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FAILURE LINE METHOD APPLIED TO WALLS WITH OPENINGS

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ABSTRACT

The capacity of unreinforced masonry (URM) walls subjected to out-of-plane bending has been investigated in many parts of the world over the past 3 decades. This research has led to incorporation of design provisions in masonry codes in Australia, Britain and Canada. Despite a large body of test data, the majority of data available is for walls without openings and the existing design provisions are only for solid walls. Frequently in design, however, URM walls feature openings of various sizes due to doors and windows. There is, consequently, a need to investigate the out-of-plane capacity of these walls to improve understanding and develop design approaches.

This paper presents information on applying the Failure Line Method (FLM) to unreinforced concrete block walls with openings that are supported on all four edges and subject to pressure normal to the wall surface. The effects of location and size of openings were analyzed using FLM and these results were compared to experimentally determined capacities. It is shown that the FLM can be a slightly conservative but easy to apply design tool to rationally take advantage of two way bending and the inherent reserve strength available after first cracking of a masonry wall.

The FLM, recently introduced into Canadian masonry design practice for walls without openings, can, in accordance with CSA S304.1-04 [1], be extended for design of walls with openings.

KEYWORDS: URM, failure line, out-of-plane bending

INTRODUCTION

The new CSA Standard, Masonry Design for Buildings [1] CSA S304.1-04 contains new provisions for the analysis and design for out-of-plane loads of unreinforced masonry walls supported on more than two edges. These provisions introduce the use of the Failure Line Method (FLM) which includes the moment equations

$$M_{fpa} = \beta_f w_f l^2 \qquad \text{Equation 1}$$

for bending in the horizontal direction, and

$$M_{fn} = \mu \beta_f w_f l^2 \qquad \text{Equation 2}$$

for bending in the vertical direction. A table of design moment coefficients, β_f, is available for different wall aspect ratios height/length (h/L), and orthogonal strength ratios μ_m. This strength ratio is $\left(\phi_m f_{tn} + P_f / A_e\right) / \phi_m f_{tp}$, where f_{tn} and f_{tp} are the flexural tensile strengths normal and parallel to the bed joints, respectively. P_f / A_e introduces the benefit of compression due to axial load and $\phi_m = 0.6$ is the resistance factor for masonry. The equations and coefficients are intended for use only with solid walls subjected to uniform loads. However, the following note provides guidance for other conditions including walls with openings. As the use of the FLM for solid walls is new, it is clear that for designers to apply the method to walls with openings, some additional information should be made available. The purpose of this paper is to bridge the gap between applying FLM to solid walls and applying FLM to walls with openings. Several examples are given.

REVIEW OF FAILURE LINE CONCEPTS

The FLM was developed at McMaster University over the past several years as a variation of the "yield-line" [2] and "fracture-line" [3] methods incorporated into British masonry design [4]. A masonry wall is considered to act as a group of rigid plates connected along crack lines sufficient to form a failure mechanism as depicted in Figure 1. Thus, this crack pattern defines the "failure line" mechanism. Although the FLM is a plastic approach, what sets it apart from other plastic-based approaches such as the yield line methods is the more rational treatment of first cracking. Early on in loading, most masonry walls will form an initial crack that can be predicted by elastic plate analysis. Most walls will not fail at this point and, in fact, can carry a significantly larger load than that causing the first crack. The observation was made that, at failure, little or no moment can be transferred across this pre-existing first crack. Therefore, moment resistance across this crack can and should be ignored in analysis.

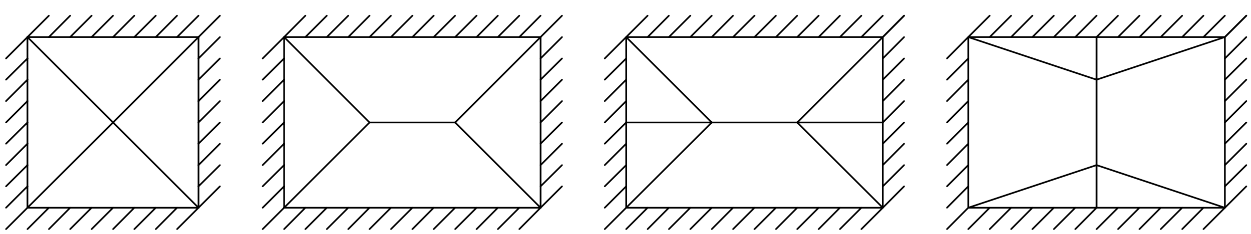


Figure 1 - Failure Lines and Crack Patterns for Solid Walls Supported on Four Edges

In terms of calculation, the virtual work approach is used. In this method, the wall is given a virtual displacement which, through rigid body motion defines displacements at all other points including rotations along failure lines. The external work is calculated as the resultant wind load times the displacements at the centroid of each rigid plate. The internal work is calculated as the moment capacities along failure lines times the corresponding rotations. Equating the external work and internal work produces an expression that can be used to determine the failure lines representing the critical cracking pattern [5]. Although this procedure is relatively straightforward for solid wall panels, complications arise when extending the method to walls with openings.

EXTENDING FLM TO ACCOMMODATE WALL OPENINGS

In modern construction, many masonry walls will have some form of opening. This being the case, it is important to note that it isn't sufficient at this time to lump all masonry walls with openings into a single category of analysis, as there is an almost infinite combination of opening sizes, location and number. For this reason, we have divided our discussion into several parts, based on selected types of wall openings. These sections will include person-sized door openings, small-window openings, large-window openings, multiple small-window openings and insignificant openings. Each of these will be considered in turn beginning with insignificant openings. Before this can be done, however, it is necessary to understand the main modification of the FLM for walls with openings: the treatment of matching deflections and the distribution of load from the window or door to the wall.

ADJUSTMENT OF WALL PANEL DEFLECTIONS

Considering the typical FLM pattern in Figure 2, the horizontal first crack is known to occur quite early in loading. Formation of the diagonal cracks which propagate from this central crack to the wall corners occurs after further loading and generally leads quickly to failure as a failure mechanism is formed. The deflections at the centroids of the resulting rigid plates are based on an initial deflection that can be labelled Δ. Now assuming that the diagonal cracks from the left side of the wall form a perfect triangle of height γL, the deflection at the centroid of the triangle is Δ/3, as the centroid is at 1/3 of the height of the triangle measured from the base, or edge of the wall. The apex of this triangle meets with and must match the deflection, Δ, along the horizontal line representing the first crack. The deflection at the centroids of other rigid body elements can be similarly determined by geometric compatibility.

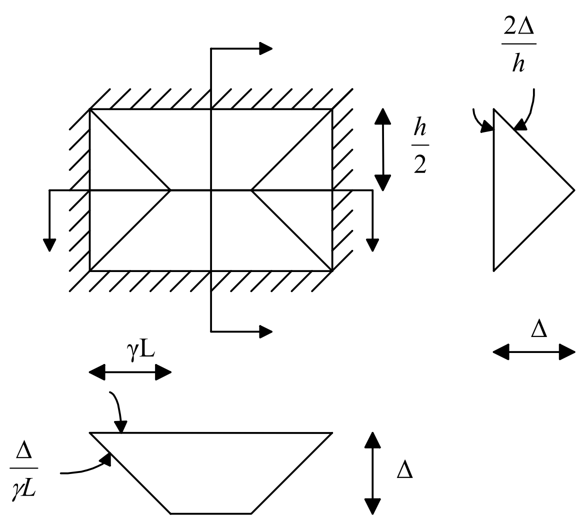


Figure 2 - Relationship between Deflections and Rotations for Failure Line Mechanisms

Now, for comparison, consider the wall in Figure 3 which is supported along the top and bottom as well as the left side. The 2.8 m square wall has a 1 m square opening located 1.1 m from the left side and 0.6 m from the top edge. The key to using the Failure Line Method is to investigate every possible crack pattern that can produce a failure mechanism. By evaluating each one using virtual work, the critical failure line mechanism is the one that gives the lowest resistance to load. For example, assuming the failure line pattern depicted in Figure 3, with the line labelled 1 representing the first crack, it follows naturally from the discussion of solid walls that for the line labelled 2 to match deflection Δ_1 along the first crack, it must be multiplied by the ratio of vertical distances of the two line as rigid panel A rotates about the top support. Therefore with reference to Figure 3

$$\Delta_2 = \Delta_1 \times \frac{0.6}{y} \qquad \text{Equation 3}$$

As the line labelled 3 meets Line 2 at a common point, their deflection at that point must be the same. The deflections are constant along the lengths of Lines 2 and 3 because the entire lengths of each line are at a constant distance from the point of rotation at the adjacent edge of the wall panel. Therefore,

$$\Delta_3 = \Delta_2 = \Delta_1 \times \frac{0.6}{y} \qquad \text{Equation 4}$$

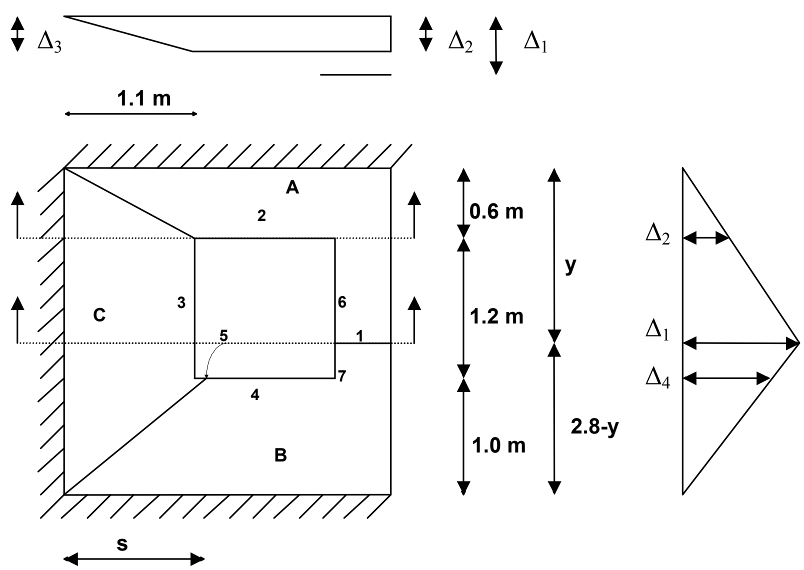


Figure 3 - Failure Lines for a Wall Supported along Three Edges and Containing a Window

For the deflection of failure Line 4, which is assumed to intersect the bottom edge of the window at a distance '*s*' from the left side support, the deflections along both the horizontal and vertical sides of the window must match. For compatibility vertically,

$$\Delta_4 = \frac{1.0}{2.8 - y}\Delta_1 \qquad \text{Equation 5}$$

and the deflection along Line 4 are constant because the entire line is a constant distance from the bottom support which serves as the rotation point for rigid segment B. To relate distance *s* to distance *y*, consider rotation of rigid segment C about the left support. From geometry,

$$\Delta_4 = \Delta_3 \frac{s}{1.1} = \frac{1.0}{2.8 - y} \Delta_1 \qquad \text{Equation 6}$$

It should be noted that deflection along Line 5 varies linearly from Δ_3 to Δ_4. Similarly, deflection along Line 6 varies linearly from Δ_1 to Δ_2 and deflection along Line 7 varies linearly from Δ_1 to Δ_4. Alternative failure line patterns could include having the left diagonal failure lines intersecting different edges of the window or intersecting a horizontal line originating from Line 3.

WALLS WITH INSIGNIFICANT OPENINGS

It should be clear that, in certain cases, an opening may have negligible effect on the strength of a wall. For instance, removing a single block from the wall would not be expected to have any major strength effect. Small openings are often provided for exhaust fans. Although the decision is within the discretion of the designer, there is still a quick check that can be done.

Treating the wall with an opening as a solid wall, the FLM can be applied to determine the critical combination of first cracks and subsequent cracks acting as failure lines for the final collapse mechanism. If the opening is small and does not intercept any of the secondary failure lines, it most likely will not affect the calculated wall strength.

SMALL WINDOW OPENINGS

Small windows are defined for this investigation as openings that comprise not more than 10% of the wall area, but are positioned over failure lines. For the purpose of relating this discussion to the experimental data [6] available, a 2.8 m high by 5.8 m long wall with a 1.2 m by 1.2 m square opening is considered. The opening is positioned vertically 5 courses from the base of the wall, and both a centred position and a 1.2 m off-centre position are considered. The walls are 20 cm hollow concrete block construction with flexural tensile strengths of 0.315 MPa and 0.988 MPa normal and parallel to the bed joints, respectively. Although the window does not contribute to the strength of the wall, it does collect out-of-plane loads and transfer forces to the wall in the form of line loads around the perimeter of the window.

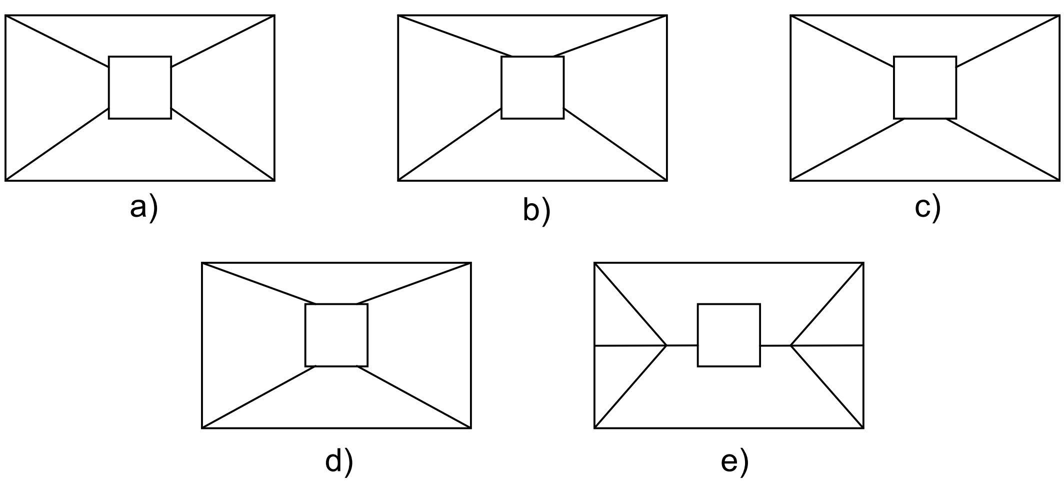


Figure 4 - Potential Collapse Mechanisms for the Failure Line Method

Beginning with a case of a centred 1.2 m by 1.2 m opening, the use of the FLM demands that all possible reasonable crack patterns be considered. This very quickly leads to the five symmetrical cracking patterns depicted in Figure 4, which must be considered in turn. Note that the diagonal failure lines make contact with the opening sides at variable points along these sides. Therefore, as illustrated earlier, the virtual work equations must be formulated to allow the critical pattern corresponding to the least calculated wall strength to be found.

In this particular case, the pattern in Figure 4(a) is the governing case. External work terms are summed up as the distributed wind pressure multiplied over the respective rigid body areas acting through deflections at the centroid of each rigid body element. Depending on the details of attaching the window, the pressure on the window can be distributed as load to the adjacent edges of the opening. In this case, the framing of the opening transferred most of the pressure to the top and bottom edges of the opening. These loads times the relative virtual deflections along the edges are easily included. Internal work is calculated as moment capacity across a failure line acting through a virtual rotation, which, from small angle rotation, can be expressed in terms of virtual deflection. Failure lines that cross the window opening do not contribute any internal work. The final step is to equate the sum of external work to the sum of internal work, and rearrange the equation to determine the critical failure line geometry for the assumed flexural strength ratio. In the cases of walls with openings, these equations can become quite complex. However, a spreadsheet model or mathematical software package can be easily adapted to solve the problem.

Following this method, 2.85 kPa was calculated for the strength of this wall whereas the corresponding test wall strength was found to be 3.57 kPa. This conservative 20% under estimation of strength is typical for the FLM [5,6]. It should be noted that tests of "identical walls" often differ [7, 8] by more than 20% because of the highly variable nature of mortar bond. It was noted that the predicted critical failure line pattern closely matched the observed cracking pattern.

Applying this methodology to a wall with the 1.2 m by 1.2 m opening at 1.2 m off centre, the governing failure line pattern shown in Figure 5 was found. Note that the dashed line in Figure 5 represented the first crack. Because it occurred at a lower load, it is not considered to provide failure line resistance at failure of the wall panel. Use of the FLM leads to a predicted strength of 2.56 kPa, compared to a corresponding test value of 3.22 kPa. Again, the predicted value is about 20% lower than the test value and the predicted failure line pattern closely matched the observed crack pattern.

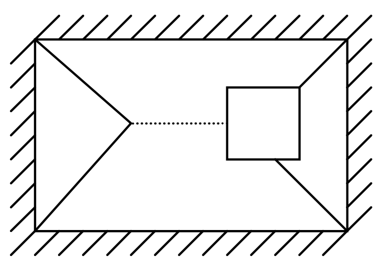


Figure 5 - Critical Failure Line Pattern for Off-Centre Small Window

MULTIPLE SMALL-WINDOW OPENINGS

With conservative results from single small openings, the natural progression was to investigate multiple openings. The problem of two small openings was considered using the same 2.8 m high by 5.8 m long wall. The two 1.2 m square openings were spaced 3 blocks apart and were considered to be at the same height and centred horizontally on the wall. The critical failure line pattern was determined to be that in Figure 6.

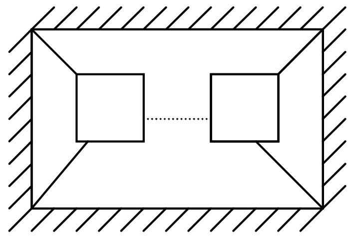
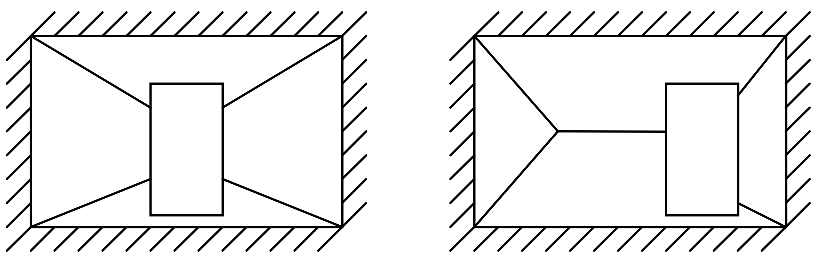


Figure 6 - Critical Failure Line Pattern for Multiple Small Window Openings

The FLM predicted capacity of 1.92 kPa, compared to the test value of 2.79 kPa, is about 31% conservative. The failure line pattern closely matched the observed crack pattern.

PERSON-SIZED DOORS

A common opening in masonry wall construction is a person-sized door. Using the same methodology as for small openings, two locations were considered in the same 2.8 m by 5.8 m wall: a door centred in the wall panel (Figure 7-a), and a door at 1.2 m off-centre (Figure 7-b). The opening was sized as 1.2 m long by 2.0 m high and started after the first course (to match the test wall), leaving three courses over the top.



a) b)

Figure 7 - Door Centred in Wall Panel and 1.2 m Off-Centre in Wall Panel

For the centrally located door, the governing failure line pattern is shown in Figure 7-a, where any pre-existing horizontal bed-joint cracks near mid-height would not affect the failure line analysis. The FLM predicted capacity corresponding to this failure line pattern is 3.15 kPa, compared to 3.61 kPa for a test wall. Therefore the predicted value is about 13% conservative. The wall with a 1.2 m off-centre door has a governing failure line pattern as shown in Figure 7-b, that results in a predicted strength of 2.55 kPa, compared to a test value of 3.65 kPa; the prediction is 30% conservative. The predicted failure line pattern closely matched the observed crack pattern.

LARGE WINDOW OPENINGS

The final opening type considered in this investigation is a large window opening. For comparison to test data, a 1.2 m high by 3.6 m long centred opening built above the fifth course was considered in the 2.8 m high by 5.8 m long wall as shown in Figure 9. This opening comprises 33% of the total wall area and crosses critical failure lines.

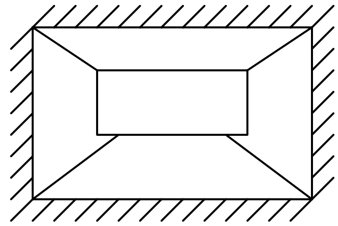


Figure 8 - Wall with Large Opening

Applying the FLM, the critical failure line pattern in Figure 8 results in a predicted capacity of 2.03 kPa, compared to a test value of 2.90 kPa. This is about 30% conservative. The predicted failure line pattern closely matched the observed crack pattern.

COMPARISON OF RESULTS

In all cases examined, the FLM provided conservative but acceptably close predictions of capacity. The previously discussed FLM predicted wall strengths and corresponding test data are shown in Figure 9. As additional information, predicted capacities using the yield-line analysis consistent with the British masonry code [4] are included. In cases when no early crack develops, the FLM and yield line analysis are identical. This raises the question of why not just use the yield-line method employed by the British for almost 30 years? The answer is the FLM provides a conceptually more rational treatment of the commonly observed pre-existing horizontal first crack. The half wall panels formed above and below this horizontal crack can be thought of as corresponding to a wall panel with a free edge along the top where no flexural resistance exists. Furthermore, for the introduction of this method as a regular design tool, it is appropriate that it typically provides an additional measure of conservatism.

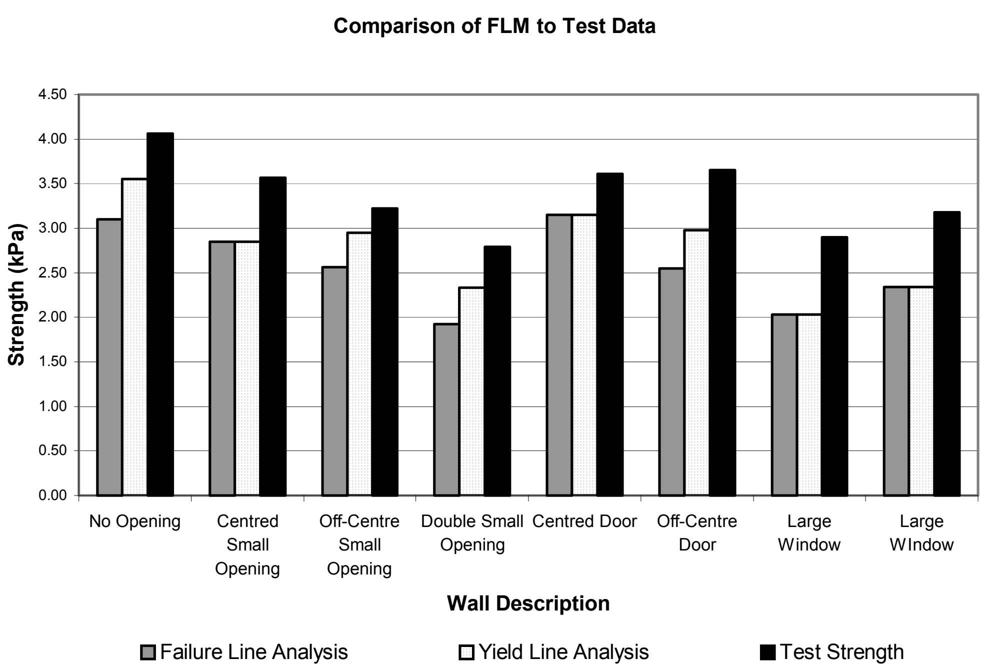


Figure 9 - The Failure Line Method Compared to Yield Line Analysis and Test Data

OTHER METHODS OF ANALYSIS

At this time, it is important to acknowledge that several other unique analytical approaches for lateral loading of URM walls have appeared in the literature recently.

In Australia, the masonry design standard now contains an approach with some similarities to that discussed here where the virtual work method is used and torsional bed-joint strength is considered [9]. Application to walls with openings has not been included.

From Portugal, plastic plate analysis with anisotropic softening has been used to model masonry. This macro model approach employs a more advanced analytical technique and is reported to have provided excellent results, allowing for the prediction of the known non-linear behaviour of the masonry walls [10].

Finally, other research at McMaster University utilizes a micro-model finite element method [11] to predict non linear behaviour. This method has the potential to provide a very detailed description of wall behaviour, and should allow designers in the future to design URM masonry walls of unusual geometry and loading conditions.

OBSERVATIONS AND CONCLUSIONS

The research reported in this paper illustrates the potential to extend the recently introduced FLM to cover design of masonry walls with openings subject to out-of-plane bending due to lateral load. Slightly conservative capacity predictions and accurate representations of critical crack patterns support this conclusion.

At this point, any designer who is familiar with the yield line method can use this knowledge to apply the FLM. It has the advantages over the other methods described above of being easily understood and applied. Designers do not need to depend on a "black box" type of solution nor do they need to possess high level modelling expertise necessary for more advanced analytical approaches.

At the current time, we are not in a position to provide tables or other simple rules to simplify design for openings in walls. Areas of opening as a percent of wall area or size of opening in terms of length of failure line eliminated have not as yet proven to be satisfactory criteria. It should be expected that narrow openings would have different effects compared to more square openings. Also, the pressure of an opening often changes the critical failure line pattern.

At the current time, equations for the FLM applied to openings such as those reported in this paper are available. A possible help to practical design is to make this available to designers in user-friendly spreadsheet formulations so that they can determine the critical conditions for their specific cases.

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