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EXAMINING THE SIGNIFICANCE OF MORTAR AND BRICK UNIT PROPERTIES ON MASONRY BOND STRENGTH USING BAYESIAN MODEL SCREENING

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ABSTRACT

Flexural bond strength of masonry is influenced by many factors such as the masonry mortar and brick units, the construction environment and the workmanship. This complex relationship has been confirmed by research work over the last three decades. What hasn't been shown is the amount of influence of these factors. Here we examine the use of a Bayesian model screening with Markov Chain Monte Carlo sampling approach to establish a ranking of significance of some of these factors, namely the mortar and the brick unit properties on bond strength of masonry. Experimental data from flexural bond strength tests performed on five-high masonry prisms cured in the laboratory (RH ≈ 20%) and in a fog room (RH =100%) are used. The experiments determined bond strength for four types of mortars with four different types of brick units creating an experimental database of 96 data tuples. This experimental database represents the complex relationship. This relationship is fitted to a number of parametric models. The best fitting model is the one that also gives a probabilistic significance ordering of the mortar and the brick unit properties on masonry bond strength

KEYWORDS: masonry bond, Bayesian model screening, Gibbs sampling

INTRODUCTION

One of the inherent limitations of masonry as a construction material is its weak bond strength that creates predefined failure planes in the structure. Several research investigations targeted enhancing masonry bond strength for example [1, 2]. However, the common challenge that faced all investigations was the fact that masonry bond strength is dependent on many interrelated parameters that can directly affect bond development (e.g. unit surface absorption, pore structure, mortar composition, mortar water retentivity and curing conditions) or indirectly affect bond strength (e.g. unit surface texture and workmanship) [3, 4, 5]. While many research investigations claimed that water absorption criteria of the brick unit might have a significant effect on developing good bond strength [6, 7], the effect of mortar quality on bond strength development has also been proven to be significant [8, 9, 10].

It is evident that the interaction between the unit surface absorption criteria and the mortar quality determines the level of competence between the two physical processes which control bond development at the unit interface: densification and dewatering [11, 12]. Lange et al. [12] explained that densification represents the consolidation of the hydration products at the mortar-unit interface, while dewatering represents the reduction of the water available to complete the hydration process at the interface as a result of unit suction. As both processes are proven to take place at the unit interface, two questions arise: which of the two processes are more responsible for the development of bond strength?; and what would affect the balance between water demand and water supply at the mortar-unit interface? These two questions lead to another important question: what constitutes the main components that are responsible for the bond strength?

While the experiments by Kampf [13] in 1963 proved that masonry bond can be fundamentally attributed to mechanical bond rather than adhesion bond, it did not report the main contributors to this mechanical bond. A few decades later, microstructural investigations of the unit-mortar interface were developed to answer this question. These investigations showed that there is a much smaller transition zone in masonry (50 μm) compared to concrete (200 μm) [12, 14]. It has also been reported that although the loose packing of the mortar particles at the unit vicinity (known as the wall effect) is much higher in the masonry interface than in concrete, the roughness of the masonry unit might counteract this wall effect [12, 15]. It was also observed that the unit mortar interface has a large content of tri-calcium- sulpho-aluminate ($3CaO.Al2O3.3CaSO4.32H2O$) known as "Ettringite" and CH crystals [16, 17]. The existence of Ettringite at later ages has been attributed to the absence of enough water for the hydration process to continue as a result of unit suction [17]. While Sugo et al. [18] and Reda and Shrive [17] showed using x-ray diffraction analysis (XRDA) and scanning electron microscopy (SEM) investigations that there is a considerable amount of CH in the interface and that quantity is substantially increased when lime is included in the mix, Lange et al. [19] did not observe a significant volume of CH crystals at the interface.

There is also no agreement on what components contribute the most to masonry bond strength. While some researchers attributed the major bond strength to the development of a wide interwoven fibrous network of C-S-H growing at the unit surface [2], others believe that the effect of CH and C-S-H in masonry bond is negligible and that most of the bond is due to the network of Ettringite that connects the unit pores to the mortar [12]. On the other hand, from the water transport point of view, many researchers argued that there is a significant relationship between the initial rate of absorption (IRA) and bond strength [20]. However, recent research [7] showed that sorptivity as a measure of water absorption is a more reliable criterion for describing the status of water transport at the mortar-unit interface for a long time period than IRA. It thus becomes obvious that all available experimental observations did not answer the questions raised above.

Statistical analysis of experimental data investigated possible means to find the group of optimal parameters that significantly affect the bond strength [2, 3]. However, a major difficulty in performing such statistical tests is the significant scatter in the test results that hinder reaching consistent statistical conclusions. Previous work in masonry has shown that bond strength is affected by many parameters. Reda Taha and Shrive [21] investigated the influence of mortar mix with pozzolanic materials on flexural bond strength by testing five-high masonry prisms cured in the laboratory. Their analysis of variance (ANOVA) studied the variational effects on twenty-four tests comprised of six different mortars with four different brick units. They revealed that masonry bond strength was not simply due to the effect of pozzolans alone. Other factors, such as mortar water retentivity/brick water suction, affect the bond strength. Thus, even with statistical analysis, the question of how much these factors influence the bond strength remains unanswered.

This paper attempts to answer this question by identifying the main effects and linear interactions of variables that contribute to bond strength. Additionally, we show how much a particular parameter contributes probabilistically to a parametric model that describes the experimental output. In this paper we propose a method to indicate the most significant contributors to flexural bond strength using Bayesian model screening. We first describe the experiments performed to examine the bond strength of masonry. We then explain the Bayesian model screening approach and how it is applied to the masonry bond database in hand. The experimental results and the results of the analysis are then presented. The paper concludes with some discussions of our findings.

EXPERIMENTAL METHODS

The experimental data analyzed here are some of those discussed earlier by Reda Taha and Shrive [21]. The following is a brief description of the experiments. Masonry bond strength was examined by testing masonry prisms made of four types of mortar and four types of brick units under two curing regimes, at three time points up to a year creating an experimental database of 96 data tuples (i.e. combinations). Mortar mix proportions and properties of the different brick units are presented in Tables 1 and 2, respectively.

Table 1 - Mix proportions by volume of the four masonry mortar used in the experimental program

Mortar group	Portland Cement	Hydrated lime	Fly ash Type (F)	Sand
A	1	0.5	0	4.5
B	1	1	0	6
C	0.8	0.4	0.3	4.5
D	0.8	0.8	0.4	6

Table 2 – Brick types and properties

Group designation	Compressive strength (MPa)	IRA ($kg/m^2/min$)	Total Absorption%	Sorptivity index
1	43.9	3.67	8.11	2347
2	38.5	6.55	7.98	5781
3	59.2	2.40	8.32	1465
4	72.0	2.28	6.7	1216

Twenty-four, five-high stack bonded prisms were constructed from each mortar type and brick unit. Each mix was used for dry (20% RH) and moist (100% RH) cured samples at 20 ^oC. Four prisms were tested from each curing condition at each of 28, 180, and 360 days of age. The masonry bond strength was examined using a bond wrench test apparatus as described in Shrive and Tilleman [22]. The top half of the brick is gripped between two neoprene pads in a clamp and a torque wrench is attached to the clamp such that the centre-line of the torque wrench arm is centred over the brick. The method complies with the basic requirements of ASTM C1072-94 [23].

BAYESIAN MODEL SCREENING

Bayesian model screening [24, 25] is an approach for identifying the most probable polynomial model to describe a phenomenon and consequently can be used to recognize the most important effects to include for estimating the observed phenomenon. Bayesian model screening thus simply represents a supplementary procedure to the classical static model fitting of least squares approximation [26] that implements Bayes updating to develop posterior probabilities for both the models and model effects [24]. Bayesian model screening is implemented using a Markov Chain Monte Carlo (MCMC) algorithm which steps through a variety of models using Gibbs sampling (the one dimensional search version of MCMC) in a search for the most likely model to represent the response data describing the phenomenon of interest.

The process begins by assuming that a polynomial model can represent the masonry bond strength as a function of its main parameters having the general form as given in equation 1.

$$y = M(\boldsymbol{b}; x) \qquad \text{Equation 1}$$

This model represents a single iteration step in the MCMC procedure and represents the output response y as a function of an unspecified number of coefficients $\boldsymbol{b}$ and the parameters x. Specifically, this model is a linear combination of k effects as shown in Equation 2.

$$y = \sum_{k=1 \ldots N} x_k \boldsymbol{b}_k = x^T \boldsymbol{b} \qquad \text{Equation 2}$$

The coefficients $\boldsymbol{b}$ fit in a least squares sense the effects x to the original data y as in Equation 3.

$$\boldsymbol{b} = (X^T X)^{-1} X^T y \qquad \text{Equation 3}$$

where y is the vector of N observations, and the matrix X consists of the m effects corresponding to the N observations as

$$y = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{Bmatrix}; \qquad X = \begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,m} \\ x_{2,1} & x_{2,2} & x_{2,m} \\ & & \\ x_{N,1} & x_{N,2} & x_{N,m} \end{bmatrix} \qquad \text{Equation 4}$$

This first iteration of the MCMC process represents a static least square polynomial model fit to N data. However, the limitation of a traditional (static) least squares fit polynomial model is that a model must already be defined. In other words, all the parameters would be assumed to contribute equally to the output y implying that all parameters are equally important. This might not be the case and this is why traditional (static) least squares might not result in the best fit. Using the MCMC approach we recognize that not every parameter is important for the best approximating model. The discernment of important and unimportant parameters is made based on the performance of a particular model. Thus the process starts by assuming a series of models that can best fit the data tuples in hand using least squares. This series of models is searched based on their performance. When a model is selected, its ability to approximate the performance is evaluated as a likelihood function, Equation 5. Kerschen et al. [25] showed that the root mean square error (RMSE) for Gaussian probability distributions is related to a likelihood function, described here as

$$L(y \mid \beta) = e^{\sum_{k=1 \ldots N} (y_k - x_k^T \beta)^2} \qquad \text{Equation 5}$$

This likelihood of a specific model is then used to determine the model posterior probability $P(\beta \mid y)$ using Bayes Theorem as follows,

$$P(\beta \mid y) = \frac{L(y \mid \beta) P(\beta)}{P(y)} \qquad \text{Equation 6}$$

Bayes Theorem states that the posterior probability $P(\beta \mid y)$ is equal to the product of the likelihood $L(y \mid \beta)$ and the prior probability $P(\beta)$ divided by the probability of the observation response $P(y)$. Here, the probability of the response data, i.e., P(y), is constant and not used in Bayes updating. If the resulting likelihood is acceptable, the posterior probability is computed and the model coefficients and its posterior probability are stored. The parameters are represented in each model by random sampling using their assumed probability distributions. Each model is generated and evaluated based on Gibbs sampling [27]. Gibbs sampling is a search technique consisting of one dimensional moves in the model space to locate a closely approximating model. This procedure iterates until all the models are evaluated and all the samples are drawn. When this occurs, the model with the highest posterior probability is selected as the most appropriate approximating model. As such, the marginal probabilities for the contributing parameters are then determined based on their frequency of use across all the models.

Here, the Bayesian model screening approach using the MCMC algorithm is applied to examine the influence of eleven parameters on the bond strength of masonry. The list of these parameters is presented in Table 3. The eleven parameters were divided into three groups; Group I including x_1 to x_4 and representing mortar parameters, Group II including x_5 to x_8 representing construction parameters, and Group III including x_9 to x_{11} representing brick unit parameters. While all the material parameters listed in Table 3 are discussed above and their values are listed in Tables 1 and 2, two additional parameters were considered in the investigation. These additional parameters include $x_{5,}$ the workability index and $x_{7,}$ the humidity index. The workability index was considered to represent the workability of the mortar mix and was chosen based on the mortar consistency and workability during construction of the prisms. The workability index ranged from 1.00 to 4.00 with 1.00 indicating a non-workable mix and 4.00 indicating a very workable mix. The last parameter is the curing index which was considered equal to the relative humidity ranging from 0.0 for zero percent humidity and 1.0 for 100% humidity. The database included 96 data-tuples with each data-tuple consisting of 11 input parameters and the bond strength.

Table 3 – List of parameters used in the investigation

Group	Parameter Group	Parameter	Symbol
I	Mortar parameters	x_1	Portland cement volume
		x_2	Lime volume
		x_3	Supplementary cementing material volume
		x_4	Sand volume
II	Construction parameters	x_5	Workability index
		x_6	Masonry age (time from construction)
		x_7	Curing index
III	Brick unit parameters	x_8	Compressive strength
		x_9	Initial rate of absorption (IRA)
		x_{10}	Total absorption
		x_{11}	Sorptivity
IV	Bond strength	**Y**	Bond Strength of Masonry

RESULTS

Experimental Results

The experimental results of the bond strength experiments are summarized in Table 4. Sample representation of the bond strength development with time for mortar groups A and D are shown for pictorial representation in Figure 1 (a) and (b), respectively.

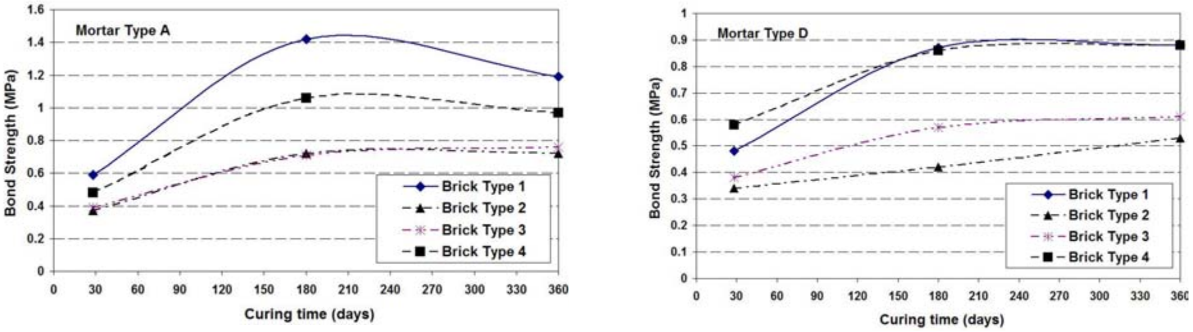
Table 4 – Bond strength results for all brick unit types and all mortar types

Mortar Mix	**Age, days**	**Brick Type**							
		1		**2**		**3**		**4**	
		Wet	**Dry**	**Wet**	**Dry**	**Wet**	**Dry**	**Wet**	**Dry**
A	28	0.59	0.47	0.37	0.35	0.39	0.46	0.48	0.45
	180	1.42	1.16	0.72	0.6	0.71	0.59	1.06	0.68
	360	1.19	0.94	0.72	0.55	0.76	0.59	0.97	0.67
B	28	0.67	0.86	0.43	0.43	0.39	0.34	0.6	0.5
	180	1.16	0.94	0.89	0.49	0.72	0.4	0.97	0.64
	360	1.2	0.95	1.01	0.55	0.78	0.38	0.99	0.69
C	28	0.63	0.62	0.42	0.36	0.44	0.33	0.59	0.51
	180	0.88	0.66	0.7	0.39	0.76	0.38	0.78	0.7
	360	0.96	0.75	0.81	0.42	0.78	0.45	0.82	0.68
D	28	0.48	0.59	0.34	0.26	0.38	0.32	0.58	0.47
	180	0.87	0.65	0.42	0.38	0.57	0.51	0.86	0.58
	360	0.88	0.67	0.53	0.36	0.61	0.47	0.88	0.58

Each value in the table represents the mean of sixteen values. Some joints broke before testing, and in some cases, the brick broke rather than the interface.

Analytical Results

An MCMC analysis was performed on the 11 masonry parameters listed in Table 3. This analysis examined models that included linear parameters with linear interaction totalling 11 linear parameters plus 55 linear interactions. The prior probability for each parameter x_i was set to be 0.25 while the prior probability for each interaction $x_i x_j$ was set to be 0.1. The interaction effect $x_i x_j$ was reduced to 0.01 when both parameters were not included as linear parameters in the model. Table 5 presents the resulting coefficients for the highest scoring approximating model with the parameters' marginal probabilities. This result showed that out of 11 parameters examined, only three parameters can be identified as significant for modelling masonry bond strength. In addition, the interaction between three other parameters representing the brick unit properties also has a significant influence on masonry bond strength. The probabilities associated with linear and interacting parameters were all found to be greater than or equal to 0.895.



(a) **(b)**

Figure 1 – Bond strength development with time for (a) Mortar type A; (b) mortar type D

Table 5 - MCMC results presenting the main linear parameters and interactions for approximating masonry bond strength

Model			Model	
Sym	***Description***	***Prior***	***C***	***P***
x_1	Portland Cement volume	0.25	0.0649	0.895
x_6	Masonry age	0.25	0.1289	1.00
x_7	Curing index	0.25	0.0873	1.00
x_6 x_7	Masonry age ***and*** Curing index	0.1	0.0873	0.950
x_8 x_{10}	Compressive strength ***and*** total absorption	0.1	-0.5267	1.00
x_8 x_{11}	Compressive strength ***and*** sorptivity	0.1	-0.3951	1.00
-	Intercept	-	0.6451	---
RMSE			***0.23***	

**** C denotes (Coefficient) and P denotes (Marginal Probability)***

DISCUSSIONS

The marginal probability (*P*) results given in Table 5 showed that the most significant factors influencing masonry bond strength are the Portland cement volume in the mortar mix, the age of the masonry, and the curing regime. Moreover, three linear interactions between the parameter pairs (masonry age and curing regime, compressive strength and total absorption, and compressive strength and sorptivity) seem to be important interactions for developing bond strength. The results confirm the finding by many researchers that masonry bond strength is a result of interaction of the brick unit characteristics and the mortar quality. While it became obvious that the Portland cement volume is the most important parameter representing the mortar quality, the method used to describe the mortar constituents might play a role in coming to this conclusion. Mortar constituent materials were represented in the analysis by considering their volume ratio while their effect on the bond might be dependent on their chemical composition. Thus, different results might be reached if the mortar is described by its chemical compositions rather than constituents. This might be the only way to represent the fact that incorporating pozzolanic materials increases the cementing material content in the mortar mix, or that lime increases the Calcium Hydroxide (CH) content in the mix. Further analysis in this direction is being examined.

Moreover, the analysis proved the fact that the effects of curing and masonry age (especially with pozzolans included in the mortar) are significant. Shrive et al. [21] came to similar conclusions by applying the ANOVA on masonry mortars including fly ash. The analysis not only showed that both masonry age and curing system are significant as separate effects, but also their interaction is significant. This finding meets many experimental and microstructural observations by numerous researchers [3, 17]. It is becoming evident that humid curing conditions and long curing periods are necessary to develop good bond and high bond strength of masonry. However, such findings also shed light on current masonry construction practices where water curing of masonry construction is not mandatory by most design codes. It is the authors' recommendation that water curing of masonry structures should be mandatory, especially for structural masonry.

Finally, the analysis revealed very interesting and controversial results showing that the two major parameters to represent the brick unit parameters are the compressive strength of the unit and its absorption criteria represented by either total absorption or sorptivity. It had been argued by many researchers that the compressive strength of the brick unit might not be the best parameter to relate directly to masonry bond strength as there is no direct relationship between the compressive strength and the mechanism by which bond develops at the interface [3, 7, 12]. The analysis showed that the interaction between the compressive strength and the water absorption criteria of the brick unit has a strong influence on masonry bond strength. However, it is worth noting that no other parameters which are typically used to describe the mechanical and fracture properties of the brick units were used in our analysis. If such data are available, other conclusions might be reached. It is also worth noting that, out of the four brick unit types, three units were extruded while one unit type was pressed. A study of further variations in the brick unit type might also be necessary before coming to general conclusions.

It is also interesting to note that our analysis did not show any sensitivity of the bond strength to IRA as a parameter to describe the water absorption criteria of brick units. On the contrary our results showed that either total absorption or sorptivity has equal opportunity in representing the water absorption criteria of brick units. These results confirm findings in early studies by Voss [28] which recommended the use of 48 hour total absorption time to represent the water absorption of masonry units. Moreover, Anderegg [29] showed that the rate of absorption rather than the total absorption is better used to describe water absorption of masonry units. Recent studies by Reda Taha et al. [7] concluded that sorptivity is a much more reliable parameter in describing water absorption of masonry units compared to either IRA or total absorption.

CONCLUSIONS

A Bayesian screening model was applied using MCMC to examine the most significant parameters affecting masonry bond strength. Eleven parameters describing the mortar, the brick units, and the construction process were used in the analysis. It is concluded that Portland cement volume, curing, and masonry age are the most important parameters affecting masonry bond strength. Evidence also shows that the interaction between mechanical properties of the brick units i.e. compressive strength and their water absorption criteria represented by the (total absorption or sorptivity) also has a strong influence on masonry bond strength and bond development.

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