cdot \theta_{cr}} \quad (7)$$

m_r residual moment

m_{cr} cracking moment

m_{t2} cracking moment depending on the flexural tensile strength f_{t2} of masonry parallel to the bed joint

m_c additional resistant moment due to the increased shear strength in the bed joint caused by the compressive stress σ_x normal to the bed joint

m_{pl} plastic moment depending on the friction coefficient in the bed joint and the corresponding compressive stress σ_x normal to the bed joint

Ω_m reduction function for the decreasing of the torque resistance in the bed joint

θ_{cr} angle of rotation of cracked masonry parallel to the bed joint

h height of brick unit including bed joint

G^θ fracture energy (see grey shaded area in Figure 5)

The masonry section fails at the cracking moment m_{cr}, which is the sum of the cracking moment m_{t2}, depending on the flexural tensile strength f_{t2} of the masonry parallel to the bed joint, and an additional resistant moment m_c caused by the increased shear strength in the bed joint due to the compressive stress σ_x normal to the bed joint. After cracking of the masonry section subjected to low vertical forces n_x, the residual moment m_r decreases until reaching the plastic moment m_{pl} (see Fig. 5). This decrease is described by the exponential function Ω_m, which depends on the angle θ_{cr}, the fracture energy G^θ and the height of the units h. The new defined fracture energy G^θ is based on the decreasing of the torque resistance in the bed joint, which results from the bond shear strength.

Load-bearing behaviour due to twisting moment m_{xy}

A complete description of non-load bearing masonry walls subjected to biaxial bending also requires consideration of the load-bearing behaviour due to the twisting moment m_{xy}. To analyse the stresses in the brick unit and bed joint, a strip model was developed for solely twisted masonry as shown in Figure 6. The strips are crossed beams whose arrangement depends on the unit size and the head joint. In flexural solid plates subjected only to twisting moments $m_{xy}= m_{yx}$ the principal moments act at an angle of 45°. The angle ψ of the strips can differ from the angle of the principal moments depending on the unit size. In this case the strips are subjected to a bending moment $M_{strip,B}$ and an additional torsional moment $M_{strip,T}$. Figure 7 presents these two strip moments depending on the ratio λ_b of unit height h and unit length l. It is shown that for $\lambda_b = 0.5$ no torsional moment $M_{strip,T}$. exists, because the inclination of the strip and principal moments is identical.

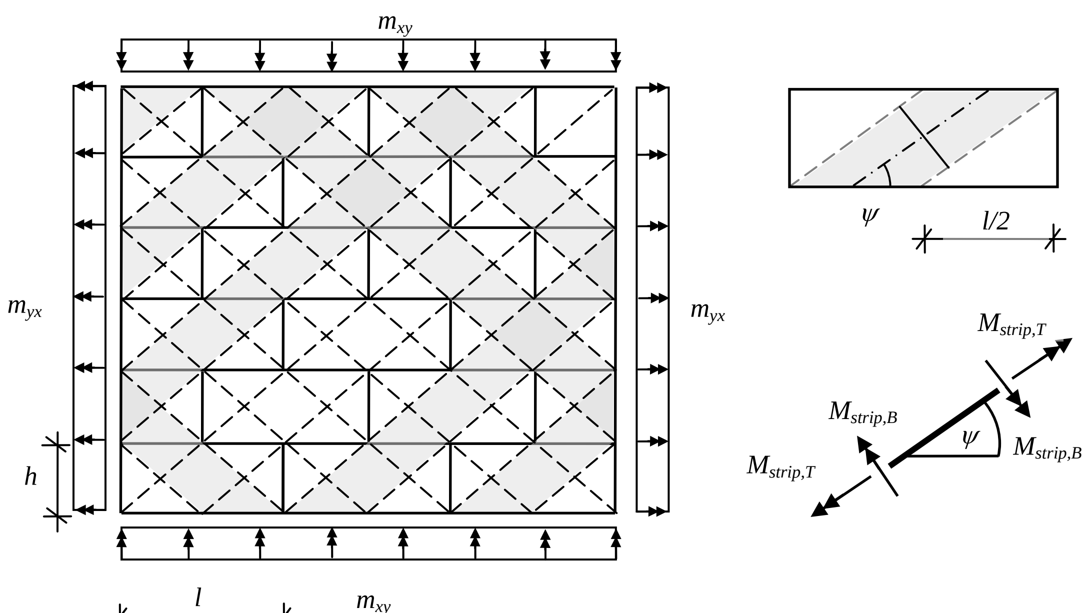


Figure 6: Strip model for solely twisted masonry

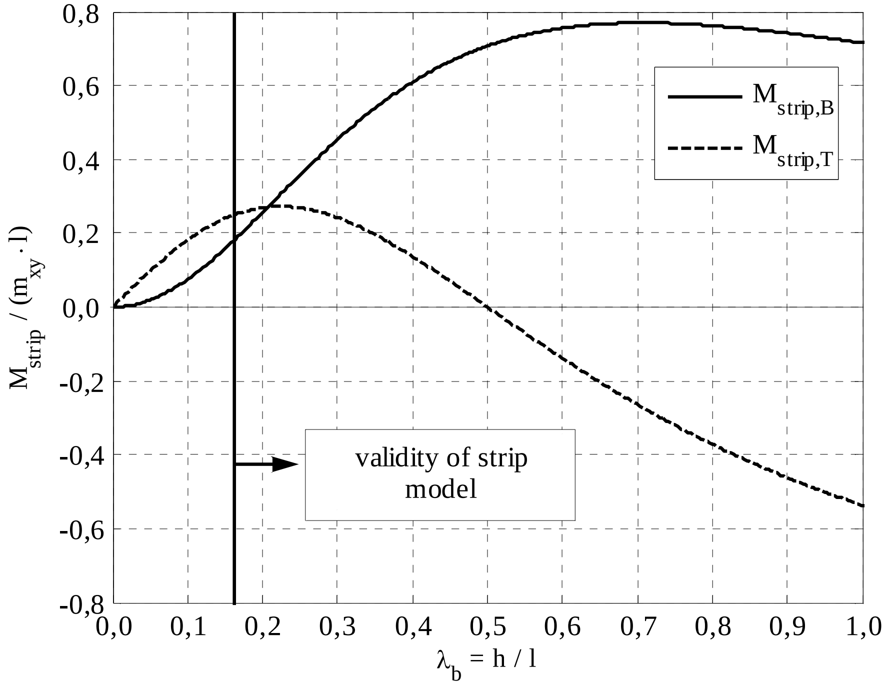


Figure 7: Bending moment $M_{strip,B}$ and torsional moment $M_{strip,T}$ depending on the ratio λ_b of unit height h and unit length l subjected to a twisting moment m_{xy}

Based on the strip moments $M_{strip,B}$ and $M_{strip,T}$ caused by the moment m_{xy}, the tensile stresses in the brick unit and the tensile and shear stresses in the mortar could be analysed. The points of interest in the analysis and the associated material strengths are shown in Figure 8.

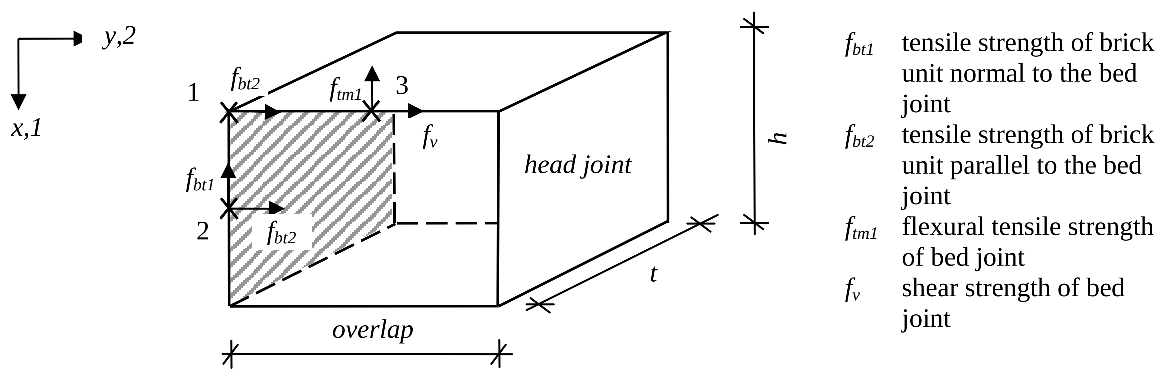


Figure 8: Analysed points with associated material strength

Combined load-bearing behaviour of masonry

Non-load bearing masonry walls subjected to out-of-plane bending combined with vertical in-plane compression due to the self weight of the wall are stressed by the forces n_x, m_x, m_y and $m_{xy}= m_{yx}$. The load-bearing behaviour of the masonry had to be described including the stresses caused by these forces. Therefore, the equations for solely twisted masonry were extended to the analytical investigation of bending in the vertical and horizontal directions. Failure conditions were obtained for uncracked masonry for each point in Figure 8 with the corresponding material strength. Additionally, equations for the prediction of line or stepped failure of masonry were developed.

Furthermore, the residual load-bearing capacity of cracked masonry was analysed for masonry with tensile failure of the brick unit (see Figure 8 points 1 and 2), as well as failure of the mortar bed joint (see Figure 8 point 3). In the case of brick unit failure, the influence of the crack path was investigated, whereas for the mortar failure, the interaction of the torque and gaping in the bed joint was analysed. For both failure modes, analytical equations were obtained for both the m_x-θ_{cr}-relationship describing the vertical spring properties and the m_y-θ_{cr}-relationship for the horizontal spring.

DESIGN METHOD FOR NON-LOAD BEARING FLEXURAL WALL PANELS

For the structural design of non-load bearing flexural wall panels, the design lateral load q_{Ed} should be smaller than the design lateral resistance q_{Rd} as shown in Equation 8. The calculation of the resistant lateral strength q_{Rd} is based on the normalized load-bearing capacity Y_w which has to be derived by a time-intensive numerical finite-element analysis. For simplified design, an approximation of the normalized load-bearing capacity Y_w is necessary. Therefore, the load bearing capacity of masonry walls was analysed for different material properties, wall dimensions and support conditions using the new numerical model. The results were approximated with Equation 9 and the related coefficients were determined. Figure 9 shows an example of the normalized load-bearing capacity Y_w for a flexural tensile strength ratio of $\mu_t = 1.0$. The required coefficients are given in Table 1 depending on the support conditions of the wall and the flexural tensile strength ratio μ_t. The load-bearing capacities for simply supported walls with a free edge on top are determined by the load bearing behaviour of the horizontal direction, significantly. Therefore, the flexural tensile strength shows only for small values an influence (see Table 1, case 3).

$$q_{Ed} \leq q_{Rd} = \frac{t^2}{A_w} \cdot \frac{f_{tk2}}{\gamma_M} \cdot \mathrm{Y}_w \qquad (8)$$

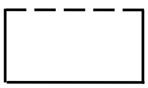
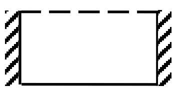
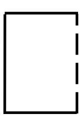
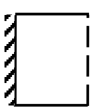
$$\text{with } \mathrm{Y}_W = \frac{a}{\lambda_w^b} + c \cdot \lambda_w^d \qquad (9)$$

f_{tk2} characteristic flexural tensile strength of masonry parallel to the bed joint
γ_M partial safety factor for material
a, b, c, d approximation coefficients depending on the support conditions of the wall and the flexural tensile strength ratio (see Table 1)

CONCLUSIONS

Based on the new analytical equations for the material behaviour, a numerical model for the calculation of flexural capacity of non-load bearing masonry walls was developed using the finite element method. The realistic description of the behaviour of masonry subjected to out-of-plane biaxial bending and the implementation in a numerical model based on the finite-element method, allows the load-bearing capacity of walls with different support conditions and all types of masonry to be determined. In addition, the simplified design model and equation presented here may be used in practise for the design of non-load bearing walls. The design equation takes into consideration the support conditions, wall dimensions and the ratio of flexural tensile strength perpendicular and parallel to the bed joint.

Table 1: Coefficients for determination of the normalized load-bearing capacity Y_w [1)] of non-load bearing masonry walls with aspect ratio of $0.3 \le \lambda_W \le 2.5$

Support conditions[2)]	Coe.	Ratio of flexural tensile strength $\mu_t = f_{tk1} / f_{tk2}$						
		0.2	0.4	0.6	0.8	1.0	1.2	1.4
	a	0,5337	0,4467	0,7524	1,4863	0,9392	0,8881	1,6164
	b	0,9440	1,4254	1,2080	-0,8152	1,3200	1,4823	1,1636
	c	0,1946	0,7997	1,0550	0,7139	1,8450	2,2124	1,7044
	d	2,5141	1,1673	1,0030	-1,4149	0,7119	0,6420	0,8594
	a	0,2427	0,3485	0,3043	0,3892	0,3292	0,3617	0,4554
	b	1,5190	1,5818	1,8700	1,8380	2,0540	2,0680	1,9620
	c	0,8185	1,5869	2,4640	2,9670	3,6570	4,1480	4,5240
	d	1,1790	1,2991	1,0400	0,9542	0,7287	0,6046	0,5324
	a	0,1450	0,3980					
	b	1,2630	0,9160					
	c	0,7224	1,2602					
	d	1,6020	1,1004					
	a	0,2517	0,1095	0,3990	1,5590	1,8390	2,2740	2,3810
	b	0,8605	1,4700	0,6452	0,0335	0,0574	0,0002	0,0211
	c	1,3370	2,4420	2,4440	1,3610	1,2110	0,8095	0,7672
	d	1,4920	0,8955	0,8431	1,1790	1,2380	1,5190	1,5550
	a	0,4359	0,6307	0,9430	1,2399	1,4570	1,7710	1,9220
	b	1,1070	1,1740	1,0892	1,0571	1,0570	1,0490	1,0740
	c	0,1010	0,1863	0,2091	0,1634	0,2760	0,1989	0,4251
	d	0,8289	0,8457	1,1658	1,4631	1,0980	1,6120	1,0930
	a	0,6544	0,5999	0,7928	1,2030	1,4160	1,6470	1,7790
	b	0,9458	1,1670	1,2140	1,0820	1,0840	1,1060	1,1320
	c	0,0074	0,4059	0,4592	0,2564	0,3982	0,4131	0,6863
	d	4,5440	0,6474	0,9412	1,6140	1,4960	1,4330	1,0820

1) Calculation of normalized load-bearing capacity: $Y_W = \frac{a}{\lambda_w^b} + c \cdot \lambda_w^d$ with $\lambda_w = \frac{H}{L}$

2) ——— simply supported ////// fixed supported — — - no support

No flexural tensile strength f_{tk1} is provided on the bottom of the wall, but a partial restraint is considered as a result of tilting of the brick

Calculation of the design lateral strength: $q_{Rd} = \frac{t^2}{H \cdot L} \cdot \frac{f_{tk2}}{\gamma_M} \cdot Y_w$

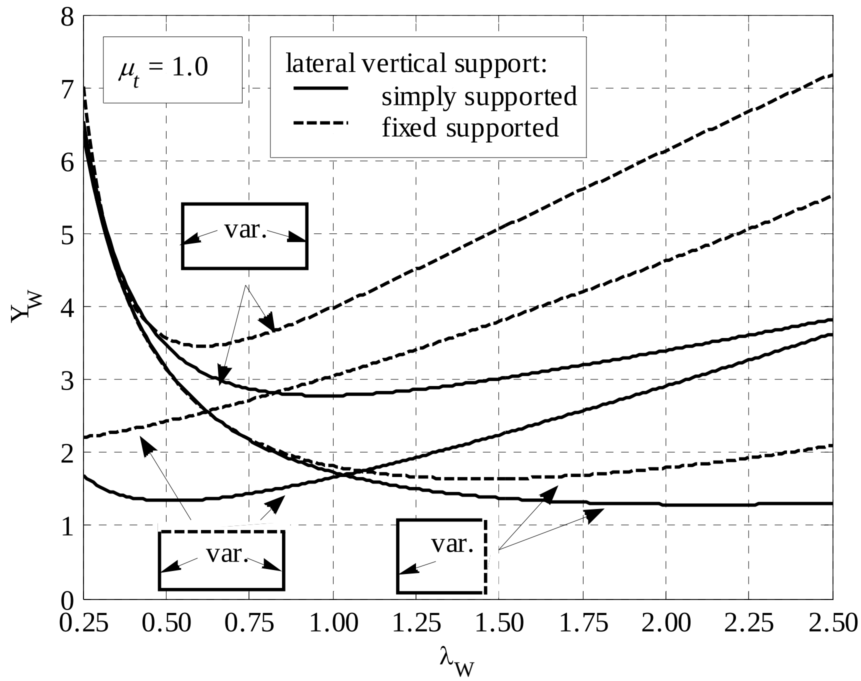


Figure 9: Normalized load-bearing capacity Y_w for different support conditions of wall panels

REFERENCES

1. AS 3700-2001 (2001) "Australian Standard – Masonry Structures" Sydney, Australia.
2. BS 5628-1 (2005) "Code of Practice for Use of Masonry – Part 1: Structural use of unreinforced Masonry" London, England.
3. CAN-CSA S304.1-04 (2004) "Masonry Design for Buildings" Mississauga, Ontario, Canada.
4. Eurocode 6-1 (2006) "Eurocode 6: Bemessung und Konstruktion von Mauerwerksbauten – Teil 1: Allgemeine Regeln für bewehrtes und unbewehrtes Mauerwerk. EN 1996-1-1:2006-01 – Deutsche Fassung EN 1996-1-1:2005" Berlin, Germany.
5. Willis, C. R. (2004) "Design of Unreinforced Masonry Walls for Out-of-plane Loading" Ph.D. Thesis, School of Civil Engineering, The University of Adelaide.
6. Jaeger, W. (2007) "Bemessung von drei- oder vierseitig gehaltenen, flächenbelasteten Mauerwerkswänden“ Mauerwerk-Kalender, 32 (2007), page 273-328, Berlin: Ernst & Sohn.
7. Lawrence, S. J. & Marshall, R. J. (1998) "The New AS 3700 Approach to Lateral Load Design" Proceedings of the 5th Australasian Masonry Conference, Gladstone, page 227-237.
8. Richter, L. (2009) "Tragfähigkeit nichttragender Wände aus Mauerwerk - Ein nichtlineares Berechnungsverfahren und Bemessungsmodell für biegebeanspruchte Innen- und Außenwänden" Dissertation, Institut für Massivbau, Technische Universität Darmstadt, Germany.