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A FINITE ELEMENT APPROACH TO CALCULATE TWO ANCIENT MASONRY VAULTS

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ABSTRACT

This paper reports on the finite element analyses of two ancient groined masonry vaults. The computations were part of an extensive study aimed at the reconstruction and retrofitting of the vaults that cover the transept of the church of Our Ladies Monastery in the city of Magdeburg, Germany. Both linear elastic and nonlinear analyses were performed to clarify the influence of several load impacts on the structural behaviour, with the sideward slope of the abutments ultimately found to be most responsible for weakening the structure. Based on the calculated results, appropriate reconstruction methods were proposed.

KEYWORDS: vault, thrust, ancient masonry, cracks, bricks, sagging

CURRENT SITUATION

Introduction

The Our Ladies Monastery in the city of Magdeburg is considered to be one of the most important Romanesque religious structures in Germany. The buildings at the site date back to the Middle Ages, between 1064 and 1200. The central part of the monastery is the church, which is built as a Christian cruciform consisting of nave and aisles with a clerestory and a large, high transept (Figure 1).

During WWII, the church was damaged by bombing, leading, e.g. to the total destruction of the vaulted ceiling above the choir. Shortly after the war and again in the 1970s, several efforts were made to repair the church. Cracks were coated with plaster, and the locally spalled ribs were remolded with gypsum plaster. The choir was not vaulted again, but was given instead a light wooden roof that has proven too weak to provide sufficiently stiff lateral support for the central transeptal vault.

Now, more than 30 years after the last repairs, the damaged condition of the remaining vaults has again become evident [1], and questions about the vaults' stability have been raised. Thus, museum tours and concert events have been restricted to a minimum, and parts of the transept have been closed to the public since spring 2003. To clarify the situation, the city chose to bring in consulting engineers to investigate the actual state of the structure in detail. Taking into account all cracks and imperfections that have been discovered so far, all efforts have been employed in computing the exact state of the vaults for the development of durable, qualified rehabilitation methods for maintaining the character of this building to avoid further operation under additional restrictions.

a) b)

Figure 1 – Church in Our Ladies Monastery
a) west-side portal, b) south side

The Vaults' Structure

The transeptal part of the cruciform crossing the nave and choir is subdivided into three parts: a main quadrangle of 8 by 8 meters (26 by 26 ft.) that is confined by northern and southern side halls with the same footprint (Figure 2). While the nave is roofed by a chain of smaller vaults, each transeptal quad is covered by its own ancient vaulted ceiling resting on arched masonry structures and exterior walls.

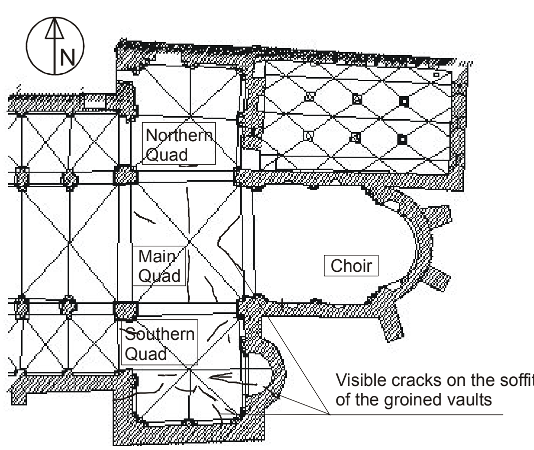


Figure 2 – Transept of the church displaying cracks on the inner surface of the vaults

The general shape of the vaults is like a cross vault, where stony ribs that cover the edges of the intersecting double-curved masonry caps meet at the apex to form their own firm arched structure. The masonry caps are approximately 40 cm (15.5 in.) thick and encapsulated by adjacent ribs and supporting masonry arches. The ribs are composed of sandstone and are principally positioned in a diagonal direction above the quad (cross ribs); in some cases, however, ribs parallel to the exterior walls were added in the southern hall (Figure 3).

a) b)

Figure 3 – Views of the soffits of the vaults
a) main quadrangle, b) southern side hall

The cross ribs measure approximately 17 meters (56 ft.) between the footing and the uppermost point and rise 3 meters (10 ft.) higher than the Romanesque and Gothic supporting arches. The ribs are works of art, 18 cm (7 in.) cantilevered on one side and 33 cm (13 in.) embedded into the masonry caps, enabling both caps and ribs to participate in the load-carrying process. At the diagonal tails, the masonry caps narrow and extend into the masonry piers, unifying caps, adjacent ribs, and piers at one point.

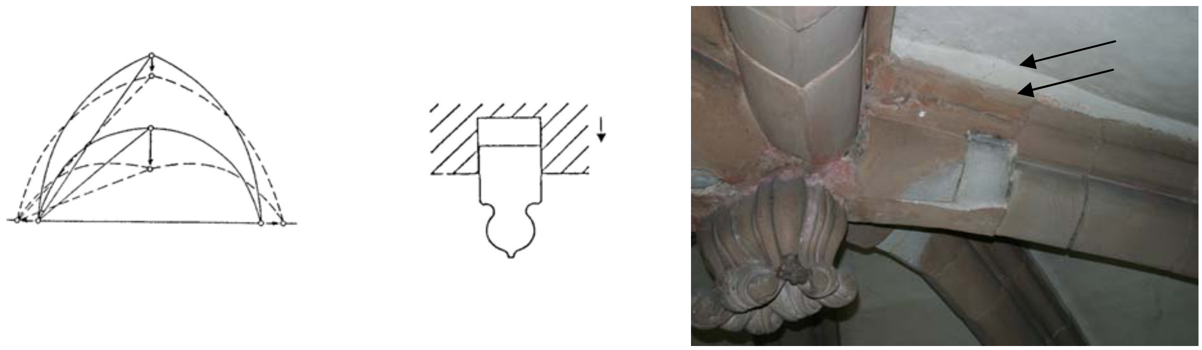
To begin the analysis, small material batches were tested. The compressive strength of the bricks was found to be 13.1 to 14.1 N/mm^2 (1.89 to 2.04 ksi), and that of the sandstone ribs between 34.3 and 36.2 N/mm^2 (4.9 and 5.2 ksi). The mortar was examined as well. Lime and sand were found to be the primary mortar constituents, providing moderate strength. According to the chemical analysis, both the bricks and mortar contain small traces of salts such as nitrate and sulfate, but no limiting factors were discovered regarding the choice of rebuilding mortar [2].

Current Condition of the Transeptal Vaults

Although the stability of the structures has been proved over the centuries, they were severely damaged by warfare during WWII. Heavy bombing not only destroyed the choir's vault, but also dramatically affected the abutment walls in the transept in general. Cracks up to 2 mm wide are visible, e.g., on the southern exterior wall.

The failure of the choir's dome also caused the loss of the horizontal support for the adjacent central vault on its east side. Even though the masonry arch on the east side bears the vault vertically, it offers almost no horizontal stiffness so that the vault's thrust is pushing it towards the side, giving it an oblique position. Furthermore, the central vault is forced to bridge a larger span since the opposite abutments remain fixed. The stiff vault is barely able to follow the arch without sagging at its apex. This situation, which is accompanied by severe bending, has caused typical cracks stemming from the ridge of the caps into the piers forming a triangular crack pattern as can be seen in Figure 2. By means of these cracks, the vaults have developed joints in order to deform freely like a statically determined arch. Pieper, who observed many flawed vaults and reported on such behaviour, derives a similar conclusion from his analyses [3].

Because of different rises between caps and ribs, the ribs with a lower rise tend to sag more than the caps as the apex is lowered. Indeed, measurements determined that the ribs have been pulled out of the masonry caps by more than 10 cm (4 in.) (Figure 4) despite the same horizontal shift. They are no longer fully mortared into the masonry and are in danger of falling down. The same potential destiny applies to the keystone at the southern apex.



a) b) c)

Figure 4 – Comparing the sagging mechanisms of ribs and caps
a) mechanism [3], b) pulling out the ribs [5], c) vault over the south side

The lowered apex also caused bending tensile stresses on the soffits, which opened once-mortared joints especially in the cantilevered ribs (Figure 5). Mortar crumbled out of the open joints. Toward the domes' base, the bending moments have changed their sign, giving rise to considerable compressive stresses on the inner surface. As a result, spallings have occurred at some cantilevered rib elements due to heavily concentrated compression forces.

a) b)

Figure 5 – Ribs in damaged condition
a) open cracks at the vertex, b) spalling of ribs due to concentrated compression

Surveying the Vaults

The age of the building of course precluded the existence of architectural drawings. Therefore, the interior surfaces of the hemispherical ceilings were measured through the use of a laser to provide a 3D net of points that created a very accurate base for a finite element analysis. Using a data grid of approximately 1 cm by 1 cm (1/2 in., squared), each vault was specified by more than 600,000 points.

FINITE ELEMENT ANALYSIS

Goals of the Analysis

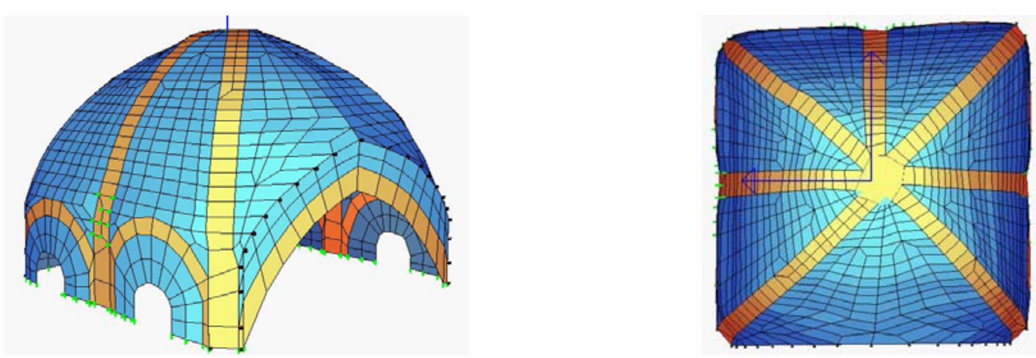
To stop the vaults' further deterioration, a static calculation based on a finite element analysis had to be done in order to infer suitable methods of preserving the ancient structures. Since controversy exists in the academic world about whether or not the masonry caps are involved in the load-carrying behavior of groined vaults [5], it was hoped that the computations could shed some light on this topic as well. Because the measured surfaces would indicate the existing deflection due to gravity loads, the first computational step was to create the vault's shape unaffected by any load impact. Using this initial step as the basis of computation, other impacts such as dead load, temperature, pier settlement, and horizontal support displacement were then added.

Shaping the FE-Model

Each of the two vaults was modeled and computed separately. Based on the 3D-data net, the analytical model adapted the actual shape of the vaults very precisely. Adjacent parts of walls, arches, and piers were adjoined to prevent keen shifts in element stiffness (Figure 6 and 7).

a) b)

Figure 6 – Vault over the main quadrangle
a) isometric view, b) ground view



a) b)

Figure 7 – Vault over the southern quadrangle
a) isometric view, b) ground view

Except at the supports, where the two vaults firmly contact adjacent vaults, the bearings were generally seen as elastic with their stiffness being recalculated from the deformation under a unit load. Modeling the east-side masonry arch in the central quad, it can be seen that it has no lateral restrictions. This can be determined by the fact that the finite element mesh automatically ensured participation in the load-carrying process. Only in one case, when the current state needed to be compared to the opposite case with fully restrained elements, was a laterally supported arch modeled.

The measured thickness of the caps was assumed to be constant at 40 cm (15.8 in.). Along the edges, where the ribs integrate in the caps' masonry, 70 cm (27.6 in.) thick elements were eccentrically embedded. Both types of elements were arranged as 4 node-quad elements using a multilayer technique. Ten layers in each element enabled each single layer to crack or to strain plastically without superseding the entire element. Compared to the radius of curvature, the caps are quite thin so that stresses perpendicular to the shell surface could principally be neglected and only the two-dimensional stress state needed further attention.

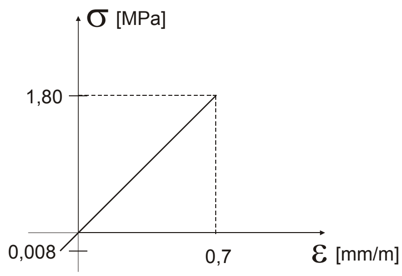
Material Model

Based on material test results, the strengths and stress-strain relationships, including tension cut-off at higher loads, were determined [2] and incorporated into the finite element analysis. A linear elastic behavior was assigned to the ancient brick masonry, while the stony ribs were modeled to be linear elastic-ideally plastic. A constant and quite conservative tensile strength of 0.008 N/mm² (1.16 psi) were assigned both materials. The Poisson's ratios and specific weights were laid out in correspondence to reference values [3]. Table 1 summarizes the single values being applied.

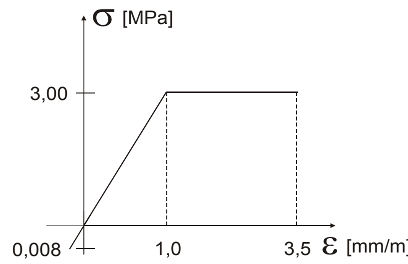
Table 1 – Material parameters applied (US standard measures in parentheses)

Material Parameters		Brick Masonry of Caps	Masonry of Stony Ribs
Compressive strength	N/mm²	1.80 (261 psi)	3.00 (435 psi)
Tensile strength	N/mm²	0.008 (1.16 psi)	0.008 (1.16 psi)
Elastic modulus	N/mm²	2,500 (362 ksi)	3,000 (435 ksi)
Poisson's ratio	--	0.25	0.20
Specific weight	kN/m³	20 (127 lb/ft³)	27 (172 lb/ft³)

Since the shells are actually subjected to multi-axial stresses, adequate strength values had to be taken into account. Despite differing from uniaxial test data, multi-axial strength values may be inferred from these data (Table 1). This inference was made automatically by the computer program in each load step according to the procedure by Kupfer [6].



a)



b)

Figure 8 – Simplified stress-strain relationships that were applied in the computation a) masonry of the caps, b) masonry of the stony ribs

Impacts and Combinations

Both groined masonry vault models were subjected to multiple load impacts. While the linear elastic analysis was primarily used to check the plausibility of the model, to find precarious loading conditions, and finally to mark potentially damaging load combinations in advance, the nonlinear analysis was employed to investigate the actual structural behavior of the vaults to infer suitable preservation methods. In the nonlinear analysis, the dead load as the primary load case was superimposed on all other impacts such as temperature, pier settlement, and horizontal support displacement prior to the nonlinear calculation. Ultimately, more than two dozen load combinations were considered for each vault.

RESULTS OF THE ANALYSIS

General Aspects

Based on the results of the calculations, the influence of each single load impact was illustrated and explained by means of stress distribution, stress flow diagrams, crack pattern, and deformation analysis. It was found that the horizontal support displacements had tremendous impact on the vaults' stability. Particularly the shift toward the east of the central vault and the shift toward the south of the southern vault emerged as the most significant load cases. They have drawn attention to the harm that might befall certain parts of the structure if the two domes are not fixed in a timely manner. To demonstrate possible results if a repair is delayed, corresponding load cases were extended by increasing the horizontal shift up to 3 mm toward the outside, while the changes in stress distribution and crack propagation were closely observed.

Gravity as the Fundamental Impact

The stress distribution for gravity loads was nearly in the compression range and varied between 0.05 and 0.25 N/mm² (7.25 and 36.25 psi). Although the ribs had become apparent in the stress plots, no significant differences between the bearing behaviour of caps and ribs were apparent, yet the deformation found along the east-side edge of the vault indicated that there were increasing strength and stiffness degradations. That is, areas near the horizontally unsupported east-side masonry arch had started to hang on the stiffer ribs (Figure 9), relieving the cap's load because an unfavourable superposition of membrane and bending stresses inside the cap initiated cracks, at first, and then weakened the cap's ability to carry loads, forcing the loads to bypass the arch. Indeed, tests demonstrated that stresses on the soffit along the east-side arch had partly changed their sign into tensile stresses.

The stress flow diagram reveals a ring of compressive stresses surrounding the vertex, but stretching more to the south and north as the cap started to sag. The east-west direction is relieved of stresses to an extent not really equal to that at which the north-south direction takes higher loads. Thus, the load-carrying concept, where the thrust originating from the vaults is normally distributed in all directions, no longer applies.

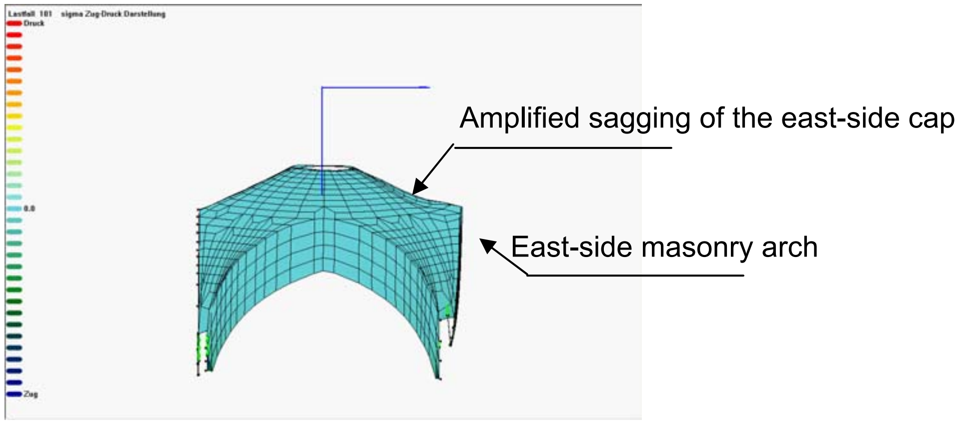


Figure 9 – Deformed vault structure under dead load

Superposition of Single Load Impacts

It was found that primarily sideward abutment shifts had a tremendous impact on the structure; hence, this issue received the greatest focus. Subjected to a load combination of gravity loads and horizontal support displacement in an eastward direction, the firm and statically undetermined structure of the vaults responded very sensitively. Much more than under gravity loads alone, the east-side arch was pushed aside under an additional support displacement in that direction.

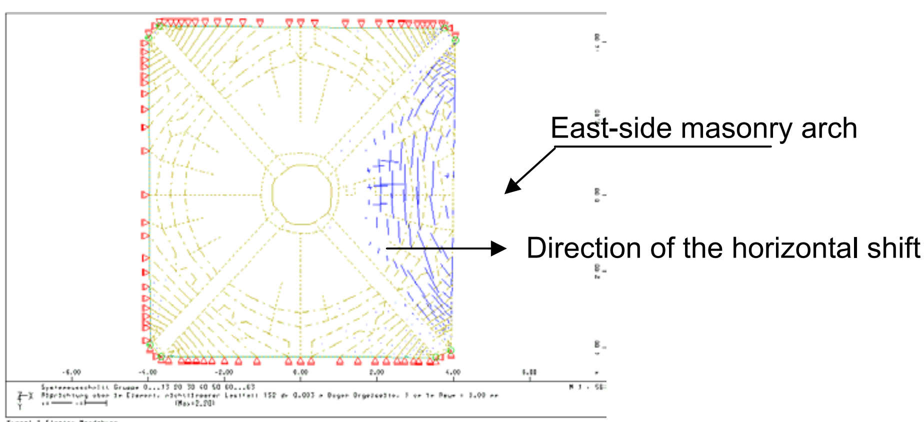
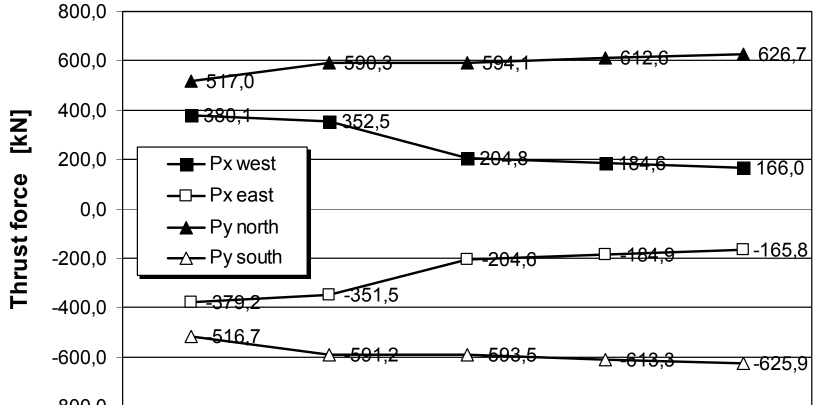


Figure 10 – Crack propagation under a combined loading of gravity and horizontal shift

As can be seen in Figure 10 (scale 1:56), the crack propagation on the inner an outer side expanded into the structure as the horizontal shift grew. At a displacement of 2 mm, the typical triangular crack in the eastern cap had already developed almost entirely [7]. Superimposing the crack patterns of the inner and outer surfaces, it becomes evident that many cracks have spread through the whole shell thickness. The sagging of the vaults not only led to higher stresses (appr. 70 percent of allowable stresses at a shift of 2 mm) but also caused a nominal enhancement of thrusts. As the eastern arch leaned back and took less loads, the vault's thrust had to be redirected mostly in a north-south direction, causing a stress increase of almost 20 percent at a shift of 3 mm. Meanwhile, the east-west direction of the vault's thrust was relieved by a 43 percent stress reduction compared to its state with firmly fixed abutments (Figure 11).



	fully restrained	dead load	dead load + 1 mm shift	dead load + 2 mm shift	dead load + 3 mm shift
Px west	380,1	352,5	204,8	184,6	166,0
Px east	-379,2	-351,5	-204,6	-184,9	-165,8
Py north	517,0	590,3	594,1	612,6	626,7
Py south	-516,7	-591,2	-593,5	-613,3	-625,9

Displacement [mm]

Figure 11 – Development of the vault's thrust depending on horizontal support displacement in eastward direction

The diagram in Figure 11 displays the development of the central vault's thrusts under gravity loads with increasing shifts of the east-side masonry arch. Starting from a state in which all abutments were in fixed positions, the horizontal forces changed slightly when the east-side restrainers were removed. Whereas the initial state suggests an ideal case with undistorted loading conditions, the second state denotes the situation created by the horizontally unsupported east-side arch. With enhanced horizontal shift, the east-west reaction force dropped rapidly, especially within the first millimeter when the drop was approximately 54 percent compared to the thrust under fully restrained conditions. The current situation is estimated at somewhere between a 2 and 3 mm horizontal shift, which serves as a warning that delaying retrofitting measures could lead to further damage to the structure.

REHABILITATION AND RESTORATION MEASURES

All measures taken to rehabilitate and restore the ancient groined masonry vaults must focus on maintaining the current state of the structures. The actual stresses, strains, and imperfections have to be considered since it is clear that they cannot be altered. Nevertheless, in order to reactivate the vaults' favorable spatial behavior, rehabilitation measures should attempt to create fully restrained structures so that the structures can react favorably when additional impacts might occur. That is, radial and tangential compression forces should both be well established to transfer loads directly to the immediate supports. Because the caps do not sag further, they are able to carry the remaining loads by themselves. Additional redirection of internal forces, therefore, does not need to occur, while stress concentrations at the edges, where caps, ribs, and piers unify, should not be higher than they are currently.

First, all open joints and cracks have to be carefully filled with mortar. The second major step should be the strengthening and stiffening of the abutments. Specifically, they have to be retrofitted with grouted stainless steel tendons that are slightly posttensioned. Stiffening the abutments with precompression serves as a important restraining element to fix the vaults' base to prevent further degradation. Because the analysis implied that the vaults may not endure any additional sideward shifts, the stiffening structures themselves as well as the process of transferring the bracing forces into the ground must restrict all deformation to a minimum.

CONCLUSIONS AND OUTLOOK

The apparently insufficient structural conditions of these two ancient groined masonry vaults forced an investigative study of the actual bearing behavior of the vaults prior to analyzing their structural safety and inferring suitable rehabilitation measures. The outcome of the analysis not only confirmed the observed crack patterns, but also clarified the reasons for the crack locations and marked critical loading conditions. Whereas temperature impacts primarily led the stiff vaults to crack immediately, such impacts did not undermine the ultimate loading state. The load impacts resulting from a horizontal support displacement, however, had significant consequences. Accompanied by typical crack patterns, an increased support displacement led to an amplified sagging of the vaults. The horizontal reaction force dropped in the drift direction and rose in the direction perpendicular to the drift. It has been proved that spatial masonry vault structures are most stable when they are firmly supported laterally. Consequently, cracks and horizontal displacements that weaken the lateral support of the church's vaults must be fixed.

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