NON-DIMENSIONAL MASONRY FAILURE CRITERION UNDER BIAXIAL STRESS STATE

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ABSTRACT

The masonry under biaxial stress state is the most common case of walls subjected to complex systems of in-plane loads. Masonry and especially brickwork is a material, which exhibits distinct directional properties because of its anisotropic nature where the mortar joints act as planes of weakness. Taking into account the numerous uncertainties of the problem, an analytical mathematical model describing the masonry failure surface in a simple manner should be an efficient tool for the investigation of the behaviour of masonry structures. To define failure under biaxial stress, a three-dimensional surface in terms of the two normal stresses and the shear stress (or the two principal stresses and their orientation to the bed joints) is required. This paper describes a method to define a general non-dimensional anisotropic (orthotropic) failure surface of masonry under biaxial stress, using a cubic tensor polynomial. The evaluation of strength parameters is performed using existing experimental data via a least squares approach. The derived failure surface is shown to be in good agreement with classical experimental results.

KEYWORDS: anisotropy, failure criterion, failure surface, masonry, cubic tensor polynomial.

INTRODUCTION

For the purpose of masonry characterization, analysis, and design, an operationally simple strength criterion is essential. Masonry, one of the older structural materials, has a mechanical behaviour, which has not yet been fully investigated. Systematic experimental and/or analytical investigations on the response of masonry and its failure modes have been conducted in the last two decades.

There have been numerous analytical criteria for masonry structures [1-3]. The main disadvantage of existing criteria is that they ignore the distinct anisotropic nature of masonry; even if they do not ignore that, they consist of more than one type of surface leading to additional effort in the analysis process of the masonry structures [4]. According to Zienkiewicz, Valliappan and King [5] the computation of singular points ("corners") on failure surfaces may be avoided by a suitable choice of a continuous surface, which usually can represent, with a good degree of accuracy, the true condition.

Since reliable experimental data in the combined-stress state are emerging rapidly [6-8], it is, therefore, timely to examine the validity and utility of existing criteria, and to propose a failure surface of convex shape suitable for the anisotropic nature of masonry material. According to Hill [9] and Prager [10] the failure surface for a stable material must be convex. This, in mathematical terms, is valid if the total Gaussian curvature K of the failure surface is positive.

SHORT LITERATURE SURVEY

Masonry exhibits distinct directional properties due to the influence of the mortar joints. Depending upon the orientation of the joints to the stress directions, failure can occur in the joints alone or simultaneously in the joints and the blocks. The failure of masonry under uniaxial and biaxial stress state has been studied experimentally in the past by many researchers but there have been few attempts to obtain a general analytical failure criterion. The following is a brief review of the most representative experimental and analytical investigations.

Researchers have long been aware of the significance of the bed joint angle to the applied load. Johnson and Thompson [11] carried out compression tests on brick masonry discs to produce indirect tensile stresses on joints inclined at various angles to the vertical compressive load. Differences in failure patterns of the specimens were evident with the disc bed joints at various angles. The highest strength of the masonry has been observed for the cases when the compressive load was perpendicular to the bed joints or when the principal tensile stress at the centre of the disc was parallel to the bed joints. In this case failure occurred through bricks and perpendicular joints. The lowest strength has been observed when the compressive load was parallel to the bed joints or when the principal tensile stress at the centre of the disc was perpendicular to the bed joints. In this case failure occurred along the interface of brick and mortar joint.

The most complete experimental investigation of half scale clay brick masonry specimens under biaxial stress state was performed by Page [8]. A total of 180 panels, with five different bed joint orientations, were tested for a range of principal stress ratios. A minimum of four tests were performed for each combination of σ_1, σ_2 and θ. The results for each bed joint angle are illustrated in Figure 1. A few points are mentioned to highlight the nature of masonry. First, a large dispersion of mechanical characteristics of brick masonry can be seen, despite the fact that, according to the researchers, all the panels have been made by the same bricklayers and under the same environmental conditions. Second, the intense anisotropic nature of this masonry material is depicted. These results will be used in this paper as data to the development of the proposed analytical anisotropic failure criterion.

A failure surface for brick masonry, in the tension-tension principal stress region, has been derived analytically by Page [7]. The shape of this failure surface was found to be critically dependent on the bed joint orientation and the relationship between the shear and tensile bond strengths of the mortar joints.

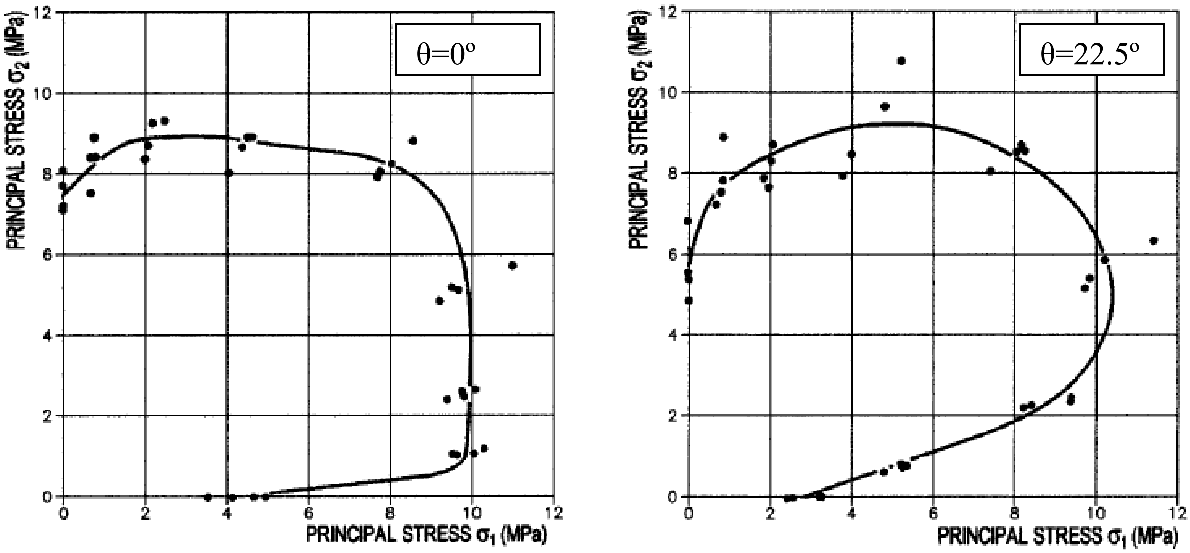

Figure 1: Failure Curves of Brick Masonry under Biaxial Compression in Terms of the Principal Stresses $\sigma_1, \sigma_2, \theta$ [Page 1981]

There have been few attempts to obtain a general failure criterion for masonry because of the difficulties in developing a representative biaxial test and the large number of tests involved. The problem has been discussed by Yokel and Fatal [12], with reference to the failure of shear walls. Dhanasekar et al. [1] interpolated the test data of Page [8] by means of three elliptic cones, which, however, as the authors mentioned, do not correspond with the observed distinct modes of failure. The elliptic cones have been expressed by a second order tensor polynomial. A wide review of the subject can be found in Samarasinghe [6] and Hendry [13].

More recently, Bortolotti et al. [3] using previous discussed experimental results by Page [8], have proposed a generalized failure criterion in the form of an elliptical curve valid for masonry and concrete but only for the case of biaxial compression state.

THE ANALYTICAL MODEL

For the expression of an analytical failure model of masonry, a tensor polynomial has been proposed. This failure surface, in the stress space, can be described by the equation [14-17]:

$$\left(\sigma_\ell\right) = F_i\sigma_i + F_{ij}\sigma_i\sigma_j + F_{ijk}\sigma_i\sigma_j\sigma_k + \cdots - 1 = 0 \qquad (1)$$

In this equation σ_ℓ ($\ell = 1, 2, \ldots, 6$) are the components of stresses and F_i, F_{ij}, F_{ijk} (i, j, k = 1, 2,..., 6) are strength tensors of the second, fourth and sixth rank, respectively.

If one restricts the analysis to a plane stress state and considers that a cubic formulation is a reasonably accurate representation of the failure surface -with suitable assumptions mainly based on the symmetry and orthotropic nature that presents the material of brick masonry [17-18]- and using the notations $\left(\sigma_x, \sigma_y, \tau\right)$ instead of $\left(\sigma_1, \sigma_2, \sigma_6\right)$, Equation 1 reduces to:

$$f(\sigma_x, \sigma_y, \tau) = F_1\sigma_x + F_2\sigma_y + F_{11}\sigma_x^2 + F_{22}\sigma_y^2 + F_{66}\tau^2 + 2F_{12}\sigma_x\sigma_y + 3F_{112}\sigma_x^2\sigma_y + \\ + 3F_{122}\sigma_x\sigma_y^2 + 3F_{166}\sigma_x\tau^2 + 3F_{266}\sigma_y\tau^2 - 1 = 0 \quad (2)$$

EVALUATION OF THE STRENGTH PARAMETERS

The determination of the strength parameters is made in two steps. In the first step, as it is presented extensively and in depth immediately below, the determination of Principal Strength Tensor Components (F_i, F_{ii}) are made, followed, in the second stage, by the determination of the Interaction Strength Tensor Components (F_{ij}, F_{ijk}).

First Step: The Principal Strength Tensor Components F_i and F_{ii}, could be determined using the experimental monoaxial tensile and compressive failure stresses across the axis x and y, respectively, as well as the shear failure stresses in the plane xy. Monoaxial strengths of the wall in tension and compression across the x axis are used, noted as X and X′ respectively. For the case of masonry, the two points (X, 0, 0) and (-X′, 0, 0) intersecting the axis x with the failure surface, are determined. For these points, Equation 2 takes the form:

$$F_1 X + F_{11} X^2 = 1, \qquad - F_1 X' + F_{11} X'^2 = 1 \quad (3)$$

The solution of the system of Equations 3 are giving the values:

$$F_1 = \frac{1}{X} - \frac{1}{X'}, \qquad F_{11} = \frac{1}{XX'} \quad (4)$$

The monoaxial tests across the y axis, lead respectively to the values of:

$$F_2 = \frac{1}{Y} - \frac{1}{Y'}, \qquad F_{22} = \frac{1}{YY'} \quad (5)$$

The points of failure surface (0, 0, S) and (0, 0, -S) are determined by the test of the masonry panel in pure shear. Using Equation 2 it results for these points that:

$$F_{66} = \frac{1}{S^2} \quad (6)$$

Second Step: In order to define the masonry failure criterion under biaxial stress state (Equation 2) the values of the Interaction Strength Tensor Components F_{12}, F_{112}, F_{122}, F_{166} and F_{266} have to be determined, using the least squares method. These constants are calculated through the system of equations:

$$\frac{\partial E_v}{\partial F_{12}} = 0, \quad \frac{\partial E_v}{\partial F_{112}} = 0, \quad \frac{\partial E_v}{\partial F_{122}} = 0, \quad \frac{\partial E_v}{\partial F_{166}} = 0, \quad \frac{\partial E_v}{\partial F_{266}} = 0 \quad (7)$$

where:

$$E_\nu = \sum_{i=1}^{\nu}\Big(F_1\sigma_{xi} + F_2\sigma_{yi} + F_{11}\sigma_{xi}^2 + F_{22}\sigma_{yi}^2 + F_{66}\tau_i^2 + 2F_{12}\sigma_{xi}\sigma_{yi} + + 3F_{112}\sigma_{xi}^2\sigma_{yi} + 3F_{166}\sigma_{xi}\tau_i^2 + + 3F_{266}\sigma_{yi}\tau_i^2 - 1\Big)^2 \qquad (8)$$

The equations to be used for ν groups of values ($\sigma_{xi}, \sigma_{yi}, \tau_i$) (i = 1, 2,..., ν), properly chosen, can be also written in the form:

$$\begin{bmatrix} 8S_{220} & 12S_{320} & 12S_{230} & 12S_{212} & 12S_{122} \\ 12S_{320} & 18S_{420} & 18S_{330} & 18S_{312} & 18S_{222} \\ 12S_{230} & 18S_{330} & 18S_{240} & 18S_{222} & 18S_{132} \\ 12S_{212} & 18S_{312} & 18S_{222} & 18S_{204} & 18S_{114} \\ 12S_{122} & 18S_{222} & 18S_{132} & 18S_{114} & 18S_{024} \end{bmatrix} \times \begin{bmatrix} F_{12} \\ F_{112} \\ F_{122} \\ F_{166} \\ F_{266} \end{bmatrix} = \begin{bmatrix} -4\sum_{i=1}^{\nu}\sigma_{xi}\sigma_{yi}A_i \\ -6\sum_{i=1}^{\nu}\sigma_{xi}^2\sigma_{yi}A_i \\ -6\sum_{i=1}^{\nu}\sigma_{xi}\sigma_{yi}^2A_i \\ -6\sum_{i=1}^{\nu}\sigma_{xi}\tau_i^2A_i \\ -6\sum_{i=1}^{\nu}\sigma_{yi}\tau_i^2A_i \end{bmatrix} \qquad (9)$$

where:

$$S_{jkl} = \sum_{i=1}^{\nu}\sigma_{xi}^{j}\sigma_{yi}^{k}\tau_i^{l}, \; (j, k, l=0,1,2,3,4)$$

and:

$$A_i = F_1\sigma_{xi} + F_2\sigma_{yi} + F_{11}\sigma_{xi}^2 + F_{22}\sigma_{yi}^2 + F_{66}\tau_i^2 - 1$$

The surface corresponding to these values F_{12}, F_{112}, F_{122}, F_{166} and F_{266} should be checked for its closed form and convex shape. According to Hill [9] and Prager [10] the failure surface for a stable material must be convex. The surface is closed if the total Gaussian curvature K of the failure surface:

$$K = -\frac{1}{\left(\partial f/\partial\sigma_x\right)^2 + \left(\partial f/\partial\sigma_y\right)^2 + \left(\partial f/\partial\tau\right)^2} D > 0 \qquad (10)$$

is positive [19-20], or, as the denominator is always positive if:

$$D = \begin{vmatrix} \partial^2 f/\partial\sigma_x^2 & \partial^2 f/\partial\sigma_x\partial\sigma_y & \partial^2 f/\partial\sigma_x\partial\tau & \partial f/\partial\sigma_x \\ \partial^2 f/\partial\sigma_x\partial\sigma_y & \partial^2 f/\partial\sigma_y^2 & \partial^2 f/\partial\sigma_y\partial\tau & \partial f/\partial\sigma_y \\ \partial^2 f/\partial\sigma_x\partial\tau & \partial^2 f/\partial\sigma_y\partial\tau & \partial^2 f/\partial\tau^2 & \partial f/\partial\tau \\ \partial f/\partial\sigma_x & \partial f/\partial\sigma_y & \partial f/\partial\tau & 0 \end{vmatrix} \langle \ 0 \qquad (11)$$

If this condition is not fulfilled, i.e. the solution does not correspond to a closed failure surface, the areas of local minimum extremes have to be used. The limits of these areas are determined through a parametric investigation for any one of these five constants, e.g. for constant F_{12}. Using various values for F_{12} ($-\infty \leq F_{12} \leq +\infty$) the equivalent values for the other four constants are calculated [17].

Through various values of the function (8) are calculated, and verification of condition 11 is also checked for each step. This verification leads to the determination of the limits inside which a closed failure surface is secured. The five values of the constants F_{12}, F_{112}, F_{122}, F_{166} and F_{266} fulfilling the requirement of the closed failure surface and at the same time minimizing the value of the function 8, are selected as the solution of the problem.

APPLICATION

In the case presented, the method has been applied through a special-purpose computer program developed by the authors. With this program the failure surface is determined for a real case of a masonry material studied already experimentally [8]. This data has been used by many other researchers [1-3]. As discussed above the determination of the strength parameters is made in two steps.

First, the Principal Strength Tensor Components (F_i, F_{ii}) are estimated via Equations 4-6 using the experimental values of material monoaxial failure strength depicted in Table 1. The coefficients (F_i, F_{ii}) are given in Table 2.

Table 1: Monoaxial failure strengths for masonry material [8]

X (MPa)	X′ (MPa)	Y (MPa)	Y′ (MPa)	S = S′ (Mpa)
0,40	4,3625	0,10	7,555	0,40

Table 2: Principal Strength Tensor Components (F_i, F_{ii})

F_1 $(Mpa)^{-1}$	F_{11} $(Mpa)^{-2}$	F_2 $(Mpa)^{-1}$	F_{22} $(Mpa)^{-2}$	F_{66} $(MPa)^{-2}$
0.227E+01	0.573E+00	0.987E+01	0.132E+01	0.625E+01

Table 3: Data of Biaxial Tests

Test No	σ_x (MPa)	σ_y (MPa)	τ (MPa)
1	-0.727	-7.542	0.000
2	-0.727	-8.417	0.000
3	-2.272	-9.250	0.000
4	-2.181	-8.750	0.000
5	-4.545	-8.667	0.000
6	-7.909	-7.791	0.000
7	-8.818	-8.750	0.000
8	-9.454	-4.792	0.000
9	-9.590	-2.333	0.000
10	-11.273	-5.583	0.000
11	-9.272	-1.000	0.000
Note: These values have been estimated from graphs [8].			

Table 4: Data of Biaxial Tests

Test No	σ_x (MPa)	σ_y (MPa)	τ (MPa)
1	-4.181	-8.000	0.000
2	-9.909	-5.042	0.000
3	-8.308	-8.475	0.084
4	-4.555	-1.310	-1.622
5	-5.821	-5.821	3.571
6	-6.620	-6.620	2.120
7	-5.821	-5.821	-3.571
8	-6.620	-6.620	-2.120
9	-8.273	-8.475	-0.084
10	-5.227	-1.310	1.622
11	-4.181	-8.000	0.000
12	-9.909	-5.042	0.000
Note: These values have been estimated from graphs [8].			

Second, the determination of the Interaction Strength Tensor Components (F_{ij}, F_{ijk}) is made by the solution of Equation 9 using experimental data of Tables 3 and 4. The results corresponding to these data are F_{12}=-0.150 (Mpa)$^{-2}$, F_{112}=0.3195E-02 (Mpa)$^{-3}$, F_{122}=0.1045E-02 (Mpa)$^{-3}$, F_{166}=0.9466E-01 (Mpa)$^{-3}$, F_{266}=0.1563E+00 (MPa)$^{-3}$.

With the above-mentioned calculated values of the coefficients, the failure surface for the masonry is described by the equation:

$$2.27\sigma_x + 9.87\sigma_y + 0.573\sigma_x^2 + 1.32\sigma_y^2 + 6.25\tau^2 - 0.30\sigma_x\sigma_y + \\ + 0.009585\sigma_x^2\sigma_y + 0.003135\sigma_x\sigma_y^2 + 0.28398\sigma_x\tau^2 + 0.4689\sigma_y\tau^2 = 1 \qquad (12)$$

The main disadvantage of this anisotropic failure criterion is that it applies only to the specific masonry material that was studied by Page [8]. This disadvantage could be reversed if this criterion is expressed in a non-dimensional form, and, as such, can be applied more generally to a plethora of masonry materials. This can be achieved by dividing and multiplying (at the same time) each term in Equation 12 by one material monoaxial strength raised in the sum of the exponents of the variables σ_x, σ_y, τ (as appeared in each term). We select to use the uniaxial compressive strength Y' across the y-axis, which, in terms of the masonry material corresponds to the uniaxial compressive strength denoted with the symbol $f_{wc}^{90°}$.

Equation 12 can thus be recast as:

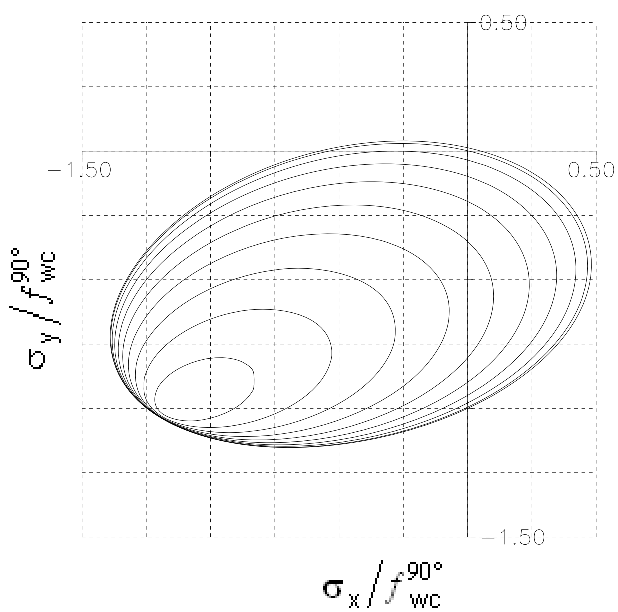

Figure 2: Non-Dimensional Failure Surface of Masonry in Normal Stress Terms ($\tau / f_{wc}^{90°}$ =0.00 up to 0.45 by step=0.05)

$$17.15\left(\frac{\sigma_x}{f_{wc}^{90°}}\right)+74.57\left(\frac{\sigma_y}{f_{wc}^{90°}}\right)+32.71\left(\frac{\sigma_x}{f_{wc}^{90°}}\right)^2+75.34\left(\frac{\sigma_y}{f_{wc}^{90°}}\right)^2+356.74\left(\frac{\tau}{f_{wc}^{90°}}\right)^2-17.12\left(\frac{\sigma_x}{f_{wc}^{90°}}\right)\left(\frac{\sigma_y}{f_{wc}^{90°}}\right)+$$
$$+4.13\left(\frac{\sigma_x}{f_{wc}^{90°}}\right)^2\left(\frac{\sigma_y}{f_{wc}^{90°}}\right)+1.35\left(\frac{\sigma_x}{f_{wc}^{90°}}\right)\left(\frac{\sigma_y}{f_{wc}^{90°}}\right)^2+122.46\left(\frac{\sigma_x}{f_{wc}^{90°}}\right)\left(\frac{\tau}{f_{wc}^{90°}}\right)^2+202.20\left(\frac{\sigma_y}{f_{wc}^{90°}}\right)\left(\frac{\tau}{f_{wc}^{90°}}\right)^2=1 \quad (13)$$

Figure 2 depicts the contour map of Equation 13, that is the non-dimensional failure surface of masonry in normal stress terms (with $\tau / f_{wc}^{90°}$ taking values of 0 up to 0.45 by steps of 0.05).

The validity of the method is demonstrated by comparing the derived analytical failure surface of Equation 13 with existing experimental results [8]. Figures 3 and 4 present both the analytical curves for the failure surface of Equation 13 and the superimposed more than 75 experimental data-points. The good agreement of the experimental and analytical data can be seen.

CONCLUSIONS

In this paper, a non-dimensional anisotropic masonry failure criterion under biaxial stress state is presented. Main advantages of this criterion constitute: a) the ability to ensure the closed and convex shape of the failure surface (the failure surface for a stable material must be convex), b) the expression of the failure by a *single*, general-purpose surface, which can be used for all possible combinations of plane stress in order to make easier its inclusion to existing software for the non-linear analysis of masonry structures, c) the good agreement of the proposed criterion with the results of the real masonry behaviour (experimental data) under failure conditions.

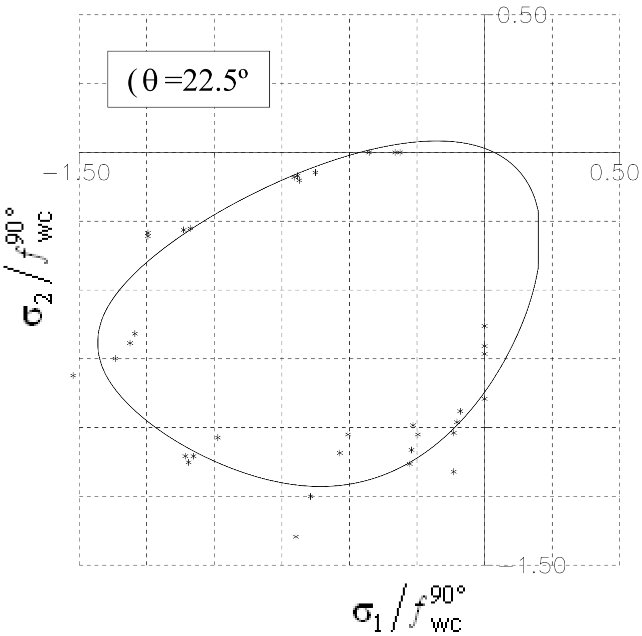

Figure 3: Non-Dimensional Failure Curve of Masonry in Principal Stress Terms

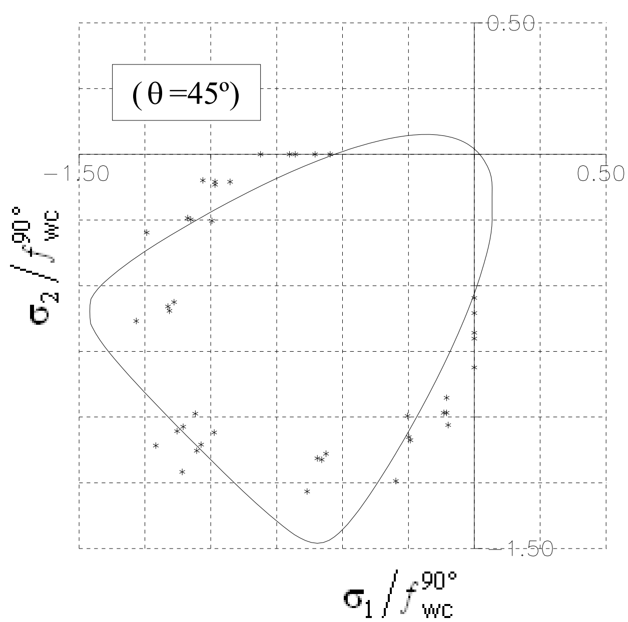

Figure 4: Non-Dimensional Failure Curve of Masonry in Principal Stress Terms

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